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PRICING EQUITY-LINKED DEBT USING THE VASICEK MODEL

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Abstract. We consider equity-linked debt where the holder receives both interest payments and payments linked to the performance of an equity index. We use a Green's function approach to value such instruments under the assumption that the equity index obeys a lognormal random walk and the risk-free interest rate is given by the Vasicek model.

1. Introduction

Equity-linked debt is a term used to refer to debt securities, such as bonds or bank-issued certificates of deposit (CDs), whose payment stream is linked to the value of an equity security or, more commonly, an index of such securities [**11, 7**]. Typically with such structures, the payment stream will consist of two distinct part: one part resembles the payment stream of a bond and consisting of the return of some percentage of the principal at the maturity of the bond and may or may not also include the payment of interest coupons at intermediate dates, while the other part resembles the pay-off from a call option, paying some percentage of the excess of the index over a pre-specified strike price. In effect, the price of the bond is split into two pieces, with a portion of the price being the present value of the payment streams from the bond-like element while the other portion is the value of the option. Equity-linked debt can also involve a put option rather than a call option, in which case the pay-off from the option-like element will be linked to the shortfall of the index relative to the strike price. In the case where these securities are in the form of CDs targetted at retail investors, they are frequently referred to as bull CDs (for the call) and bear CDs (for the put). Equity-linked notes are attractive to investors because they combine the conservative aspects of fixed income investing with the aggressive nature of options, they have the attraction that, if issued by a US federally chartered bank, they may be insured by the Federal Deposit Insurance Corporation or FDIC, and they can provide equity exposure for entities restricted from investing directly in equities, whether by legislation or by investment mandate. They are attractive to issuers because they enable the issuer to lower their funding costs or attract new sources of funds.

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The advantages of equity-linked notes to both investors and issuers are explained more fully in both [**11**] and in Chapter 9 of [**7**].

According to [**11**], the earliest such instrument was a $100 million SPIN or Standard & Poor's 500 Indexed Subordinated Note issued by Salomon Brothers in August 1986, the S&P 500 being a US equity index. The SPIN was a four year note, paying a semi-annual coupon of 2%, while at maturity the holder received both the principal and additional amount equal to 92.2% of any increase in the S&P500 above 108.5% of the value of that index on the issue date. In this example, 92.2% is referred to as the participation rate, while the strike price is 108.5% of the original index value. In March 1997, Chase Manhattan offered the first equity-linked product offered by a commercial bank and targetted at retail investors. The initial offering of the Chase Manhattan Market Index Investment came in maturities of up to 12 months, and investors could choose between a guaranteed minimum return of 4% and a low participation percentage in the S&P 500 index return or a guaranteed return of 0% (meaning the return of principal without interest payments) together with a higher participation percentage in the index return. The Chase offering, which included notes with embedded puts as well as calls, was controversial at the time as there was concern that it might be in violation of Depression-era banking laws, specifically the 1933 Glass-Steagall Act which restricted the activities of commercial banks in the securities arena, but it was ultimately given the go-ahead by the banking regulators and insured by the FDIC. Other offerings followed, as detailed in [**11**], but the stock market crash of October 19, 1987, brought the issuance of equity-linked debt to a standstill, with the next major offering not coming until January 1991, when Goldman Sachs issued a $100 million SIGN or Stock Index Growth Note, which was notable firstly for being backed by the full faith and credit of the Republic of Austria and secondly for being traded on the New York Stock Exchange, or NYSE, in $10 denominations. This last feature meant that there was a secondary market in SIGNs, in distinct contrast to the equity-linked CDs where the holder was either locked in until maturity or had to pay a severe penalty for early withdrawal. Many more equity-linked notes have since been issued, and some of the more notable ones are described in [**11**]. The S&P 500 has been the index of choice for these notes, but notes have also been issued on other indices, including indices in both the US and other countries, and also sector indices, such as the S&P pharmaceuticals index, and even on individual stocks.

