ON CERTAIN OPERATIONAL FORMULA FOR MULTIVARIABLE BASIC HYPERGEOMETRIC FUNCTIONS

S. D. PUROHIT AND R. K. RAINA

ABSTRACT. In the present paper certain operational formulae involving Riemann-Liouville and Kober fractional q-integral operators for an analytic function are derived. The usefulness of the main results are exhibited by considering some examples which also yield q-extensions of some known results for ordinary hypergeometric functions of one and more variables.

1. INTRODUCTION

Several authors have used certain fractional q-calculus operators to obtain various operational and transformation formulae involving basic hypergeometric functions (see, for instance, [1]–[4], [7], [10]–[12]). Motivated by the interesting outcome of some of the earlier works and a possible scope for their applications in evaluation and solution of the types of q-integral equations, we further determine certain operational formulae involving the Riemann-Liouville and Kober type fractional q-integral operators.

A q-analogue of the familiar Riemann-Liouville fractional integral operator of a function $f(x)$ is defined by ([1])

$$(1.1) \qquad I_q^\mu \{f(x)\} = \frac{1}{\Gamma_q(\mu)} \int_0^x (x - tq)_{\mu-1} f(t) \mathrm{d}(t; q)$$

$$(\mathrm{Re}(\mu) > 0; \quad |q| < 1).$$

Also, in [1] the basic analogue of the Kober fractional integral operator is defined by

$$(1.2) \qquad I_q^{\eta,\mu} \{f(x)\} = \frac{x^{-\eta-\mu}}{\Gamma_q(\mu)} \int_0^x (x - tq)_{\mu-1} t^\eta f(t) \mathrm{d}(t; q)$$

$$(\mathrm{Re}(\mu) > 0; \quad |q| < 1; \quad \eta \in \mathbb{R}).$$

We shall make use of the following notations and definitions in the sequel.
For real or complex a and $|q| < 1$, the q-shifted factorial is defined by

$$(1.3) \qquad (a; q)_n = \begin{cases} 1, & \text{if } n = 0 \\ (1 - a)(1 - aq) \dots (1 - aq^{n-1}) & \text{if } n \in \mathbb{N}, \end{cases}$$

and in terms of the q-gamma function (1.3) can be expressed as

$$(1.4) \qquad (q^a; q)_n = \frac{\Gamma_q(a + n)(1 - q)^n}{\Gamma_q(a)}, \qquad n > 0,$$

where the q-gamma function (cf. Gasper and Rahman [3]) is given by

$$(1.5) \qquad \Gamma_q(a) = \frac{(q; q)_\infty}{(q^a; q)_\infty (1 - q)^{a-1}} = \frac{(1; q)_{a-1}}{(1 - q)^{a-1}},$$

provided that $a \neq 0, -1, -2, \ldots$

Further,

$$(1.6) \qquad (x; y)_\nu = x^\nu \prod_{n=0}^{\infty} \left[\frac{1 - (y/x)q^n}{1 - (y/x)q^{\nu+n}} \right] = x^\nu \, {}_1\Phi_0 \left[\begin{array}{c} q^{-\nu} \\ - \end{array} ; q, yq^\nu/x \right].$$

The multiple basic hypergeometric function (cf. Srivastava and Karlsson [9]) is defined by

$$\Phi_{C:D';\ldots;D^{(n)}}^{A:B';\ldots;B^{(n)}} \left(\begin{array}{c} [(a) : \theta', \ldots, \theta^{(n)}] : [(b') : \phi']; \ldots; [(b^{(n)}) : \phi^{(n)}] \\ [(c) : \psi', \ldots, \psi^{(n)}] : [(d') : \delta']; \ldots; [(d^{(n)}) : \delta^{(n)}] \end{array} ; q, z_1, \ldots, z_n \right)$$

