Let K_ω denote the complete graph on the natural numbers. It is shown that if the edges of K_ω are colored with 2 colors, then there is a monochromatic infinite path P with upper density $\geq 2/3$, where the upper density for P is $\limsup_{n\to\infty} |V(P) \cap \{1, 2, \ldots, n\}|/n$. It is also shown that there is a monochromatic infinite path P such that the set $\{1, 2, \ldots, n\}$ contains at least the first $.21n$ vertices of the path P. Corresponding results are obtained for coloring K_ω with r colors for $r \geq 3$, and upper bounds are given for various density conditions on infinite monochromatic paths. For example it is shown that the edges of K_ω can be colored with r colors such that every $(r-1)$-colored path has upper density $\leq 1 - (2^r - 1)^{-2}$.

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