Turning to the issue of pricing, traditionally these securities have been priced by viewing the bond-like component stream and option-like payment stream as distinct and pricing them separately. Typically, the bond-like component is priced by using one of the many stochastic interest rate models available, with popular models including those due to Vasicek [**12**] and Cox-Ingersoll-Ross [**4, 5**]; a more complete list of such models can be found in most standard texts on mathematical finance, such as [**13**]. The option-like element is typically priced using the issuer's favourite option model, such as the Black-Scholes-Merton option pricing formula [**3, 10**] in which the underlying asset is assumed to follow a lognormal random walk. Alternatively, Kat [**7**] suggests that this is equivalent to buying a forward on the equity index and selling a zero coupon bond with a principal in the amount of the present value of the equity index. It should be stressed that from the issuer's point of view, the option pricing approach makes perfect sense, the more so if the issuer acquires the two payment streams from different sources and is merely bundling them together, as might happen if for example a bank buys an over-the-counter (OTC) option from an options dealer and bundles that cash flow together with an interest component. The holder, however, will likely take a very different view and consider the cash flow from the security to be one stream rather two distinct components, and from the holder's viewpoint, elements of this payment stream received at the same time are being discounted at different rates. This is all the more important when we observe that many of the investors targetted by equity-linked notes are conservative investors whose normal habitat is fixed income securities, and for such investors, the discounting of payment streams is an important detail.

In mathematical terms, the value of the bond component will be the present value of the payment scheme discounted at the continuously variable spot interest rate $r(t)$, which obeys the stochastic differential equation (sde)

$$dr = u(r,t)dt + w(r,t)dX_1, \tag{1}$$

where the functional forms of $u(r,t)$ and $w(r,t)$ determine the behaviour of the spot interest rate r, while the option component is the present value of the expected pay-off from the option, which is arrived at using the assumption that the price S of the underlying index (or stock in the case of an equity-linked note on a single underlying) obeys a lognormal random walk given by the sde

$$dS = \mu S dt + \sigma S dX_2, \tag{2}$$

where σ is the volatility of the stock price and μ is the drift. In the above, dX_1 and dX_2 are both normally distributed random processes with zero mean and variance dt and may be correlated, with

$$E\left[dX_1 dX_2\right] = \rho dt \tag{3}$$

and $-1 \leq \rho(r,S,t) \leq 1$. For equity options, the arbitrage arguments lead us to set the drift of the index price equal to the risk-free interest rate, r, minus any dividends paid by the index. From the holder's viewpoint, the inconsistency is that the interest rate r is assumed to be stochastic in pricing the bond-like component using (1) but constant in pricing the option-like component using (2). For consistency, we would argue that the interest rate used to price the equity component should also be stochastic, so the interest rate used in (2) is that given by (1). This approach has been applied in the past to convertible bonds, which are fixed income securities which can be exchanged for the stock of the issuer under certain circumstances (if the bond can be exchanged for a stock other than that of the issuer, it is termed an exchangeable bond). The contingent claims analysis of convertible bonds dates back at least as far as the works of [**6, 1**], with extensions by [**2**] to include the stochastic interest rates discussed above and also by [**9**] to use the value of the issuing firm's stock rather than the value of the firm as an underlying variable. Following this path, it is possible to construct a risk-free portfolio containing a convertible bond, and this leads to the following partial differential equation (PDE) for the value V of a convertible,

$$\begin{aligned} &\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \rho\sigma S w \frac{\partial^2 V}{\partial S \partial r} + \frac{1}{2} w^2 \frac{\partial^2 V}{\partial r^2} \\ + \quad &(r - D_0)\, S \frac{\partial V}{\partial S} + (u - \lambda w)\, \frac{\partial V}{\partial r} - rV = 0, \end{aligned} \tag{4}$$

the derivation of which can be found in [**13**]. In (4), D_0 is the constant dividend yield on the equity, $\lambda(r, S, t)$ is the market price of interest rate risk and $u - \lambda w$ is the risk adjusted drift, and this equation is typically valid for $t \leq T$, where T is the time at which the note matures. Almost all of the work on convertibles discussed above led to a numerical rather than an analytical solution for the value of a convertible, typically using binomial trees. Recently however [**8**] a Green's function for the price of a convertible was derived under the assumption that the the interest rate followed a random walk given by the Vasicek [**12**] model. [**8**] were able to solve the PDE (4) by using a double integral transform, taking a Laplace transform in time and a Mellin transform with respect to the asset price, and thereby transforming (4) into an ordinary differential equation (ODE) which they then solved. Inverting the transforms then led to a Green's function for the price of a convertible.

