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Partitions into parts which are unequal and large. (In English)

Number theory, Proc. 15th Journ. Arith., Ulm/FRG 1987, Lect. Notes Math. 1380, 19-30 (1989).

[For the entire collection see Zbl 667.00007.]

Let $q(n)$ be the number of partitions of n into unequal parts, and let $\rho(n,m)$ be the number of partitions of n into unequal parts $\geq m$. The first and third authors have previously shown that $\rho(n,m) = (1+o(1))q(n)/2^{m-1}$ for $m = o(n^{1/5})$ [Colloq. Math. Soc. János Bolyai 34, 397-450 (1984; Zbl 548.10010)]. Three additional theorems giving estimates for $\rho(n,m)$ are now obtained.

Theorem 1: For all $n \geq 1$ and m such that $1 \leq m \leq n$, we have (i) $q(n)/2^{m-1} \leq \rho(n,m) \leq q(n+m(m-1)/2)/2^{m-1}$ and (ii) $\rho(n,m) \leq q(n+[m(m-1)/4])/2^{m-2}$, where $[x]$ is the integral part of x.

Theorem 2: When n tends to infinity, and $m = o(n/\log n)^{1/3}$, we have

$$\rho(n,m) = (1+o(1))q(n+[m(m-1)/4])/2^{m-1}.$$

Theorem 3: For fixed ϵ, with $0 < \epsilon < 10^{-2}$ and for $m = m(n)$, $1 \leq m \leq n^{3/8-\epsilon}$, and $n \to \infty$,

$$\rho(n,m) = (1+o(1))q(n)/\prod_{1\leq j\leq m-1}(1+\exp(-\pi j/2\sqrt{3n})).$$

The paper concludes with a table of values for $\rho(n,m)$ with $1 \leq n \leq 100$ and $1 \leq m \leq \min(n,12)$.

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11P81 Elementary theory of partitions

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partitions with unequal parts; number of partitions; table