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The chromatic number of the graph of large distances. (In English)

Combinatorics, Proc. 7th Hung. Colloq., Eger/Hung. 1987, Colloq. Math. Soc. János Bolyai 52, 547-551 (1988).

[For the entire collection see Zbl 673.00009.]

Let S be a set of n points in $\mathbb{R}^d$. Let $d_1 > d_2 > ... > d_k > ...$ be the distances between the points in S. We assign the following graph $G(S, \leq k)$ tothe set S: the vertices of $G(S, \leq k)$ correspond to the points in S, two vertices being connected iff the distance of the corresponding points is at least d_k. In a previous paper [Discrete Comput. Geom. 4, No.6, 541-549 (1989; Zbl 694.05031)], the authors studied the chromatic number $\chi(G(S, \leq k))$ of this graph in the plane. In this paper the problem is studied in higher dimensions and it is proved that if S is a set of n points in $\mathbb{R}^d$ such that no d of its elements are contained in a $(d-2)$-dimensional subspace, and $n > 2(d+1)^d k^d$, then $\chi(G(S, \leq k)) \leq g(d) + d - 1$, where $g(d)$ denotes the least number of parts into which the d-dimensional unit ball can be cut so that the diameter of each part is at most 1. Moreover, for every $d \geq 2$ there exists a k and there exist arbitrarily large (even infinite) sets of points in $\mathbb{R}^d$ such that no d of their elements are contained in a $(d-2)$-dimensional subspace and $\chi(G(S, \leq k) = g(d) + d - 1$.

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