In the present work, we are considering equity-linked notes rather than convertible bonds, but we would argue that the value V of such a note will obey the same PDE as the convertible bond (4), and the pay-off for an equity-linked note with an embedded call option can actually be written as the combination of the pay-off from a discount bond and that from a convertible. It follows that if we use the same interest rate model as [**8**], specifically the Vasicek model, then we can use their Green's function to write down an expression for the value of an equity linked debt, and that is precisely the path which we will take in the next section.

2. Analysis

In this section we will discuss the value V of an equity-linked note, such as a bull or a bear CD, which we assume is a function of S, the level of the underlying equity index, or the price of the underlying stock if the note is on an individual security, and r, the spot interest rate. We assume that S obeys a lognormal random walk (2), while r obeys the sde (1). As we mentioned in the previous section, if we apply arbitrage arguments to a security whose value depends upon both the value of an equity and the level of interest rates, we arrive at the PDE [**13**]

$$\begin{aligned} &\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \rho\sigma S w \frac{\partial^2 V}{\partial S \partial r} \\ + \quad &\frac{1}{2} w^2 \frac{\partial^2 V}{\partial r^2} + (r - D_0)\, S \frac{\partial V}{\partial S} + (u - \lambda w)\, \frac{\partial V}{\partial r} - rV = 0. \end{aligned} \tag{5}$$

In deriving this PDE, the only assumption made about the spot interest rate is that it obeys the sde (1), and in order to solve (5), it is necessary to make some assumptions about $u-\lambda w$ and w, meaning it is necessary to select a specific interest rate model. Many of the popular one-factor interest rate models are special cases of the general affine model for which $u-\lambda w = a(t) - b(t)r$ and $w = (c(t)r - d(t))^{1/2}$. One such special case is the Vasicek model, with $u - \lambda w = a - br$ and $w = c$, where a, b and c are constants, as opposed to functions of t in the general affine model, The Vasicek model allows interest rates to become negative. Several other special cases of the general affine model are listed in Chapter 46 of [**13**]. If we specialize to the Vasicek model, and further make the transformation $t = T - \tau$, so that τ is the remaining life of the note, the PDE above becomes

$$\begin{aligned} \frac{\partial V}{\partial \tau} &= \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \rho\sigma c S \frac{\partial^2 V}{\partial S \partial r} + \frac{1}{2}c^2 \frac{\partial^2 V}{\partial r^2} \\ &+ (r - D_0)\, S \frac{\partial V}{\partial S} + (a - br)\frac{\partial V}{\partial r} - rV. \end{aligned} \tag{6}$$

In [**8**] (6) was solved by taking a Laplace transform with respect to τ, the time until maturity, and a Mellin transform with respect to S, the equity price. Assuming the pay-off at maturity was $V_0(S,r)$, the solution in [**8**] was of the form

$$V(S,r,\tau) = \int_{-\infty}^{\infty}\int_0^{\infty} G\left(S,\tilde{S},r,\tilde{r},\tau\right) V_0(\tilde{S},\tilde{r}) d\tilde{S} d\tilde{r}, \tag{7}$$

where

$$\begin{aligned} G\left(S,\tilde{S},r,\tilde{r},\tau\right) &= \frac{\tilde{S}^{-1}\sqrt{b}}{\pi c\sqrt{1-e^{-2b\tau}}} \\ &\times \left(\alpha_+^{-1/2}\exp\left[\gamma_+ - \frac{\left(\beta_+ - \log(S/\tilde{S})\right)^2}{4\alpha_+}\right]\right. \\ &\left. + - \alpha_-^{-1/2}\exp\left[\gamma_- - \frac{\left(\beta_- - \log(S/\tilde{S})\right)^2}{4\alpha_-}\right]\right), \end{aligned} \tag{8}$$

is the Green's function. In (8), $\alpha_\pm$, $\beta_\pm$ and $\gamma_\pm$ are functions of r, $\tilde{r}$ and τ, and are given by

