

SOME GEOMETRIC PROPERTIES OF A GENERALIZED CESÀRO SEQUENCE SPACE

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ABSTRACT. In this paper we define the generalized Cesàro sequence space $Ces_p(q)$ and exhibit some geometrical properties of the space. The results herein proved are exhibit analogous to those by Y. A. Cui [Southeast Asian Bull. Math. **24** (2000), 201–210] for the Cesàro sequence space Ces_p .

1. INTRODUCTION

Let $(X, \|\cdot\|)$ be a real Banach space. By $B(X)$ and $S(X)$, we denote the closed unit ball and the unit sphere of X , respectively. For any subset A of X , χ_A represents a characteristics function of A .

A norm $\|\cdot\|$ is called uniformly convex (UC) (cf. [2]) if, for each $\varepsilon > 0$, there exists $\delta > 0$ such that, for $x, y \in S(X)$, $\|x - y\| > \varepsilon$ implies

$$(1.0.1) \quad \left\| \frac{1}{2}(x + y) \right\| < 1 - \delta.$$

A Banach space X is said to have the *Banach-Saks property* if every bounded sequence (x_n) in X admits a subsequence $\{z_n\}$ such that the sequence $\{t_k(z)\}$ is convergent in norm in X (see [1]),

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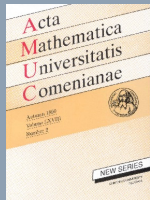


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where

$$t_k(z) = \frac{1}{k}(z_1 + z_2 + \cdots + z_k).$$

Every Banach space X with the Banach-Saks property is reflexive, but the converse is not true (see [4, 5]). Kakutani [6] proved that any uniformly convex Banach space X has the Banach-Saks property. Moreover, he also proved that if X is a reflexive Banach space and $\theta \in (0, 2)$ such that for every sequence (x_n) in $S(X)$ weakly convergent to zero, there exist $n_1, n_2 \in \mathbb{N}$ satisfying the Banach-Saks property.

For a sequence $(x_n) \subset X$, we define

$$A(x_n) = \lim_{n \rightarrow \infty} \inf \{ \|x_i + x_j\| : i, j \geq n, i \neq j \}.$$

In [6], the following new geometric constant connected with the packing constant (see [7]) and with the Banach-Saks property was defined

$$C(X) = \sup \{ A(x_n) : (x_n) \text{ is a weakly null sequence in } S(X) \}.$$

Recall that a sequence (x_n) is said to be an ε -separated sequence if, for some $\varepsilon > 0$,

$$\text{sep}(x_n) = \inf \{ \|x_n - x_m\| : n \neq m \} > \varepsilon.$$

A Banach space X is said to satisfy *property* (β) if and only if, for every $\varepsilon > 0$, there exists $\delta > 0$ such that, for each element $x \in B(X)$ and each sequence (x_n) in $B(X)$ with $\text{sep}(x_n) \geq \varepsilon$, there is an index k such that

$$\left\| \frac{x + x_k}{2} \right\| \geq 1 - \delta, \quad \text{for some } k \in \mathbb{N}.$$

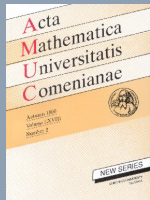


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In this paper, we define the generalized Cesàro sequence space as follows: let $p \in [1, \infty)$ and q be a bounded sequence of positive real numbers such that

$$Q_n = \sum_{k=0}^n q_k, \quad (n \in \mathbb{N}),$$
$$\text{Ces}_p(q) = \left\{ x = (x(i)) : \|x\| = \left(\sum_{n=1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i x(i)| \right)^p \right)^{1/p} < \infty \right\}.$$

If $q_i = 1$ for all $i \in \mathbb{N}$, then $\text{Ces}_p(q)$ is reduced to Ces_p (cf. [3, 8, 9]).

Lemma 1.1. *Let $x, y \in \text{Ces}_p(q)$. Then for any $\varepsilon > 0$ and $L > 0$, $\exists \delta > 0$ such that*

$$| \|x + y\|^p - \|x\|^p | < \varepsilon$$

whenever

$$\|x\|^p \leq L \quad \text{and} \quad \|y\|^p \leq \delta.$$

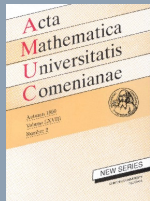


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Proof. For any fix $\varepsilon > 0$ and $L > 0$, take $\beta = 2^{-p}L^{-1}\varepsilon$ and $\delta = 2^{-p}\beta^{p-1}\varepsilon$. Then for any $x, y \in \text{Ces}_p(q)$ with $\|x\|^p \leq L$ and $\|y\|^p \leq \delta$, we have

