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On elements of sumsets with many prime factors. (In English)

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Let $\nu(n)$ be the number of distinct prime factors of n. The following problem is studied in the paper. Having two finite sets of positive integers $\mathcal{A}$ and $\mathcal{B}$ how big is $\nu(n)$ on the sumset $\mathcal{A}+\mathcal{B}$? Suppose that $\mathcal{A}$ and $\mathcal{B}$ are subsets of $\{n \leq N/2\}$. Then certainly $\max \nu(a+b) \leq m$ where $m = m(N)$ is the maximal value of $\nu(n)$ for $n \leq N$. It is shown that for dense sets this upper bound is almost attained, more precisely, for each $\varepsilon > 0$ there is a $c(\varepsilon)$ such that if $|\mathcal{A}||\mathcal{B}| > \varepsilon N^2$ then we have $\max \nu(a+b) > m - c(\varepsilon)\sqrt{m}$. It is also shown that this result is close to best possible. The proof has both probabilistic and combinatorial flavour.

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11N25 Distribution of integers with specified multiplicative constraints
11B75 Combinatorial number theory
11N56 Rate of growth of arithmetic functions

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hybrid theorems; multiplicative properties of sumsets