$$(1.7)$$
$$= \sum_{m_1, \cdots, m_n=0}^{\infty} \frac{\prod\limits_{j=1}^{A} (a_j; q)_{m_1 \theta'_j + \ldots + m_n \theta^{(n)}_j} \prod\limits_{j=1}^{B'} (b'_j; q)_{m_1 \phi'_j} \cdots \prod\limits_{j=1}^{B^{(n)}} (b^{(n)}_j; q)_{m_n \phi^{(n)}_j}}{\prod\limits_{j=1}^{C} (c_j; q)_{m_1 \psi'_j + \ldots + m_n \psi^{(n)}_j} \prod\limits_{j=1}^{D'} (d'_j; q)_{m_1 \delta'_j} \cdots \prod\limits_{j=1}^{D^{(n)}} (d^{(n)}_j; q)_{m_n \delta^{(n)}_j}}$$

$$\cdot \frac{z_1^{m_1}}{(q; q)_{m_1}} \cdots \frac{z_n^{m_n}}{(q; q)_{m_n}},$$

where the arguments $z_1, \cdots, z_n$, and the complex parameters

$$\left\{ \begin{array}{ll} a_j, \ j = 1, \ldots, A; & b_j^{(k)}, \ j = 1, \ldots, B_j^{(k)}; \\ c_j, \ j = 1, \ldots, C; & d_j^{(k)}, \ j = 1, \ldots, D_j^{(k)}; \quad k = 1, \ldots, n \ \text{(primes)}, \end{array} \right.$$

and the associated coefficients

$$\left\{ \begin{array}{ll} \theta_j^{(k)}, \ j = 1, \ldots, A; & \phi_j^{(k)}, \ j = 1, \ldots, B_j^{(k)}; \\ \psi_j^{(k)}, \ j = 1, \ldots, C; & \delta_j^{(k)}, \ j = 1, \ldots, D_j^{(k)} \quad k = 1, \ldots, n \text{(primes)}, \end{array} \right.$$

are so constrained that the multiple series (1.7) converges.

For $\theta_j^{(k)} = 1$ $(j = 1, \ldots, A)$, $\phi_j^{(k)} = 1$ $(j = 1, \ldots, B_j^{(k)})$, $\psi_j^{(k)} = 1$ $(j = 1, \ldots, C)$, $\delta_j^{(k)} = 1$ $(j = 1, \ldots, D_j^{(k)})$ for all $k = 1, \ldots, n$ (primes) the definition (1.7) reduces to the q-analogue of the generalized Kampé de Fériet function of n variables given by

$$\Phi_{C:D';\ldots;D^{(n)}}^{A:B';\ldots;B^{(n)}} \left(\begin{array}{c} (a) : (b'); \ldots ; (b^{(n)}) \\ (c) : (d'); \ldots ; (d^{(n)}) \end{array} ; q, z_1, \ldots, z_n \right)$$

$$(1.8) \qquad = \sum_{m_1, \cdots, m_n = 0}^{\infty} \frac{\prod\limits_{j=1}^{A} (a_j; q)_{m_1 + \ldots + m_n} \prod\limits_{j=1}^{B'} (b'_j; q)_{m_1} \cdots \prod\limits_{j=1}^{B^{(n)}} (b_j^{(n)}; q)_{m_n}}{\prod\limits_{j=1}^{C} (c_j; q)_{m_1 + \ldots + m_n} \prod\limits_{j=1}^{D'} (d'_j; q)_{m_1} \cdots \prod\limits_{j=1}^{D^{(n)}} (d_j^{(n)}; q)_{m_n}}$$

$$\cdot \frac{z_1^{m_1}}{(q; q)_{m_1}} \cdots \frac{z_n^{m_n}}{(q; q)_{m_n}}.$$

The generalized basic hypergeometric series (cf. Slater [7] is given by

$$(1.9) \qquad {}_r\Phi_s \left[\begin{array}{c} a_1, \ldots, a_r \\ b_1, \ldots, b_s \end{array} ; q, z \right] = \sum_{n=0}^{\infty} \frac{(a_1, \ldots, a_r; q)_n}{(q, b_1, \ldots, b_s; q)_n} z^n,$$

where, for convergence, $|q| < 1$ and $|z| < 1$ if $r = s + 1$, and for any z if $r \le s$.