$$\begin{aligned}
 \|x + y\|^p &= \sum_{n=1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i x(i) + q_i y(i)| \right)^p \\
 &= \sum_{n=1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n \left| (1 - \beta) q_i x(i) + \beta \left(q_i x(i) + \frac{q_i y(i)}{\beta} \right) \right| \right)^p \\
 &\leq \sum_{n=1}^{\infty} \left((1 - \beta) \frac{1}{Q_n} \sum_{i=1}^n |q_i x(i)| + \beta \frac{1}{Q_n} \sum_{i=1}^n \left| q_i x(i) + \frac{q_i y(i)}{\beta} \right| \right)^p \\
 &\leq (1 - \beta) \sum_{n=1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i x(i)| \right)^p + \beta \sum_{n=1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n \left| q_i x(i) + \frac{q_i y(i)}{\beta} \right| \right)^p \\
 &\leq \sum_{n=1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i x(i)| \right)^p + \frac{\beta}{2} \sum_{n=1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n |2q_i x(i)| \right)^p \\
 &\quad + \frac{\beta}{2} \sum_{n=1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n \left| \frac{2q_i y(i)}{\beta} \right| \right)^p \\
 &\leq \|x\|^p + \varepsilon/2 + (2/\beta)^{p-1} \sum_{n=1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i y(i)| \right)^p \\
 &\leq \|x\|^p + \varepsilon.
 \end{aligned}$$

□

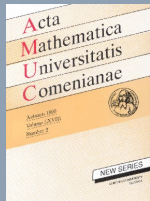


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2. MAIN RESULTS.

Theorem 2.1. *The space $Ces_p(q)$ satisfies the property (β) .*

Proof. Let $Ces_p(q)$ not have property (β) . Then there exists $\varepsilon_0 > 0$ such that, for any $\delta \in (0, \varepsilon_0/(1 + 2^{1+p}))$, there is a sequence $(x_n) \subset S(Ces_p(q))$ with $\text{sep}(x_n) > \varepsilon_0^{1/p}$ and an element $x_0 \in S(Ces_p(q))$ such that

$$\left\| \frac{x_n + x_0}{2} \right\|^p > 1 - \delta \quad \text{for any } n \in \mathbb{N}.$$

Fix $\delta \in (0, \varepsilon_0/(1 + 2^{1+p}))$. We want to show that

$$(2.1.1) \quad \limsup_{j \rightarrow \infty} \sup_k \sum_{n=j+1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i x_k(i)| \right)^p \leq \frac{2^{1+p} \delta}{2^p - 1}.$$

Otherwise, without loss of generality, we can assume that there exists a sequence (j_k) such that $j_k \rightarrow \infty$ as $k \rightarrow \infty$ and

$$(2.1.2) \quad \sum_{n=j_k+1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i x_k(i)| \right)^p > \frac{2^{1+p} \delta}{2^p - 1} \quad \text{for every } k \in \mathbb{N}.$$

Let $\delta > 0$ be a real number corresponding to $\varepsilon = \delta$ and $L = 1$ in Lemma 1. By absolute continuity of the norm of x_0 , there exists a positive integer n_1 such that

$$\|x_0 \chi_{n_1, n_1+1, n_1+2, \dots}\|^p = \sum_{n=n_1+1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i x_0(i)| \right)^p < \delta_1.$$



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Choose k so large that $j_k > n_1$. By the Lemma 1 and (2.1.2), we have

$$\begin{aligned}
 1 - \delta &< \sum_{n=1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n \left| \frac{q_i x_k(i) + q_i x_0(i)}{2} \right| \right)^p \\
 &= \sum_{n=1}^{n_1} \left(\frac{1}{Q_n} \sum_{i=1}^n \left| \frac{q_i x_k(i) + q_i x_0(i)}{2} \right| \right)^p + \sum_{n=n_1+1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n \left| \frac{q_i x_k(i) + q_i x_0(i)}{2} \right| \right)^p \\
 &\leq \frac{1}{2} \sum_{n=1}^{n_1} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i x_0(i)| \right)^p + \frac{1}{2} \sum_{n=1}^{n_1} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i x_k(i)| \right)^p \\
 &\quad + \sum_{n=n_1+1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n \left| \frac{q_i x_k(i)}{2} \right| \right)^p + \delta \\
 &\leq \frac{1}{2} + \frac{1}{2} \sum_{n=1}^{n_1} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i x_k(i)| \right)^p + \frac{1}{2^p} \sum_{n=n_1+1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i x_k(i)| \right)^p + \delta \\
 &\leq \frac{1}{2} + \frac{1}{2} \sum_{n=1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i x_k(i)| \right)^p - \frac{2^p - 1}{2^p} \sum_{n=n_1+1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i x_k(i)| \right)^p + \delta \\
 &< 1 - 2\delta + \delta = 1 - \delta.
 \end{aligned}$$

