Zbl 688.10043
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On prime-additive numbers. (In English)
Stud. Sci. Math. Hung. 27, No.1-2, 207-212 (1992). [0081-6906]

Let $n = \prod_{i=1}^{k} p_i^{\alpha_{p_i}}$. The authors call n strongly prime-additive if $n = \sum_{k=1}^{k} p_i^{\beta_i}$, $p_i^{\beta_i} < n \leq p_i^{\beta_i+1}$. We only know three strongly prime additive numbers 228, 3115, 190233. n is prime additive if $n = \sum_{i=1}^{k} p_i^{\gamma_i}$, $0 < \gamma_i \leq \beta_i$. We do not know if there are infinitely many prime additive numbers. n is weakly prime additive if it is not power of a prime, and $n = \sum p_{i_r}^{\delta_r}$, $0 < \delta_r$ where $p_{i_1}, \ldots$ is a subset of the prime factor of n.
We prove that there are infinitely many weakly prime-additive numbers. Denote by A(x) the number of the weakly prime-additive numbers not exceeding x. We prove

$$(1) \quad c \log^3 x < A(x) < x / \exp(\log x)^{1/2-\epsilon}.$$

A. Balog and C. Pomerance proved (2) $A(x) > \log^k x$ for every k. It might be of some interest to get better inequalities for $A(x)$.

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Classification:
11P32 Additive questions involving primes
11A41 Elemementary prime number theory

Keywords:
counting functions; strongly prime additive numbers; weakly prime- additive numbers; de Bruijn's function; Hardy-Ramanujan theorem; representation of a number by powers of its prime divisors