The purpose of this paper is to obtain certain operational formulae involving the Riemann-Liouville and Kober type of fractional q-integral operators of an analytic function. The applications yield examples of image formula under these above operators, thereby illustrating the usefulness of the main results.
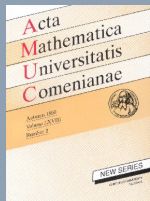
2. Main Results

Suppose that a function $f(z_1, \ldots, z_n)$ is analytic in the domain $D = D_1 \times D_2 \times \cdots \times D_n$ $(z_i \in D_i,$ $i = 1, \cdots, n)$ possessing the power series expansion

$$(2.1) \qquad f(z_1, \ldots, z_n) = \sum_{m_1, \ldots, m_n = 0}^{\infty} C(m_1, \ldots, m_n) \prod_{i=1}^{n} z_i^{m_i},$$

where $|z_i| < R_i$ $(R_i > 0, i = 1, \ldots, n)$, and $C(m_1, \ldots, m_n)$ is a bounded sequence of real or complex numbers.

For an analytic function $f(z_1, \ldots, z_n)$ defined by (2.1), we derive the following two operational formula involving the fractional q-integral operators for a real variable x and complex variables $z_1, \ldots, z_n$.

Theorem 1. *Corresponding to the bounded sequence $C(m_1, \ldots, m_n)$ let the function $f(z_1, \ldots, z_n)$ be defined by (2.1), then*

$$\Omega \left\{ f(x^{k_1} z_1, \ldots, x^{k_n} z_n) \right\} = x^{\alpha_p - 1} \sum_{m_1, \ldots, m_n = 0}^{\infty} C(m_1, \ldots, m_n) \prod_{i=1}^{n} (x^{k_i} z_i)^{m_i}$$

$$(2.2) \qquad \qquad \cdot \prod_{j=1}^{p} \left\{ \frac{\Gamma_q(\alpha_j + \mu_j + M)}{\Gamma_q(\alpha_j + \mu_j + \lambda_j + M)} \right\},$$

where $\mathrm{Re}(\alpha_j + \mu_j) > 0$ $(j = 1, \ldots, p)$, $\max \left\{ \left| x^{k_1} z_1 \right|, \ldots, \left| x^{k_n} z_n \right| \right\} < R$ $(R > 0)$, *for arbitrary k_i* $(i = 1, \ldots, n)$, Ω *is a chain of fractional q-calculus operators defined by*

$$(2.3) \qquad \Omega = I_q^{\mu_p, \lambda_p} x^{\alpha_p - \alpha_{p-1}} \ldots I_q^{\mu_2, \lambda_2} x^{\alpha_2 - \alpha_1} I_q^{\mu_1, \lambda_1} x^{\alpha_1 - 1},$$
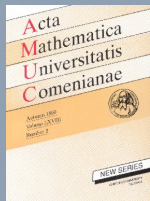
provided that both sides of (2.2) *exist, and*

$$(2.4) \qquad M = k_1 m_1 + \ldots + k_n m_n.$$

Proof. In view of (2.1) and (2.3), we obtain by replacing each z_i by $x^{k_i} z_i$ $(i = 1, \ldots, n)$:

$$\Omega \left\{ f(x^{k_1} z_1, \ldots, x^{k_n} z_n) \right\} = \Omega \left\{ \sum_{m_1, \ldots, m_n = 0}^{\infty} C(m_1, \ldots, m_n) x^M \prod_{i=1}^{n} z_i^{m_i} \right\}.$$

