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A PRIORI BOUNDS FOR GLOBAL SOLUTIONS OF A SEMILINEAR PARABOLIC PROBLEM

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Introduction

Consider the problem

$$
\text{(P)} \qquad \begin{cases} u_t = \Delta u + |u|^{p-1}u, & x \in \Omega,\ t \in (0,\infty), \\ u = 0, & x \in \partial\Omega,\ t \in (0,\infty), \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases}
$$

where Ω is a smoothly bounded domain in $\mathbb{R}^n$, $n \geq 2$ and $1 < p < p_S := \frac{n+2}{n-2}$ ($p_S = \infty$ if $n = 2$). It is known that global solutions of this problem are bounded (see [**CL**] or [**FL**]). Moreover, the corresponding bound is known to depend only on some suitable norm of the initial condition u_0 provided $u_0 \geq 0$ (cf. [**G**]) or $p < p_{CL}$ (cf. [**CL**]), where

$$
p_{CL} = \frac{3n+8}{3n-4} < p_S.
$$

The main purpose of this paper is to improve the result of Cazenave and Lions in [**CL**] by proving the same a priori bound for $p < p_S$.

The proof of our result is based on the original proof of Cazenave and Lions combined with a bootstrap argument and maximal regularity estimates. If $n > 3$ then we are able to prove our result also by another method which does not use the property of maximal regularity. Consequently, the second method can be generalized also to problems where the maximal regularity is not known (or does not hold).

The a priori bounds that we study seem to play a crucial role in several applications. Let us mention two of them. The first one is concerned with the study of blow-up rates of solutions of some classes of parabolic problems; in this context, the exponent p_{CL} appears in [**GK**], [**FM**], [**M**], for example. The second application consists in the dynamical proof of existence of sign changing stationary solutions (see [**Q2**]).

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A priori estimates for global solutions of problem (P) were first shown in [**NST**] for Ω convex, $u_0 \geq 0$ and $p < (n+2)/n$. In this paper, the corresponding bound does not depend even on the norm of the initial condition. Such an estimate cannot be obtained for signed solutions since problem (P) possesses stationary solutions whose norms (and energies) are arbitrarily large.

If we consider the set D of all initial conditions u_0 (in an appropriate function space) for which the solution u of (P) exists globally and tends to zero as $t \to \infty$, then the ω-limit set of any bounded solution starting on the boundary of D consists of nontrivial stationary solutions of (P). Since there are no positive stationary solutions for $p \geq p_S$, the a priori estimates for global (positive) solutions of (P) cannot be true in this case.

Main Result

We denote by $\|\cdot\|_q$, $\|\cdot\|_{k,q}$ or $\|\cdot\|_{C^k}$ the usual norms in $L_q(\Omega)$, $W^k_q(\Omega)$ or $C^k(\bar{\Omega})$, respectively. We shall often use the fact that $\|u\|_{1,2} \leq C\|\nabla u\|_2$ for $u \in X := \{v \in W^1_2(\Omega) : v = 0 \text{ on } \partial\Omega\}$. We also denote by A the operator $-\Delta$ with homogeneous Dirichlet boundary conditions on $\partial\Omega$. It is well known that A considered as an operator in $L_q(\Omega)$ has bounded imaginary powers for any $1 < q < \infty$.

By a global solution of (P) we mean a function $u \in C\big([0,\infty), X\big)$ which is a classical solution of (P) for $t > 0$ and fulfills $u(0) = u_0$. The variation-of-constants formula and standard bootstrap arguments show that the following Lemma holds.

Lemma. *(i) For any $C > 0$ there exists $\delta > 0$ such that any solution u satisfying $\|u(t_0)\|_{1,2} \leq C$ fulfills $\|u(t)\|_{1,2} \leq 2C$ for any $t \in [t_0, t_0+\delta]$.*

(ii) For any $C, \delta > 0$ there exists $\tilde{C} > 0$ such that any solution u satisfying $\|u(t)\|_{1,2} \leq C$ on $[t_0, t_0+\delta]$ fulfills $\|u(t)\|_{C^2} \leq \tilde{C}$ for any $t \in [t_0+\delta/2, t_0+\delta]$.

