Zbl 668.05037
Erdős, Paul; Hajnal, András
On the number of distinct induced subgraphs of a graph. (In English)
Discrete Math. 75, No.1-3, 145-154 (1989). [0012-365X]

Let $i(G)$ be the number of pairwise non-isomorphic induced subgraphs of graph $G =< V, E >$. The graph $G =< V, E >$ is ℓ-canonical if there is a partition $< A_i \subset V : 0 \leq i < \ell >$ such that $\{x, y\} \in E \Leftrightarrow \{x', y'\} \in E$ for all $i, j < 1$, $x, x' \in A$, $y, y' \in A$. The graph $G =< V, E >$ is (ℓ, m)-almost canonical if there is an ℓ-canonical graph $G_0 =< V, E_0 >$ such that all components of symmetric difference of G and G_0 (denoted by $G_\Delta G_0 =< V, E_\Delta E_0 >$) have sizes at most m.
The authors and (independently) N. Alon and B. Bollobás prove the following result. Let $i(G) = o(n^2)$. Then one can omit $o(n)$ vertices of G in such a way that the remaining graph is either complete or empty. In the paper the following stronger theorem is proved.
Theorem 1. For all $\epsilon > 0$ and for all $k \geq 1$ there exists a $\delta > 0$ such that for all n and for all G with n vertices $i(G) \leq \delta n^{k+1}$ it follows that these exists a $W \subset V$, $|W| \leq \epsilon n$, such that $G[V \setminus W]$ is (ℓ, m)-almost canonical for some ℓ, m satisfying $\ell + m$, $k + 1$. In addition the following estimation is obtained.
Theorem 2. Let G be a graph with n vertices, $c > 0$, $k > 2c \log 2$ and $K_{c \log n, c \log n} \not\subset G, \bar{G}$, where $K_{n,m}$ is bipartite graph, $\bar{G}$ is the complement of G. Then for every sufficiently large n, $i(G) \geq 2^{n/4k}$.

Yu.M. Voloshin

Classification:
05C35 Extremal problems (graph theory)
05C30 Enumeration of graphs and maps

Keywords:
induced subgraphs