On interchanging the order of summation and the chain of fractional q-integral operator Ω (which is valid under the conditions given in (2.1) and in the hypothesis of Theorem 1), we get

$$(2.5) \qquad \Omega \left\{ f(x^{k_1} z_1, \ldots, x^{k_n} z_n) \right\} = \sum_{m_1, \ldots, m_n = 0}^{\infty} C(m_1, \ldots, m_n) \prod_{i=1}^{n} z_i^{m_i} \Omega \left\{ x^M \right\}.$$

Applying the fractional q-integral formula due to Yadav and Purohit [**12**, p. 440, eqn. (19)]

$$(2.6) \qquad I_q^{\eta, \mu} \left\{ x^{\nu-1} \right\} = \frac{\Gamma_q(\nu + \eta)}{\Gamma_q(\nu + \eta + \mu)} x^{\nu-1},$$

$$(\mathrm{Re}(\nu + \eta) > 0, \ |q| < 1)$$

succesively p times on the right-hand side of (2.5), we arrive at the desired result (2.2) of Theorem 1.
$\square$

If we set $k_1 = \ldots = k_n = 1$ in the Theorem 1, then we obtain the following corollary:

Corollary 1. *Corresponding to the bounded sequence $C(m_1, \ldots, m_n)$, let the function $f(z_1, \ldots, z_n)$ be defined by* (2.1), *then*

$$
\Omega\{f(xz_1, \ldots, xz_n)\} = x^{\alpha_p - 1} \sum_{m_1, \ldots, m_n = 0}^{\infty} C(m_1, \ldots, m_n) \prod_{i=1}^{n} (xz_i)^{m_i}
$$

(2.7)

$$
\cdot \prod_{j=1}^{p} \left\{ \frac{\Gamma_q(\alpha_j + \mu_j + M_1)}{\Gamma_q(\alpha_j + \mu_j + \lambda_j + M_1)} \right\},
$$

where $\operatorname{Re}(\alpha_j + \mu_j) > 0 \ (j = 1, \ldots, p)$, $\max\{|xz_1|, \ldots, |xz_n|\} < R_1 \ (R_1 > 0)$, Ω *is defined by equation* (2.3), *and*

(2.8)
$$
M_1 = m_1 + \ldots + m_n.
$$

Theorem 2. *For the bounded sequence $C(m_1, \ldots, m_n)$, let the function $f(z_1, \ldots, z_n)$ be defined by* (2.1), *then*

$$
\Omega^* \{f(x^{k_1} z_1, \ldots, x^{k_n} z_n)\} = x^{\beta_p - 1} \sum_{m_1, \ldots, m_n = 0}^{\infty} C(m_1, \ldots, m_n) \prod_{i=1}^{n} (x^{k_i} z_i)^{m_i}
$$

(2.9)

$$
\cdot \prod_{j=1}^{p} \left\{ \frac{\Gamma_q(\alpha_j + M)}{\Gamma_q(\beta_j + M)} \right\},
$$

where $\operatorname{Re}(\alpha_j) > 0 \ (j = 1, \ldots, p)$, $\max\{|x^{k_1} z_1|, \ldots, |x^{k_n} z_n|\} < R \ (R > 0)$, *for arbitrary k_i ($i = 1, \ldots, n$), Ω^* is a chain of fractional q-calculus operators defined by*

(2.10)
$$
\Omega^* = I_q^{\beta_p - \alpha_p} x^{\alpha_p - \beta_p - 1} \ldots I_q^{\beta_2 - \alpha_2} x^{\alpha_2 - \beta_1} I_q^{\beta_1 - \alpha_1} x^{\alpha_1 - 1},
$$

provided that both sides of (2.9) *exist, and M is given by* (2.4).