The main result of this paper is the following

Theorem. *Let $p < p_S$ and $\delta_0, C_0 > 0$. Then there exists a constant $C > 0$ (depending only on p, δ_0, C_0) with the following property: If $u(t, u_0)$ is a global solution with the initial condition $u_0 \in X$ satisfying $\|u_0\|_{1,2} \leq C_0$ then $\|u(t,u_0)\|_{C^1} \leq C$ for any $t \geq \delta_0$.*

Remark 1. Instead of initial conditions u_0 in X we could consider initial functions in $L_q(\Omega)$ and require $\|u_0\|_q \leq C_0$, where $q > (p-1)n/2$. This follows from [**BC**].

Proof of Theorem. By $\delta < \delta_0$ and C we shall denote various constants which may vary from step to step and which may depend on δ_0, C_0 but which are independent of u_0.

Fix u_0 with $\|u_0\|_{1,2} \leq C_0$ and assume that the corresponding solution $u(t) = u(t, u_0)$ exists globally. Then $u(t) \in C^2(\bar{\Omega})$ for $t > 0$ and $\sup_{t \geq \delta} \|u(t)\|_{C^2} < \infty$

(see [**CL**, Proposition 6]). Denote

$$E(u) = \frac{1}{2}\int_\Omega |\nabla u|^2\,dx - \frac{1}{p+1}\int_\Omega |u|^{p+1}\,dx.$$

It is well known that

$$\frac{d}{dt}E\big(u(t)\big) = -\int_\Omega u_t^2(t)\,dx, \qquad t>0. \tag{1}$$

Consequently, $E\big(u(t)\big) \le E(u_0) \le C$.

Multiplying the equation $u_t = \Delta u + |u|^{p-1}u$ by u, integrating by parts and using Hölder inequality we obtain

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\int_\Omega u^2(t)\,dx &= \int_\Omega u(t)u_t(t)\,dx = -\int_\Omega |\nabla u(t)|^2\,dx + \int_\Omega |u(t)|^{p+1}\,dx\\ &= -2E\big(u(t)\big) + \frac{p-1}{p+1}\int_\Omega |u(t)|^{p+1}\,dx\\ &\ge -2E\big(u(t_0)\big) + c\Big(\int_\Omega |u(t)|^2\,dx\Big)^{(p+1)/2}, \quad \text{for any } t\ge t_0,\end{aligned}\tag{2}$$

where c is a positive constant. This estimate implies both $E\big(u(t)\big)\ge 0$ and

$$\sup_{t\ge 0}\|u(t)\|_2 < C \tag{3}$$

(otherwise u has to blow up in finite time in $L_2(\Omega)$-norm). Now the estimate $0\le E\big(u(t)\big)\le C$ can be written in the form

$$0 \le \frac{1}{2}\int_\Omega |\nabla u(t)|^2\,dx - \frac{1}{p+1}\int_\Omega |u(t)|^{p+1}\,dx \le C \tag{4}$$

and, due to (1), we have also

$$\int_0^\infty \|u_t(t)\|_2^2\,dt < C. \tag{5}$$

Similarly as in (2) we obtain also

$$\int_\Omega u(t)u_t(t)\,dx = -(p+1)E\big(u(t)\big) + \frac{p-1}{2}\int_\Omega |\nabla u(t)|^2\,dx.$$

which implies

$$\|u(t)\|_{1,2}^2 \le C\big(1+\|u(t)u_t(t)\|_1\big) \qquad \text{for } t>0. \tag{6}$$

