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Erdős, Paul; Sárközy, A.
On a problem of Straus. (In English)
Disorder in physical systems, Vol. in Honour of J. M. Hammersley 70th Birthday, 55-66 (1990).

[For the entire collection see Zbl 714.00019.]

If $\mathcal{A}$ is a set of integers with the property that no element a_i is the average of any subset of $\mathcal{A}$ consisting of two or more elements, then $\mathcal{A}$ is said to be non-averaging. Let $f(N)$ denote the maximum cardinality of a non-averaging subset of $\{0,1,2,...,N\}$. The best estimates for $f(N)$ are due to *Á.P.Bosznay* (1989; Zbl 682.10049) and *P.Erdős* and *E.G.Straus* (1970; Zbl 216.01503) who showed that $f(N) \gg N^{1/4}$ and $f(N) \ll N^{2/3}$ respectively. In this paper, the authors improve this last estimate to: For $N > N_0$, $f(N) < 403(N\log N)^{1/2}$. Denote by $\mathcal{P}(\mathcal{A})$ the set of distinct integers n which can be represented in the form $n = \sum_{a\in\mathcal{A}} \epsilon_a a$ where $\epsilon_a = 0$ or 1 for all a and $0 < \sum_{a\in\mathcal{A}} \epsilon_a < \infty$. The above estimate for $f(N)$ is obtained via a bound for $F(N)$ which is defined to be the largest k such that there exist two subsets $\mathcal{A} = \{a_1,...,a_k\}$, $\mathcal{B} = \{b_1,...,b_k\}$ of $\{0,1,...,N\}$ with $\mathcal{P}(\mathcal{A}) \cap \mathcal{P}(\mathcal{B}) = \emptyset$. The infinite analogue of this problem is: If $\mathcal{A},\mathcal{B}$ are infinite sets of positive integers with $\mathcal{P}(\mathcal{A}) \cap \mathcal{P}(\mathcal{B}) = \emptyset$ then how large can $\min(A(x),B(x))$ be? Here $A(x)$, $B(x)$ are the counting functions of $\mathcal{A},\mathcal{B}$ respectively. The authors conjecture that

$$\liminf_{x\to\infty} \frac{\min(A(x),B(x))}{x^{1/2}} = 0$$

and show by an interesting construction that the $x^{1/2}$ in the conjecture cannot be replaced by $x^{1/2}(\log x)^{-1/2-\epsilon}$.

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00A07 Problem books

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non-averaging sets; maximum cardinality