Proof. Proceeding as in Theorem 1, we can write

$$(2.11) \qquad \Omega^* \left\{ f(x^{k_1}z_1, \ldots, x^{k_n}z_n) \right\} = \sum_{m_1, \ldots, m_n = 0}^{\infty} C(m_1, \ldots, m_n) \prod_{i=1}^{n} z_i^{m_i} \Omega^* \left\{ x^M \right\}.$$

Then applying the formula due to Agarwal [1]:

$$(2.12) \qquad I_q^{\mu} \left\{ x^{\nu - 1} \right\} = \frac{\Gamma_q(\nu)}{\Gamma_q(\nu + \mu)} x^{\mu + \nu - 1},$$

$$(\text{Re}(\nu) > 0; \ |q| < 1)$$

successively p times on the right-hand side of (2.11), we obtain (2.9) of Theorem 2. □

For $k_1 = \ldots = k_n = 1$, the Theorem 2 reduces to the following corollary:

Corollary 2. *For the bounded sequence $C(m_1, \ldots, m_n)$, let the function $f(z_1, \ldots, z_n)$ be defined by (2.1), then*

$$\Omega^* \left\{ f(xz_1, \ldots, xz_n) \right\} = x^{\beta_p - 1} \sum_{m_1, \ldots, m_n = 0}^{\infty} C(m_1, \ldots, m_n) \prod_{i=1}^{n} (xz_i)^{m_i}$$

$$(2.13)$$

$$\cdot \prod_{j=1}^{p} \left\{ \frac{\Gamma_q(\alpha_j + M_1)}{\Gamma_q(\beta_j + M_1)} \right\},$$

where $\text{Re}(\alpha_j) > 0 \ (j = 1, \ldots, p)$, $\max \left\{ |xz_1|, \ldots, |xz_n| \right\} < R_1 \ (R_1 > 0)$, Ω^* is a chain of fractional q-calculus operators defined by equation (2.10), and M_1 is given by (2.8).

3. Applications of the Main Results

In this section, we consider some consequences and applications of the results derived in Section 2. It is interesting to observe that in view of the following limiting cases:

$$(3.1) \qquad \lim_{q \to 1^-} \Gamma_q(a) = \Gamma(a) \qquad \text{and} \qquad \lim_{q \to 1^-} \frac{(q^a; q)_n}{(1-q)^n} = (a)_n,$$

where

$$(3.2) \qquad (a)_n = a(a+1)\ldots(a+n-1),$$

the operational formulae (2.7) of Corollary 1 and (2.13) of Corollary 2 above provide, respectively, the q-extensions of the known results due to Raina [5, p. 52, eqns. (27) and (26)].

By assigning suitable special values to the arbitrary sequence $C(m_1, \ldots, m_n)$, our main results (Theorems 1 and 2) can be applied to derive certain operational formulae for a basic hypergeometric function of several variables involving Riemann- -Liouville and Kober fractional q-integral operators. To illustrate that we consider the following examples.

Example 1. Let us set

$$(3.3) \qquad C(m_1, \ldots, m_n) = \frac{\prod_{j=1}^{A} (a_j; q)_{m_1 \theta'_j + \ldots + m_n \theta_j^{(n)}} \prod_{j=1}^{B'} (b'_j; q)_{m_1 \phi'_j} \cdots \prod_{j=1}^{B^{(n)}} (b_j^{(n)}; q)_{m_n \phi_j^{(n)}}}{\prod_{j=1}^{C} (c_j; q)_{m_1 \psi'_j + \ldots + m_n \psi_j^{(n)}} \prod_{j=1}^{D'} (d'_j; q)_{m_1 \delta'_j} \cdots \prod_{j=1}^{D^{(n)}} (d_j^{(n)}; q)_{m_n \delta_j^{(n)}}}$$

$$\cdot \frac{1}{(q; q)_{m_1}} \cdots \frac{1}{(q; q)_{m_n}},$$

the multivariable basic hypergeometric function:

$$\Omega\left\{\Phi_{C:D';\ldots;D^{(n)}}^{A:B';\ldots;B^{(n)}}\left(\begin{array}{c}[(a):\theta',\ldots,\theta^{(n)}]:[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}]\\{}[(c):\psi',\ldots,\psi^{(n)}]:[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}]\end{array};q,x^{k_1}z_1,\ldots,x^{k_n}z_n\right)\right\}$$