Now Hölder inequality yields

$$\|u(t)u_t(t)\|_1 \le \|u(t)\|_2 \|u_t(t)\|_2,$$

so that (6) and (3) guarantee $\|u(t)\|_{1,2}^4 \le C\bigl(1 + \|u_t(t)\|_2^2\bigr)$. Using (5) we arrive at

$$\sup_{t\ge\delta} \int_t^{t+1} \|u(s)\|_{1,2}^{2q}\, ds < C \tag{7}$$

for $q = 2$ (and $\delta = 0$). This estimate and (4) show also

$$\sup_{t\ge\delta} \int_t^{t+1} \|u(s)\|_{p+1}^{(p+1)q}\, ds < C. \tag{8}$$

Now (5), (7) and the imbedding $W_2^1(\Omega) \hookrightarrow L_{2^*}(\Omega)$ (where $2^* = 2n/(n-2)$ if $n > 2$, $2^* > 1$ is arbitrary if $n = 2$) yield

$$\sup_{t\ge\delta} \int_t^{t+1} \bigl(\|u_t(s)\|_2^2 + \|u(s)\|_{2^*}^{2q}\bigr)\, ds < C. \tag{9}$$

The interpolation theorem in [**CL**, Appendice] (cf. also the proof of Proposition 2 in [**CL**]) and (9) imply

$$\sup_{t\ge\delta} \|u(t)\|_\lambda < C \tag{10}$$

for any $\lambda < 2^*(2q+2)/(2q+2^*)$, hence for any

$$\lambda < \lambda_1(q) := \frac{2n(q+1)}{q(n-2)+n}. \tag{11}$$

Similarly, estimates (5), (8) and the interpolation theorem mentioned above imply (10) for any

$$\lambda < \lambda_2(q) := \frac{(p+1)q+2}{q+1} = p+1-\frac{p-1}{q+1}. \tag{12}$$

Put

$$\lambda(q) = \max\{\lambda_1(q), \lambda_2(q)\}.$$

Then (10) is true for any $\lambda < \lambda(q)$. Notice that the function $q \mapsto \lambda(q)$ is (strictly) increasing and $\lambda(q) \to p_S + 1$ as $q \to \infty$.

It is well known (see [**A2**, Theorem 15.2] and [**Q1**], for example) that (10) implies a bound for $\|u(t)\|_{1,2}$ (and, consequently, for $\|u(t)\|_{C^1}$) provided $p < 1 + \frac{2\lambda}{n}$, i.e. if

$$p < 1 + \frac{2\lambda_1(q)}{n} \qquad \text{or} \qquad p < 1 + \frac{2\lambda_2(q)}{n}. \tag{13}$$

Notice that each of the conditions in (13) is equivalent to

$$p < p(q) := \frac{(n+2)q+n+4}{(n-2)q+n}, \tag{14}$$

where the function $q \mapsto p(q)$ is increasing and $p(q) \to p_S$ as $q \to \infty$. Note also that in our case $q = 2$ and $p(2) = p_{CL}$.

Our aim is to improve the step (6)$\Longrightarrow$(7). Using a priori bounds for $u(t)$ in L_λ with $\lambda > 2$ (instead of (3)), we obtain bound (7) with larger values of q and this will increase the upper bounds $\lambda(q)$ and $p(q)$.

From now on, we shall proceed by two different methods. In the first one, we shall use maximal regularity estimates. Then we shall reprove our main result (for $n > 3$) by another method which does not require the maximal regularity property.

Proof based on maximal regularity

In the proof, we shall use a bootstrap argument. We know (7) for $q = 2$. We shall show that the validity of (7) for some $q \geq 2$ implies (7) for some $\tilde{q} > q$. Moreover, the difference $\tilde{q} - q$ will be bounded below by a positive constant, so that, after finitely many steps, we end up with some $\hat{q}$ for which $p(\hat{q}) > p$. As already mentioned above, this will prove the assertion.

