**Zbl 689.10061**
**Erdős, Paul; Sárközy, A.; Sós, V.T. (Turan Sós, V.)**
*On a conjecture of Roth and some related problems. I.* (In English)
**Irregularities of partitions, Pap. Meet., Fertod/Hung. 1986, Algorithms Comb. 8, 47-59 (1989).**

[For the entire collection see Zbl 682.00006.]
For a fixed partition of the set of positive integers into $k$ classes, let $C$ be the set of those integers that can be represented as a sum of two different integers from the same class. Proving (in a much stronger form) a conjecture of K. F. Roth, the authors show that
(i) the number of even integers $\leq M$ not in $C$ is $O(M^{1-2^{-k-1}})$,
(ii) for $k = 2$ this can be improved to $O(\log M)$,
(iii) but there is a 2-partition such that $2^n \notin C$ for all n.
The partition into odd and even numbers shows that not much more can be expected. (i)-(ii) yield a lower bound for the total number of elements represented; the authors improve this to $M/2 + O(1)$ for $k < 4$, and show that it can be $< M/2 - ck \log M$ if $k \geq 4$. If every class is supposed to contain both even and odd integers, then, however, an improvement to $(+1/(2k) - \epsilon)M$ is possible.
The representation of special elements (squares, primes) and more general representations are also considered.
[For part II, cf. Zbl 699.10068]

*I.Z.Ruzsa*

Classification:
11B05 Topology etc. of sets of numbers

Keywords:
partitions; addition of sets; combinatorial number theory