$$(3.4) \qquad =\prod_{j=1}^{p}\left\{\frac{\Gamma_q(\alpha_j+\mu_j)}{\Gamma_q(\alpha_j+\mu_j+\lambda_j)}\right\}x^{\alpha_p-1}\Phi_{C+p:D';\ldots;D^{(n)}}^{A+p:B';\ldots;B^{(n)}}\left(\begin{array}{c}[(a):\theta',\ldots,\theta^{(n)}];\\{}[(c):\psi',\ldots,\psi^{(n)}];\end{array}\right.$$

$$\left.\begin{array}{c}[(\alpha_p+\mu_p):k_1,\ldots,k_n]:[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}]\\{}[(\alpha_p+\mu_p+\lambda_p):k_1,\ldots,k_n]:[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}]\end{array};q,x^{k_1}z_1,\ldots,x^{k_n}z_n\right),$$

and

$$\Omega^{*}\left\{\Phi_{C:D';\ldots;D^{(n)}}^{A:B';\ldots;B^{(n)}}\left(\begin{array}{c}[(a):\theta',\ldots,\theta^{(n)}]:[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}]\\{}[(c):\psi',\ldots,\psi^{(n)}]:[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}]\end{array};q,x^{k_1}z_1,\ldots,x^{k_n}z_n\right)\right\}$$

$$(3.5) \qquad =\prod_{j=1}^{p}\left\{\frac{\Gamma_q(\alpha_j)}{\Gamma_q(\beta_j)}\right\}x^{\beta_p-1}\Phi_{C+p:D';\ldots;D^{(n)}}^{A+p:B';\ldots;B^{(n)}}\left(\begin{array}{c}[(a):\theta',\ldots,\theta^{(n)}];\\{}[(c):\psi',\ldots,\psi^{(n)}];\end{array}\right.$$

$$\left.\begin{array}{c}[(\alpha_p):k_1,\ldots,k_n]:[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}]\\{}[(\beta_p):k_1,\ldots,k_n]:[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}]\end{array};q,x^{k_1}z_1,\ldots,x^{k_n}z_n\right).$$

It is may be noted that for $p=1$ the results (3.4) and (3.5) correspond respectively to the known results due to Yadav, Purohit and Kalla [**13**, p. 60, eqn. (17) and p. 70, eqn. (47)].

Example 2. Next, if we set

$$(3.6) \qquad C(m_1, \ldots, m_n) = \prod_{j=1}^{n} \left\{ \frac{(\gamma_j; q)_{m_j}}{(q; q)_{m_j}} \right\}$$

in (2.1), then

$$(3.7) \qquad f(xz_1, \ldots, xz_n) = \prod_{j=1}^{n} {}_1\Phi_0 \left[\begin{array}{c} \gamma_j \\ - \end{array} ; q, xz_j \right],$$

then the results (2.7) of Corollary 1 and (2.13) of Corollary 2 yield respectively the following operational formulae involving the basic generalized Kampé de Fériet function of n variables (1.8)

$$(3.8) \qquad \Omega \left\{ \prod_{j=1}^{n} {}_1\Phi_0 \left[\begin{array}{c} \gamma_j \\ - \end{array} ; q, xz_j \right] \right\} = \prod_{j=1}^{p} \left\{ \frac{\Gamma_q(\alpha_j + \mu_j)}{\Gamma_q(\alpha_j + \mu_j + \lambda_j)} \right\} x^{\alpha_p - 1}$$

$$\Phi_{p:0;\ldots;0}^{p:1;\ldots;1} \left(\begin{array}{c} (\alpha_p + \mu_p) : \gamma_1; \ldots; \gamma_n \\ (\alpha_p + \mu_p + \lambda_p) : - ; \ldots; - \end{array} ; q, xz_1, \ldots, xz_n \right),$$

