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Grandes valeurs de fonctions liées aux diviseurs premiers consécutifs d'un entier.

Large values of functions connected to consecutive prime divisors of an integer. (In French)

Théorie des nombres, C. R. Conf. Int., Québec/Can. 1987, 169-200 (1989).

[For the entire collection see Zbl 674.00008.]

Let $n = q_1^{\alpha_1}...q_k^{\alpha_k}$ be the standard factorization of n into primes. The authors are interested in large values of the two functions

$$f(n) = \sum_{i=1}^{k-1} q_i/q_{i+1}, \quad F(n) = \sum_{i=1}^{k-1}(1 - q_i/q_{i+1}).$$

The main results are as follows: (i) There exists a constant $C > 0$ such that, as $n \to \infty$, F(n)$\leq \sqrt{\log n} - C + o(1)$, with equality holding for infinitely many n. (ii) Call an integer $N > 1$ an f-champion if $f(N) > f(n)$ for every $n < N$. Then, for every sufficiently large k, the number $N_k = p_1...p_k$, where p_i denotes the ith prime, is an f-champion. Moreover, under the assumption of Crámer's conjecture $p_{i+1} - p_i \ll (\log p_i)^2$, every sufficiently large f- champion is of the form N_k or N_k/p for some prime factor p of N_k.

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