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Erdős, Paul; Ivić, Aleksandar

On the iterates of the enumerating function of finite Abelian groups. (In English)

Bull., Cl. Sci. Math. Nat., Sci. Math. 17, 13-22 (1989). [0001-4184]

Let $a(n)$ denote the number of non-isomorphic Abelian groups of order n. It was proved by the reviewer [Q. J. Math., Oxf. II. Ser. 21, 273-275 (1970; Zbl 206.03402)] that

$$\limsup_{n\to\infty} \frac{\log a(n) \log\log n}{\log n} = \frac{\log 5}{4}.$$

Now the authors investigate the iterates of a(n), which are defined by $a^{(r)}(n) = a(a^{(r-1)}(n))$, $a^{(1)}(n) = a(n)$, $r = 2, 3, \ldots.$ The main result is

$$a^{(2)}(n) \ll \exp\{B(\log n)^{7/8}/(\log\log n)^{19/16}\}$$

with a positive constant B and $\log a^{(r)}(n) \ll (\log n)^{c_r}$ with $c_1 = 1$, $c_2 = 7/8$ and $c_r \le (1/2)c_{r-1} + (3/8)c_{r-2}$ for $r \ge 3$.
Furthermore, let $K(n) = \min\{r : a^{(r)}(n) = 1\}$. Then an asymptotic representation for the mean value of $K(n)$ is established.

E.Krätzel

Classification:
11N45 Asymptotic results on counting functions for other structures

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arithmetic functions; finite abelian groups; number of non-isomorphic Abelian groups of order n; iterates; asymptotic representation; mean value