and

$$(3.9) \qquad \Omega^* \left\{ \prod_{j=1}^{n} {}_1\Phi_0 \left[\begin{array}{c} \gamma_j \\ - \end{array} ; q, xz_j \right] \right\} = \prod_{j=1}^{p} \left\{ \frac{\Gamma_q(\alpha_j)}{\Gamma_q(\beta_j)} \right\} x^{\beta_p - 1}$$

$$\Phi_{p:0;\ldots;0}^{p:1;\ldots;1} \left(\begin{array}{c} (\alpha_p) : \gamma_1; \ldots; \gamma_n \\ (\beta_p) : -; \ldots; - \end{array} ; q, xz_1, \ldots, xz_n \right).$$

Further, if we put $p = 1$ in (3.9) and replace α_1 and β_1 by $1 + k$ and $1 + k + \alpha$, respectively, then we are led to the result

(3.10)
$$I_q^\alpha \left\{ x^k \prod_{j=1}^n {}_1\Phi_0 \left[\begin{array}{c} \gamma_j \\ - \end{array} ; q, xz_j \right] \right\} = \frac{\Gamma_q(1+k)}{\Gamma_q(1+k+\alpha)} x^{k+\alpha}$$
$$\Phi_D^{(n)} \left[1 + k : \gamma_1; \ldots; \gamma_n; 1 + k + \alpha; \; q, xz_1, \ldots, xz_n \right],$$

provided that $\mathrm{Re}(\alpha) > 0$, $q < 1$ and $\max \{|xz_1|, \ldots, |xz_n|\} < 1$, where the function $\Phi_D^{(n)}(\cdot)$ denotes the basic Lauricella function defined by

(3.11)
$$\Phi_D^{(n)} \left[a, b_1, \ldots, b_n; c; q; z_1, \ldots, z_n \right]$$
$$= \sum_{m_1,\ldots,m_n \geq 0} \frac{(a; q)_{m_1+\ldots+m_n}}{(c; q)_{m_1+\ldots+m_n}} \prod_{j=1}^n \left\{ \frac{(b_j; q)_{m_j} z_j^{m_j}}{(q; q)_{m_j}} \right\},$$

where for convergence $|z_1| < 1, \ldots, |z_n| < 1, |q| < 1$. The result (3.10) is the q-extension of the known result due to Srivastava and Goyal [8, p. 649, eqn. (3.6)] (see also Saigo and Raina [6]).

Example 3. Finally, if we set

(3.12)
$$C(m_1, \ldots, m_n) = \frac{(\alpha; q)_{M_1}}{(\mu; q)_{M_1}} \prod_{j=1}^n \left\{ \frac{(\sigma_j; q)_{m_j}}{(q; q)_{m_j}} \right\},$$

where as before M_1 is given by (2.8), then (2.7) of Corollary 1 and (2.13) of Corollary 2 yield, respectively the following operational formulae involving the basic Lauricella function $\Phi_D^{(n)}(\cdot)$ defined

by (3.11) and the basic Kampé de Fériet function of n variables defined by (1.8)

$$
\Omega\left\{\Phi_D^{(n)}\left[\alpha, \sigma_1, \ldots, \sigma_n; \mu; q; xz_1, \ldots, xz_n\right]\right\} = \prod_{j=1}^{p}\left\{\frac{\Gamma_q(\alpha_j + \mu_j)}{\Gamma_q(\alpha_j + \mu_j + \lambda_j)}\right\} x^{\alpha_p - 1}
$$