Hence, let (7) be true for some $q \geq 2$. Then (10) is true for $\lambda < \lambda_2(q)$. Choose $\lambda \in \big(2, \lambda_2(q)\big)$. Then $\lambda < p+1$. Denote

$$\theta = \frac{p+1}{p-1}\frac{\lambda-2}{\lambda}, \qquad \lambda' = \frac{\lambda}{\lambda-1} \qquad \text{and} \qquad p_1 = \frac{p+1}{p}.$$

Then $\theta \in (0,1)$ and using (6), Hölder inequality, (10) and interpolation, we obtain

$$\begin{aligned} \|u(t)\|_{1,2}^2 &\leq C\big(1 + \|u(t)u_t(t)\|_1\big) \leq C\big(1 + \|u_t(t)\|_{\lambda'}\big) \\ &\leq C\big(1 + \|u_t(t)\|_{p_1}^{\theta}\|u_t(t)\|_2^{1-\theta}\big), \end{aligned} \tag{15}$$

since $\frac{\theta}{p_1} + \frac{1-\theta}{2} = \frac{1}{\lambda'}$.

Inequality (7) implies $\sup_{t\geq\delta}\int_t^{t+\delta}\|u(s)\|_{1,2}^{2q} < C$, so that

$$\sup_{t\geq\delta}\inf_{s\in(t,t+\delta)}\|u(s)\|_{1,2} < C$$

(with a new constant C depending on δ) and, consequently, enlarging δ and C and using Lemma we get

$$\sup_{t\geq\delta}\inf_{s\in(t,t+\delta)}\|u(s)\|_{C^2} < C.$$

Fix $t \geq 2\delta$ and let $\tau \in (t-\delta, t)$ be such that

$$\|u(\tau)\|_{C^2} < C. \tag{16}$$

We have $1-\theta = \frac{2}{p-1}\left(\frac{p+1}{\lambda}-1\right) < \frac{2}{q}$ for λ sufficiently close to $\lambda_2(q)$ since the last inequality is satisfied for $\lambda = \lambda_2(q)$. Now choose $\tilde{q} > q$ such that

$$\beta := \frac{2}{(1-\theta)\tilde{q}} > 1$$

and notice that $\theta\tilde{q}\beta' > 1$ where $\beta' = \beta/(\beta-1)$. We raise (15) to the power $\tilde{q}$, integrate it from τ to $(t+1)$, use Hölder inequality, (5), maximal Sobolev regularity (see [**A1**, Theorem III.4.10.7]), (16) and (4) to get

$$\begin{aligned}
\int_\tau^{t+1} \|u(s)\|_{1,2}^{2\tilde{q}} &\leq C\Big(1 + \int_\tau^{t+1} \|u_t(s)\|_{p_1}^{\theta\tilde{q}} \|u_t(s)\|_2^{(1-\theta)\tilde{q}}\, ds\Big) \\
&\leq C\Big(1 + \Big(\int_\tau^{t+1} \|u_t(s)\|_{p_1}^{\theta\tilde{q}\beta'}\, ds\Big)^{1/\beta'} \underbrace{\Big(\int_\tau^{t+1} \|u_t(s)\|_2^2\, ds\Big)^{1/\beta}}_{\leq C}\Big) \\
&\leq C\Big(1 + \Big(\int_\tau^{t+1} \||u(s)|^p\|_{p_1}^{\theta\tilde{q}\beta'}\, ds\Big)^{1/\beta'} + \|u(\tau)\|_{C^2}^{\theta\tilde{q}}\Big) \\
&\leq C\Big(1 + \Big(\int_\tau^{t+1} \|u(s)\|_{p+1}^{p\theta\tilde{q}\beta'}\, ds\Big)^{1/\beta'}\Big) \\
&\leq C\Big(1 + \Big(\int_\tau^{t+1} \|u(s)\|_{1,2}^{2p\theta\tilde{q}\beta'/(p+1)}\, ds\Big)^{1/\beta'}\Big).
\end{aligned}$$

Now we see that the last estimate implies (7) with $\tilde{q}$ instead of q provided $\frac{2p\theta\tilde{q}\beta'}{p+1} \leq 2\tilde{q}$, that is if $\theta\beta' \leq p_1$. This condition is equivalent to

$$p \leq \frac{\lambda(\tilde{q}-1)-\tilde{q}}{\tilde{q}-2}. \tag{17}$$

Considering $\tilde{q} \to q+$ and $\lambda \to \lambda_2(q)-$ we see that it is sufficient to verify

$$p(q-2) < \lambda_2(q)(q-1) - q,$$

which is equivalent to $(p-1)2q > 0$. Consequently, the sufficient condition for bootstrap is satisfied and we are done (the uniform lower estimate for $\tilde{q}-q$ follows by an obvious contradiction argument).