That is a contradiction. Hence (2.1.1) must hold. Since

$$\left(\frac{1}{Q_{n_1}} \sum_{i=1}^{n_1} |q_i x_k(i)| \right)^p \leq \sum_{n=1}^{n_1} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i x_k(i)| \right)^p \leq 1,$$

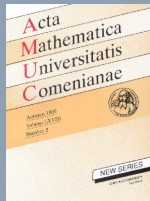


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we have $|q_i x_k(i)| \leq Q_{n_1}$ for $k \in \mathbb{N}$ and $i = 1, 2, \dots, n_1$. Hence there is a subsequence (z_n) of (x_n) and a sequence (a_n) of real numbers such that

$$\lim_{k \rightarrow \infty} q_i z_k(i) = a_i, \quad \text{for } i = 1, 2, \dots, n_1.$$

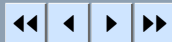
Therefore,

$$\sum_{n=1}^{n_1} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i z_k(i) - q_i z_m(i)| \right)^p < \delta \text{ for } n, m \text{ sufficiently large.}$$

Consequently,

$$\begin{aligned} & \|z_k - z_m\|^p \\ &= \sum_{n=1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i z_k(i) - q_i z_m(i)| \right)^p \\ &= \sum_{n=1}^{n_1} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i z_k(i) - q_i z_m(i)| \right)^p + \sum_{n=n_1+1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i z_k(i) - q_i z_m(i)| \right)^p \\ &\leq \sum_{n=1}^{n_1} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i z_k(i) - q_i z_m(i)| \right)^p \\ &\quad + 2^p \left(\sum_{n=n_1+1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i z_k(i)| \right)^p + \left(\sum_{n=n_1+1}^{\infty} \left(\frac{1}{Q_n} \sum_{i=1}^n |q_i z_m(i)| \right)^p \right) \right) \\ &\leq \delta + 2^{p+1} \delta < \varepsilon_0, \end{aligned}$$

i.e., $\text{sep}(x_n) \leq \text{sep}(z_n) < \varepsilon_0^{1/p}$. This is a contradiction. Therefore $\text{Ces}_p(q)$ must satisfy the property (β) . \square

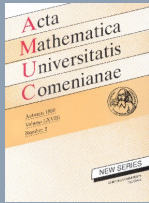


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Theorem 2.2. $C(\text{Ces}_p(q)) = 2^{1/p}$.

Proof. Let

$$K = \sup\{A(u_n) : u_n = \sum_{i=i_{n-1}+1}^{i_n} u_n(i)e_i \in S(\text{Ces}_p(q)), \\ 0 = i_0 < i_1 < i_2 < \dots, u_n \xrightarrow{\omega} 0, .\}$$

Then $C(\text{Ces}_p(q)) \geq K$. Moreover, for any $\varepsilon > 0$, there is a sequence $(x_n) \subset S(\text{Ces}_p(q))$ with $x_n \xrightarrow{\omega} 0$ such that

$$A(x_n) + \varepsilon > C(\text{Ces}_p(q)).$$

By the definition of $A(x_n)$, there exists a subsequence (y_n) of (x_n) such that

$$(2.2.1) \quad \|y_n + y_m\| + 2\varepsilon > C(\text{Ces}_p(q))$$

for any $n, m \in \mathbb{N}$ with $m \neq n$. Take $v_1 = y_1$. Then, by the absolute continuity of the norm of y_1 , there exists $i_1 \in \mathbb{N}$ such that

$$\left\| \sum_{i=i_1+1}^{\infty} v_1(i)e_i \right\| < \varepsilon.$$

Putting $z_1 = \sum_{i=1}^{i_1} v_1(i)e_i$, we have

$$\|z_1 + y_m\| = \|y_1 + y_m - \sum_{i=i_1+1}^{\infty} v_1(i)e_i\| \geq \|y_1 + y_m\| - \varepsilon \quad \text{for any } m > 1.$$

Hence by (2.2.1), we have

$$\|z_1 + y_m\| + 3\varepsilon > C(\text{Ces}_p(q)) \quad \text{for any } m > 1.$$