(3.13)

$$
\Phi_{p+1:0;\ldots;0}^{p+1:1;\ldots;1}\left(\begin{array}{c} (\alpha_p + \mu_p), \alpha : \sigma_1; \ldots; \sigma_n \\ (\alpha_p + \mu_p + \lambda_p), \mu : -; \ldots; - \end{array} ; q, xz_1, \ldots, xz_n\right)
$$

and

$$
\Omega^*\left\{\Phi_D^{(n)}\left[\alpha, \sigma_1, \ldots, \sigma_n; \mu; q; xz_1, \ldots, xz_n\right]\right\} = \prod_{j=1}^{p}\left\{\frac{\Gamma_q(\alpha_j)}{\Gamma_q(\beta_j)}\right\} x^{\beta_p - 1}
$$

(3.14)

$$
\Phi_{p+1:0;\ldots;0}^{p+1:1;\ldots;1}\left(\begin{array}{c} (\alpha_p), \alpha : \sigma_1; \ldots; \sigma_n \\ (\beta_p), \mu : - ; \ldots; - \end{array} ; q, xz_1, \ldots, xz_n\right),
$$

provided that both sides of (3.13) and (3.14) exist.

We conclude by the remark that the results established in this paper are in general forms and one can deduce several operational formulae involving the Riemann-Liouville and Kober type fractional q-integral operators associated with the basic Lauricella functions, basic Kampé de Fériet function, basic Appell functions, basic Horn's functions and basic confluent hypergeometric functions.

1. Agarwal R. P., *Certain fractional q-integrals and q-derivatives,* Proc. Camb. Phil. Soc., **66** (1969), 365–370.
2. Al-Salam W. A., *Some fractional q-integrals and q-derivatives*, Proc. Edin. Math. Soc., **15** (1966), 135–140.
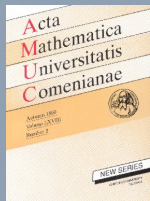3. Gasper G. and Rahman M., *Basic Hypergeometric Series*. Cambridge University Press, Cambridge, 1990.

4. Kalla S. L., Yadav R. K. and Purohit S. D., *On the Riemann-Liouville fractional q-integral operator involving a basic analogue of Fox H-function*, J. Frac. Cal. & Appl. Anal., **8(3)** (2005), 313–322.

5. Raina R. K., *On certain formulas for the multivariable hypergeometric functions*, Acta Math. Univ. Comeniance, **64(1)** (1995), 47–56.

6. Saigo M. and Raina R. K., *On the fractional calculus operator involving Gauss's series and its application to certain statistical distributions*, Rev. Tec. Ing. Univ. Zulia, **14(1)**(1991), 53–62.

7. Slater L. J., *Generalized Hypergeometric Functions*. Cambridge University Press, Cambridge, London and New York, 1966.

8. Srivastava H. M. and Goyal S. P., *Fractional derivatives of the H-function of several variables*, J. Math. Anal. Appl., **112** (1985), 641–651.

9. Srivastava H. M. and Karlsson P. W., *Multiple Gaussian Hypergeometric Series*. John Wiley and Sons (Halsted), New York, 1985.

10. Yadav R. K. and Purohit S. D., *Fractional q-derivatives and certain basic hypergeometric transformations*, South East Asian J. Math. Math. Sci., **2(2)** (2004), 37–46.

11. ———, *On fractional q-derivatives and transformations of the generalized basic hypergeometric functions*, J. Indian Acad. Math., **28(2)**(2006), 321–326.

12. ———, *On applications of Kober fractional q-integral operator to certain basic hypergeometric functions*, J. Rajasthan Acad. Phy. Sci., **5(4)** (2006), 437–448.

13. Yadav R. K., Purohit S. D. and Kalla S. L., *Kober fractional q-integral of multiple basic hypergeometric functions*, Algebras, Groups and Geom., **24(1)** (2007), 55–74.

S. D. Purohit, Department of Basic-Sciences (Mathematics), College of Technology and Engineering, M.P. University of Agriculture and Technology, Udaipur-313001, India.,
e-mail: `sunil_a_purohit@yahoo.com`

R. K. Raina, 10/11, Ganpati Vihar, Opposite Sector-5, Udaipur-313001, India.,
e-mail: `rkraina_7@hotmail.com`