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Erdős, Paul; Kroò, A.; Szabados, J.
On convergent interpolatory polynomials. (In English)
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Let $X_n : -1 \leq x_{nn} < x_{n-1,n} < ... < x_{1n} \leq 1$ $(n = 1, 2, ...)$ be a system of nodes of interpolation. Let $x_{kn} = \cos t_{kn}$, $0 \leq t_{1n} < t_{2n} < ... < t_{nn} \leq \pi$, and for an arbitrary interval $I \subseteq [0, \pi]$, denote $N_n(I) = \sum_{t_{kn} \in I} 1$. Let Π_m be the set of algebraic polynomials of degree at most m, $C[-1, 1]$ be the space of continuous functions on the interval $[-1, 1]$, and $|| \cdot ||$ be the maximum norm over $[-1, 1]$. The following theorem is proved: Theorem. For every $f(x) \in C[-1, 1]$ and $\epsilon > 0$ there exists a sequence of polynomials $p_n(x) \in \Pi_{[n(1+\epsilon)]}$ such that $p_n(x_{kn}) = f(x_{kn})$ $(k = 1, ..., n;\ n = 1, 2, ...)$ and $||f(x) - p_n(x)|| = O(E_{[n(1+\epsilon)]}(f))$ hold if and only if $\overline{\lim}_{n\to\infty} N_n(I_n)/n|I_n| \leq 1/\pi$ whenever $\lim_{n\to\infty} n|I_n| = \infty$ ($|I_n| =$ length of I_n) and $\underline{\lim}_{n\to\infty} \min_{1\leq i\leq n-1} n(t_{i+1,n} - t_{i,n}) > 0$. Here the sign O refers to $n \to \infty$ and indicates a constant depending only on ϵ; $E_n(f)$ is the best uniform approximation of $f(x)$ by polynomials of degree at most n.

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Classification:
41A05 Interpolation
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