$$\begin{aligned}
\alpha_{\pm} &= \frac{\pm\left(c+\rho\sigma b^2\right)^2 - 2\cosh b\tau(\rho\sigma b + c^2)}{2b^3\sinh b\tau} + \frac{\tau\left(\sigma^2 b^2 + c^2 + 2c\rho\sigma b\right)}{2b^2} \\
\beta_{\pm} &= \frac{\pm(c+\rho\sigma b^2)(2c^2 - 2b^2 a + b^2(r+\tilde{r})) - 4\cosh b\tau(\rho\sigma b + c)(c^2 - ba)}{2b^3 c\sinh b\tau} \\
&+ \frac{(2c\rho\sigma b + \sigma^2 b^2 + 2c^2 - 2ba + 2b^2 D_0)\tau}{2b^2} \\
&- \frac{\rho\sigma b\left(\tilde{r}e^{b\tau} + re^{-b\tau}\right) + c\left(r+\tilde{r}\right)\cosh b\tau}{bc\sinh b\tau} \\
\gamma_{\pm} &= \frac{\pm(b^2\tilde{r} - b^2 a + c^2)(b^2 r - b^2 a + c^2)}{2b^3c^2\sinh b\tau} + \frac{(c^2 - 2ba)\tau}{2b^2} \\
&- \frac{b\left(r^2 e^{-b\tau} + \tilde{r}^2 e^{b\tau}\right) - 2a\left(re^{-b\tau} + \tilde{r}e^{b\tau}\right)}{2c^2\sinh b\tau} \\
&+ \frac{2\left(c^2 b^2\left(r+\tilde{r}\right) + \left(ab - c^2\right)^2\right)\cosh b\tau}{2b^3c^2\sinh b\tau}, \qquad (9)
\end{aligned}$$

where we take the "+" signs in (9) for α_+, β_+ and γ_+, and the "−" signs for α_-, β_- and γ_-.

Although the solution (7) was originally derived for a convertible, it is equally applicable to the equity-linked note under consideration here, although of course the pay-off function V_0 will be different, and here depends only on S not r. For an equity-linked note without coupons that pays at maturity an amount B_0 with certainty together with a proportion R_0 of the level of the equity price above or below a strike level E_0 will have a pay-off

$$V_0(S) = B_0 + R_0 \max\left(S - E_0, 0\right), \qquad (10)$$

for the case of a call-like note and

$$V_0(S) = B_0 + R_0 \max\left(E_0 - S, 0\right) \qquad (11)$$

for a put-like note, and it follows that the value V of the note is given by the solution (7) with V_0 replaced by the pay-off (10) or (11), so that

$$V = \int_{-\infty}^{\infty}\int_0^{\infty} G\left(S, \tilde{S}, r, \tilde{r}, \tau\right) V_0(\tilde{S}) d\tilde{S} d\tilde{r}. \qquad (12)$$

For the call, it is worth noting that we can rewrite the pay-off in the form

$$V_0(S) = B_0 - R_0E_0 + R_0 \max\left(S, E_0\right), \qquad (13)$$

so that the pay-off is identical to the combination of a discount bond with principal $B_0 - R_0E_0$ and a quantity R_0 of a convertible bond with conversion price E_0. Similarly for the put, we can write the pay-off as

$$V_0(S) = B_0 + R_0E_0 - R_0 \min\left(S, E_0\right). \qquad (14)$$

If we separate the pay-off into the bond-like and option-like elements, we have

$$V(S, r, \tau) = B_0 B(r, \tau) + R_0 C(S, r, \tau) \qquad (15)$$

for the call-like note and

$$V(S,r,\tau) \quad = \quad B_0 B(r,\tau) + R_0 P(S,r,\tau) \tag{16}$$

for the put-like note. In the above, the bond-like element is written in terms of $B(r,\tau)$, the value of a zero coupon bond under the Vasicek model,

$$\begin{aligned} B(r,\tau) &= \int_{-\infty}^{\infty}\int_{0}^{\infty} G\left(S,\tilde{S},r,\tilde{r},\tau\right) d\tilde{S}d\tilde{r} \\ &= 2\left(\exp\left[g_0 + \frac{g_1^2}{4g_2}\right] - \exp\left[g_0 + \frac{g_1^2}{4g_2}\right]\right). \end{aligned} \tag{17}$$