Proof without maximal regularity ($n > 3$)

In this proof, we shall use a bootstrap argument again. First denote

$$C_\infty = \max\{1, \sup_{t\geq\delta} \|u(t)\|_{C^1}\} \tag{18}$$

and notice that C_∞ depends on u_0.

In the bootstrap step, we shall assume

$$\sup_{t\geq\delta}\|u(t)\|_\lambda < C_\lambda \tag{19}$$

(where C_λ may depend on u_0, $C_\lambda \geq 1$, $\lambda < p+1$) and we shall show

$$\sup_{t\geq\delta}\|u(t)\|_{\tilde\lambda} < CC_\lambda^{(p+1)/2}C_\infty^{\varepsilon p/2}, \tag{20}$$

where $\varepsilon > 0$ can be chosen arbitrarily small, $\tilde\lambda > \lambda$, $\tilde\lambda - \lambda$ is bounded below by a positive constant (which does not depend on ε) and $C = C(\varepsilon)$. Together with initial estimate (10) for $\lambda < \lambda(2)$, this will imply

$$\sup_{t\geq\delta}\|u(t)\|_\Lambda < CC_\infty^{\varepsilon M_1}, \tag{21}$$

where $\Lambda \geq p+1$, $M_1 = \frac{p}{2}\sum_{i=0}^k\big(\frac{p+1}{2}\big)^i$ and k is the number of bootstrap steps needed to get from initial λ to Λ (the number k does not depend on the value of ε). Due to (4), this implies the same bound for $u(t)$ in W_2^1 and, finally, standard bootstrap procedure yields

$$\sup_{t\geq\delta}\|u(t)\|_{C^1} \leq CC_\infty^{\varepsilon M_1 M_2}, \tag{22}$$

where M_2 is some constant depending on p and the number of bootstrap steps in the second bootstrap procedure. Although the value of δ in (22) may differ from that in (18), the uniform boundedness of $\|u(t)\|_{C^1}$ on intervals of the type $[\delta_1, \delta_2]$ (cf. Lemma) together with (18), (22) and the choice $\varepsilon < 1/(M_1M_2)$ imply our assertion.

Hence assume (19) for some $\lambda < p+1$. Since $n > 3$ implies $\lambda(2) \geq \lambda_1(2) = \frac{6n}{3n-4} \geq p_S > p$, we may assume $\lambda > p$. Put

$$\nu = \frac{\lambda}{p} - 1, \qquad \theta = \frac{1+\nu}{1-\nu}\frac{\lambda-2}{\lambda}, \qquad \lambda' = \frac{\lambda}{\lambda-1}. \tag{23}$$

Then $\nu \leq 1/(\lambda-1)$, $\theta \in (0,1)$ and using (6), Hölder inequality, (19) and interpolation, we obtain (cf. (15))

$$\begin{aligned}\|u(t)\|_{1,2}^2 &\leq C\big(1 + \|u(t)u_t(t)\|_1\big) \leq CC_\lambda\big(1 + \|u_t(t)\|_{\lambda'}\big) \\ &\leq CC_\lambda\big(1 + \|u_t(t)\|_{1+\nu}^\theta\|u_t(t)\|_2^{1-\theta}\big).\end{aligned} \tag{24}$$

An obvious estimate based on the variation of constants formula implies

$$\|u(t)\|_{2,1+\nu} \leq C\|u(\delta)\|_{2,1+\nu} + C\int_\delta^t \frac{e^{-\omega(t-\tau)}}{(t-\tau)^{1-\varepsilon/2}}\||u(s)|^p\|_{\varepsilon,1+\nu}\,ds$$

for some $\omega > 0$ and any $t > \delta$. Since $\|u(\delta)\|_{2,1+\nu} \le C$, we obtain

$$\|u(t)\|_{2,1+\nu} \le C\big(1 + \max_{s\ge\delta} \||u(s)|^p\|_{\varepsilon,1+\nu}\big). \tag{25}$$