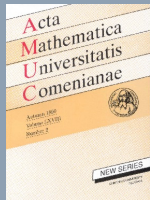


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Since $y_n(i) \rightarrow 0$ for $i = 1, 2, \dots$, there exists $n_2 \in \mathbb{N}$ with $n_2 > n_1$ such that

$$\left\| \sum_{i=1}^{i_1} y_n(i)e_i \right\| < \varepsilon \quad \text{whenever } n \geq n_2.$$

Define $v_2 = y_{n_2}$. Then there is $i_2 > i_1$ such that

$$\left\| \sum_{i=i_2+1}^{\infty} v_2(i)e_i \right\| < \varepsilon.$$

Taking $z_2 = \sum_{i=i_1+1}^{i_2} v_2(i)e_i$, we obtain

$$\begin{aligned} \|z_1 + z_2\| &= \left\| y_1 - \sum_{i=i_1+1}^{\infty} v_1(i)e_i + y_{n_2} - \sum_{i=1}^{i_1} v_2(i)e_i - \sum_{i=i_2+1}^{\infty} v_2(i)e_i \right\| \\ &\geq \|y_1 + y_{n_2}\| - 3\varepsilon. \end{aligned}$$

Hence and by (2.2.1), we immediately obtain

$$\|z_1 + z_2\| + 5\varepsilon > C(\text{Ces}_p(q)).$$

Suppose that increasing sequences $(i_j)_{j=1}^{k-1}$, $(n_j)_{j=1}^{k-1}$ of natural numbers and a sequence $(z_j)_{j=1}^{k-1}$ of elements of $\text{Ces}_p(q)$ are already defined and

$$\|z_n + z_m\| + 6\varepsilon > C(\text{Ces}_p(q)) \quad \text{for } m, n \in \{1, 2, \dots, k-1\}, \quad m \neq n.$$

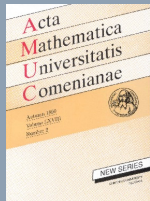


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Since $y_n(i) \rightarrow 0$ for $i = 1, 2, \dots$, there exists a natural number $n_k > n_{k-1}$ such that

$$\left\| \sum_{i=1}^{i_{k-1}} y_n(i) e_i \right\| < \varepsilon$$

provided $n \geq n_k$. Put $v = y_{n_k}$. Then there is $i_k > i_{k-1}$ such that

$$\left\| \sum_{i=i_k+1}^{\infty} v_k(i) e_i \right\| < \varepsilon.$$

Defining $z_k = \sum_{i=i_{k-1}+1}^{i_k} v_k(i) e_i$, we obtain

$$\begin{aligned} \|z_j + z_k\| &= \left\| y_{n_j} - \sum_{i=1}^{i_{j-1}} v_j(i) e_i - \sum_{i=j+1}^{\infty} v_j(i) e_i + y_{n_k} - \sum_{i=1}^{i_{k-1}} v_k(i) e_i - \sum_{i=k+1}^{\infty} v_k(i) e_i \right\| \\ &\geq \|y_{n_j} + y_{n_k}\| - 4\varepsilon \quad \text{for } j = 1, 2, \dots, k-1. \end{aligned}$$

Hence, by (2.2.1), we obtain

$$\|z_j + z_k\| + 6\varepsilon > C(\text{Ces}_p(q)) \quad \text{for } j = 1, 2, \dots, k-1.$$

Using the induction principle, we can find a sequence (z_n) satisfying the following conditions:

- (1) $z_n = \sum_{i=i_{n-1}+1} v_n(i) e_i$, where $0 = i_0 < i_1 < i_2 < \dots$;
- (2) $\|z_n + z_m\| + 6\varepsilon > C(\text{Ces}_p(q))$ for $m, n, \in \mathbb{N}$, $m \neq n$;
- (3) $\|z_n\| \leq 1$ for $n = 1, 2, \dots$;

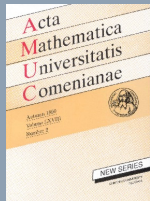


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$$(4) z_n \xrightarrow{\omega} 0 \text{ as } n \rightarrow \infty.$$

Define $u_n = z_n / \|z_n\|$ for each $n \in \mathbb{N}$. Then every $u_n \in S(\text{Ces}_p(q))$ and

$$\|u_n + u_m\| = \left\| \frac{z_n}{\|z_n\|} + \frac{z_m}{\|z_m\|} \right\| \geq \|z_n + z_m\| \geq C(\text{Ces}_p(q)) - 6\varepsilon$$

for any $m, n \in \mathbb{N}$, $m \neq n$. By the arbitrariness of ε , we have $C(\text{Ces}_p(q)) = K$.