The pay-off B_0 for the bond-like element is of course a constant and independent of the level of both interest rates $\tilde{r}$ and equity $\tilde{S}$ at the time of the pay-off, while the value of the bond-like element itself (17) will depend on the current interest rate r and on time τ but will be independent of the equity price S. The coefficients $g_{0\pm}$, $g_{1\pm}$ and $g_{2\pm}$ in (17) will be defined shortly after we have considered the behaviour of the option-like element, which is written in terms of either $C(S,r,\tau)$ or $P(S,r,\tau)$, the values of a call option and a put option respectively under the Vasicek model. These values will differ from the usual Black-Scholes formula because we make the assumption that interest rates are stochastic. For the call option, we have

$$\begin{aligned} C(S,r,\tau) &= \int_{-\infty}^{\infty}\int_{E_0}^{\infty} \left(\tilde{S} - E_0\right) G\left(S,\tilde{S},r,\tilde{r},\tau\right) d\tilde{S}d\tilde{r} \\ &= C_+(S,r,\tau) - C_-(S,r,\tau) \end{aligned} \tag{18}$$

where

$$\begin{aligned} C_\pm(S,r,\tau) &= S\pi g_{2\pm}^{-1/2} \exp\left[\frac{\left(b_{1\pm} - g_{1\pm}\right)^2}{4g_{2\pm}} + \alpha_\pm + g_{0\pm} - b_{0\pm}\right] \\ &\times \operatorname{erfc}\left[\frac{b_{1\pm}g_{1\pm} + 2b_{0\pm}g_{0\pm} - b_{1\pm}^2 - 4g_{2\pm}\alpha_\pm - 2g_{2\pm}\log\left(S/E_0\right)}{2\sqrt{g_{2\pm}\left(4g_{2\pm}\alpha_\pm + b_{1\pm}^2\right)}}\right] \\ &- E_0\pi g_{2\pm}^{-1/2} \exp\left[g_{0\pm} + \frac{g_{1\pm}^2}{4g_{2\pm}}\right] \\ &\times \operatorname{erfc}\left[\frac{b_{1\pm}g_{1\pm} + 2b_{0\pm}g_{0\pm} - 2g_{2\pm}\log\left(S/E_0\right)}{2\sqrt{g_{2\pm}\left(4g_{2\pm}\alpha_\pm + b_{1\pm}^2\right)}}\right], \end{aligned} \tag{19}$$

where erfc denotes the complementary error function, while for the put option,

$$\begin{aligned} P(S,r,\tau) &= \int_{-\infty}^{\infty}\int_{0}^{E_0} \left(\tilde{S} - E_0\right) G\left(S,\tilde{S},r,\tilde{r},\tau\right) d\tilde{S}d\tilde{r} \\ &= P_+(S,r,\tau) - P_-(S,r,\tau) \end{aligned} \tag{20}$$

where

$$
\begin{aligned}
P_\pm(S,r,\tau) &= -S\pi g_{2\pm}^{-1/2}\exp\left[\frac{(b_{1\pm}-g_{1\pm})^2}{4g_{2\pm}}+\alpha_\pm+g_{0\pm}-b_{0\pm}\right]\\
&\times\ \operatorname{erfc}\left[-\frac{b_{1\pm}g_{1\pm}+2b_{0\pm}g_{0\pm}-b_{1\pm}^2-4g_{2\pm}\alpha_\pm-2g_{2\pm}\log(S/E_0)}{2\sqrt{g_{2\pm}\left(4g_{2\pm}\alpha_\pm+b_{1\pm}^2\right)}}\right]\\
&+\ E_0\pi g_{2\pm}^{-1/2}\exp\left[g_{0\pm}+\frac{g_{1\pm}^2}{4g_{2\pm}}\right]\\
&\times\ \operatorname{erfc}\left[-\frac{b_{1\pm}g_{1\pm}+2b_{0\pm}g_{0\pm}-2g_{2\pm}\log(S/E_0)}{2\sqrt{g_{2\pm}\left(4g_{2\pm}\alpha_\pm+b_{1\pm}^2\right)}}\right].
\end{aligned}\tag{21}
$$