Now using interpolation, assumption (19) and $p(1+\nu) = \lambda$, we obtain

$$\begin{aligned}\||u(s)|^p\|_{\varepsilon,1+\nu} &\le \||u(s)|^p\|_{1+\nu}^{1-\varepsilon}\||u(s)|^p\|_{1,1+\nu}^{\varepsilon} \\ &\le C\|u(s)\|_{\lambda}^{(1-\varepsilon)p}\||u(s)|^p\|_{C^1}^{\varepsilon} \le CC_\lambda^p C_\infty^{\varepsilon p}.\end{aligned} \tag{26}$$

The equation $u_t = \Delta u + u|u|^{p-1}$ together estimates (25) and (26) imply

$$\|u_t(t)\|_{1+\nu} \le \|u(t)\|_{2,1+\nu} + \||u(t)|^p\|_{1+\nu} \le CC_\lambda^p C_\infty^{\varepsilon p}. \tag{27}$$

Put

$$q = q(\lambda) := \frac{2}{1-\theta} = \frac{(1-\nu)\lambda}{1+\nu-\nu\lambda} = \frac{2p-\lambda}{p+1-\lambda}. \tag{28}$$

Raising (24) to the power q and using (27) and (5), we obtain the following estimate

$$\begin{aligned}\int_t^{t+1} \|u(s)\|_{1,2}^{2q}\,ds &\le CC_\lambda^q\Big(1 + \sup_{t\le s\le t+1}\|u_t(s)\|_{1+\nu}^{\theta q}\int_t^{t+1}\|u(s)\|_2^2\,ds\Big) \\ &\le CC_\lambda^{q+p\theta q}C_\infty^{\varepsilon p\theta q} \le CC_\lambda^{q(p+1)}C_\infty^{\varepsilon pq}.\end{aligned}$$

The last estimate and the interpolation theorem in [**CL**, Appendice] imply (20) provided $\tilde\lambda < \lambda(q)$. Moreover, the definition of $\lambda_2(q)$ in (12) and (28) show that $\lambda(q) \ge \lambda_2(q) > \lambda$. This shows that the bootstrap is always possible.

The estimate for $\tilde\lambda - \lambda$ from below follows again by a contradiction argument: let $\{\lambda^{(k)}\}$ be an increasing sequence, $\lambda^{(k)} < p+1$, define $q^{(k)} = q(\lambda^{(k)})$ by (28) and assume $\lambda(q^{(k)}) - \lambda^{(k)} \to 0$. Denote $\lambda_\infty := \lim_{k\to\infty}\lambda^{(k)} \le p+1$. If $\lambda_\infty < p+1$ then the continuity of the functions $\lambda \mapsto q(\lambda)$ and $q \mapsto \lambda(q)$ together with $\lambda\big(q(\lambda_\infty)\big) > \lambda_\infty$ yields a contradiction. If $\lambda_\infty = p+1$ then $q^{(k)} \to \infty$ and $\lambda(q^{(k)}) \to p_S + 1 > p+1$ which yields a contradiction again. This concludes the proof. $\square$

Remark 2. The proof without maximal regularity can be repeated also for $n = 3$. Anyhow, in this case we have to restrict ourselves to $p < \lambda(2)$, that is $p < 4$. Since $4 > p_{CL}$, the method still yields an improvement of the result of Cazenave and Lions.

Remark 3. If we used estimate (10) only for $\lambda < \lambda_1(q)$ in the boostrap procedures above then we would need the following additional condition on p:

$$p < p^* := \frac{9n^2 - 4n + 16\sqrt{n(n-1)}}{(3n-4)^2} \tag{29}$$

in order to quarantee $\tilde q > q$ or $\tilde\lambda > \lambda$ for any q or λ, respectively.

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