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On the number of divisors of $n!$ (In English)
Berndt, Bruce C. (ed.) et al., Analytic number theory. Vol. 1. Proceedings of a conference in honor of Heini Halberstam, May 16-20, 1995, Urbana, IL, USA. Boston, MA: Birkhäuser, Prog. Math. 138, 337-355 (1996). [ISBN 0-8176-3824-5/hbk]

In this interesting paper, various problems concerning the number of divisors of $n!$ are investigated. The first theorem provides an asymptotic expansion for $\log d(n!)$ with first term $c_0 n(\log n)^{-1}$ for an explicit constant $c_0 > 0$. The authors show next that

$$d(n!)/d\big((n-1)!\big) = 1 + P(n)n^{-1} + O\Big(n^{-\frac{1}{2}}\Big)$$

where $P(n) = \max_{p|n} p$. This leads to an estimate for the least $K = K(n)$ such that $d((n+K)!) \geq 2d(n!)$. It follows that $K(n)/\log n$ is unbounded but that $K(n) < n^{4/9}$ for all sufficiently arge n. The final section concerns the difference $D(n) = d(n!) - d((n-1)!)$. The authors call an integer n a champ of $D(n) > D(m)$ whenever $m < n$. They show that p and $2p$ are champs for any prime p and conjecture that there are infinitely many champs not of this form.

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05A10 Combinatorial functions

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divisor functions; factorials; asymptotic results; champs