
Zbl 626.10004

Alladi, K.; Erdős, Paul; Vaaler, J.D.

Multiplicative functions and small divisors. (In English)

Analytic number theory and diophantine problems, Proc. Conf., Stillwater/Okla. 1984, Prog. Math. 70, 1-13 (1987).

[For the entire collection see Zbl 618.00005.]

The principal result of this paper states that if $k \geq 2$ and h is a nonnegative submultiplicative function satisfying $0 \leq h(p) \leq c < 1/(k-1)$ for all primes p, then

$$\sum_{d|n} h(d) \leq (1 - \frac{kc}{1+c})^{-1} \sum_{d|n,\ d \leq n^{1/k}} h(d)$$

holds for all squarefree n. By writing $g = 1 * h$, this result can be used to bound sums of the type $\sum_{n \leq x,\ n \in S} g(n)$ for certain classes of multiplicative functions g and sets of integers S. The authors sketch such an application with $g = e^{uf}$, where f is a nonnegative additive function and u a real parameter, which leads to bounds for moments of additive functions on certain sets such as the set of shifted primes $\{p+1\}$.

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Classification:
11A25 Arithmetic functions, etc.
11N37 Asymptotic results on arithmetic functions
11K65 Arithmetic functions (probabilistic number theory)

Keywords:
estimates of sums of multiplicative functions; small divisors; bounds for moments of additive functions; shifted primes