Let $\varepsilon > 0$ be given. Take $n_\varepsilon \in \mathbb{N}$ such that

$$\sum_{k=i_{n_\varepsilon}+1}^{\infty} \left(\frac{a}{Q_k}\right)^p < \varepsilon,$$

where

$$a = \sum_{i=i_{n_\varepsilon}-1+1}^{i_{n_\varepsilon}} |q_i u_{n_\varepsilon}(i)|.$$

Hence for any $m > n_\varepsilon$, we have

$$\begin{aligned} \|u_{n_\varepsilon} + u_m\|^p &= \sum_{i=i_{n_\varepsilon}-1+1}^{i_{m-1}} \left(\frac{1}{Q_k} \sum_{i=1}^k |q_i u_{n_\varepsilon}(i)|\right)^p + \sum_{k=i_{m-1}+1}^{\infty} \left(\frac{1}{Q_k} \left(a + \sum_{i=1}^k |q_i u_m(i)|\right)\right)^p \\ &\geq \sum_{i=i_{n_\varepsilon}-1+1}^{i_{m-1}} \left(\frac{1}{Q_k} \sum_{i=1}^k |q_i u_{n_\varepsilon}(i)|\right)^p + \sum_{k=i_{m-1}+1}^{\infty} \left(\frac{1}{Q_k} \left(\sum_{i=1}^k |q_i u_m(i)|\right)\right)^p \\ &= \sum_{i=i_{n_\varepsilon}-1+1}^{\infty} \left(\frac{1}{Q_k} \sum_{i=1}^k |q_i u_{n_\varepsilon}(i)|\right)^p - \sum_{k=i_{m-1}}^{\infty} \left(\frac{a}{Q_k}\right)^p + \sum_{k=i_{m-1}+1}^{\infty} \left(\frac{1}{Q_k} \left(\sum_{i=1}^k |q_i u_m(i)|\right)\right)^p \\ &> 1 - \varepsilon + 1 = 2 - \varepsilon, \end{aligned}$$

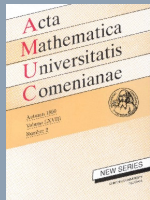


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i. e. $A(u_n) \geq (2 - \varepsilon) \geq (2 - \varepsilon)^{1/p}$.

On the other hand, for ε mentioned above, by Lemma 1.1, there exists $\delta > 0$ such that

$$\| \|x + y\|^p - \|x\|^p \| < \varepsilon$$

whenever $\|x\|^p \leq 1$ and $\|y\|^p < \delta$. Take $n_\delta \in \mathbb{N}$ such that

$$\sum_{k=i_{n_\delta}+1}^{\infty} \left(\frac{a}{Q_k}\right)^p < \delta, \quad \text{and} \quad a = \sum_{i=i_{n_\delta}+1}^{i_{n_\delta}} |q_i u_{n_\delta}(i)|.$$

Hence for any $m > n_\delta$, we have

$$\begin{aligned} & \|u_{n_\delta} + u_m\|^p \\ &= \sum_{i=i_{n_\delta}+1}^{i_{m-1}} \left(\frac{1}{Q_k} \sum_{i=1}^k |q_i u_{n_\delta}(i)|\right)^p + \sum_{k=i_{m-1}+1}^{\infty} \left(\frac{1}{Q_k} \left(a + \sum_{i=1}^k |q_i u_m(i)|\right)\right)^p \\ &\leq \sum_{i=i_{n_\delta}+1}^{\infty} \left(\frac{1}{Q_k} \sum_{i=1}^k |q_i u_{n_\delta}(i)|\right)^p + \sum_{k=i_{m-1}+1}^{\infty} \left(\frac{1}{Q_k} \left(a + \sum_{i=1}^k |q_i u_m(i)|\right)\right)^p \\ &= \|u_{n_\delta}(i)\|^p + \sum_{k=i_{m-1}+1}^{\infty} \left(\frac{1}{Q_k} \left(a + \sum_{i=1}^k |q_i u_m(i)|\right)\right)^p \\ &\quad - \sum_{k=1}^{\infty} \left(\frac{1}{Q_k} \sum_{i=1}^k |q_i u_m(i)|\right)^p + \|u_m\|^p \\ &< 2 + \varepsilon, \end{aligned}$$

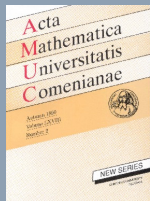


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i. e. $A((u_n)) \leq (2 + \varepsilon)^{1/p}$.

Since ε was arbitrary, we obtain $C(\text{Ces}_p(q)) = 2^{1/p}$. □

Corollary 2.1. *The space $\text{Ces}_p(q)$ satisfies the Banach-Saks property .*

Proof. The proof follows immediately from the above Theorem 1 of [3] and Theorem 2.2. □

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