In the above expressions (17,19,21), we have used the fact that $\alpha_\pm$, as given earlier in (9) is independent of r and $\tilde{r}$, while we have rewritten $\beta_\pm$ and $\gamma_\pm$ in the form

$$
\begin{aligned}
\beta_\pm &= b_{0\pm}(r,\tau)+b_{1\pm}(r,\tau)\tilde{r}\\
\gamma_\pm &= g_{0\pm}(r,\tau)+g_{1\pm}(r,\tau)\tilde{r}-g_{2\pm}(r,\tau)\tilde{r}^2,
\end{aligned}\tag{22}
$$

with the coefficients given by

$$
\begin{aligned}
b_{1\pm} &= \frac{\pm(c+\rho\sigma b^2)-2\rho\sigma be^{b\tau}-2c\cosh b\tau}{2bc\sinh b\tau}\\
b_{0\pm} &= \frac{\pm(c+\rho\sigma b^2)(2c^2-2b^2a+b^2r)-4\cosh b\tau(\rho\sigma b+c)(c^2-ba)}{2b^3c\sinh b\tau}\\
&+ \frac{(2c\rho\sigma b+\sigma^2b^2+2c^2-2ba+2b^2D_0)\tau}{2b^2}-\frac{\rho\sigma bre^{-b\tau}+cr\cosh b\tau}{bc\sinh b\tau}\\
g_{2\pm} &= \frac{be^{b\tau}}{2c^2\sinh b\tau}\\
g_{1\pm} &= \frac{\pm(b^2r-b^2a+c^2)+2abe^{b\tau}-2c^2\cosh b\tau}{2bc^2\sinh b\tau}\\
g_{0\pm} &= \frac{\pm(-b^2a+c^2)(b^2r-b^2a+c^2)}{2b^3c^2\sinh b\tau}+\frac{(c^2-2ba)\tau}{2b^2}\\
&- \frac{b^4r^2e^{-b\tau}-2ab^3re^{-b\tau}+2\left(c^2b^2r+\left(ab-c^2\right)^2\right)\cosh b\tau}{2b^3c^2\sinh b\tau},
\end{aligned}\tag{23}
$$

and we have included a negative sign in from of the $g_{2\pm}\tilde{r}^2$ term in (23) to show that that term is negative, which is essential for the evaluation of the integrals in (17,18,20).

3. Discussion

In the previous section, we considered the value of an equity-linked note that pays at maturity an amount B_0 with certainty together with a proportion R_0 of the amount by which the equity value exceeds (in the case of a call) or falls short of (in the case of a put) a strike level E_0, presenting expressions for the value in equations (15-23). Such notes are frequently referred to as bull and bear CDs when they are sold to retail investors, and are described in some detail in [**11**]. In deriving (15-23), we have taken the value of the note to be the sum of the present values of the bond-like component and the option component. Our analysis was carried out for a zero coupon or discount bond, which makes payments only at expiry, but it is straightforward to extend the analysis to include periodic coupon payments by treating the coupons as separate discount bonds and using (17) to price the coupons, so that the value of a coupon in the amount B_1 received at time τ_1 before maturity would add an amount $B_1 \times B(r, \tau - \tau_1)$ to the value of the note.

Our approach of valuing the bond-like element and the option-like element separately and then summing the value of those elements to obtain the value of the note is actually in some respects the standard way of pricing these notes. What is novel about our approach however is that we have priced the notes from the viewpoint of an investor rather than the issuer, and in particular, we have discounted the bond-like and option-like cash flows at the same (stochastic) interest rate, which we have assumed obeys the Vasicek [**12**] model, and the main consequence of this is that the value of the option-like component will depend on the interest rate as well as the level of the equity index, and this value therefore obeys the partial differential equation (4), or (6) for the interest rate model considered here, which was originally presented for a rather different class of securities dependent upon both equity values and the level of interest rates, namely convertible bonds [**13**]. Because of this, we were able to price equity-linked notes using a Green's function which was also originally presented for convertible bonds [**8**]. The traditional approach has been to ignore the effect of interest rate changes on the value of the embedded option in the note, and that approach is often the appropriate one for the issuer to take, especially if the issuer is merely assembling the note from other instruments, which could be done by purchasing a bond and an OTC option separately and then bundling the cash flows together in the note. By contrast, the holder of an equity-linked note is frequently a conservative, fixed income investor who is concerned about the interest rate sensitivity of his investments, this sensitivity being measured by rho, the derivative of the security price with respect to the interest rate. Such an investor is concerned with the present value of the cash flows from his investments, and to properly calculate that present value, an approach along the lines of that used here is the correct path.

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