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POSITIVE PERIODIC SOLUTIONS OF IMPULSIVE FUNCTIONAL DIFFERENTIAL EQUATIONS WITH A PARAMETER

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Abstract. By using a fixed point theorem of strict-set-contraction, some criteria are established for the existence of positive periodic solutions of the following impulsive functional differential equations with a parameter

$$\begin{cases} \dot{x}(t) = -a(t)f(t,x(t))x(t) + \lambda g(t,x_t,x(t-\tau(t,x(t)))), & t \in \mathbb{R}, \text{ and } t \neq t_k, \\ x(t_k^+) - x(t_k^-) = I_k(t_k, x(t_k - \tau(t_k, x(t_k)))), & k \in \mathbb{Z}. \end{cases}$$

1. Introduction

The theory and applications of impulsive functional differential equations are emerging as an important area of investigation, since it is far richer than the corresponding theory of nonimpulsive functional differential equations. Various population models characterized by the fact that sudden change of their state and process under such as population dynamics, ecology and epidemic, etc. depending on their prehistory at each moment of time can be expressed by impulsive differential equations with deviating argument. We note that the difficulties dealing with such models are that corresponding equations have deviating arguments and theirs states are discontinuous. In [**2**], Cushing pointed out that it is necessary and important to consider the models with the parameters or perturbations. This might be quite naturally exposed, for example, for such processes changing due to seasonal effects of weather, food supply, mating habit, etc.

Very recently Yan [**11**] employed a well-known fixed-point index theorem to study the existence of positive periodic solutions for the periodic impulsive functional differential equation with two parameters

$$(1.1) \qquad \begin{cases} y'(t) = h(t,y(t)) - \lambda f(t, y(t-\tau(t))), & t \in \mathbb{R},\ t \neq t_k, \\ y(t_k^+) - y(t_k) = \mu I_k(t_k, y(t_k - \tau(t_k))), & k \in \mathbb{Z}. \end{cases}$$

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By using the fixed point theorem in cones, Wu [**10**] discussed the existence of positive periodic solutions for the functional differential equation with a parameter

$$\dot{y}(t) = -a(t)f(y(t))y(t) + \lambda g(t, y(t-\tau(t))), \tag{1.2}$$

where $a(t) \in C(\mathbb{R}, [0,\infty))$, $f(\cdot) \in C([0,\infty), [0,\infty))$, $\tau(t) \in C(\mathbb{R}, \mathbb{R})$, $y \in C(\mathbb{R} \times [0,\infty), [0,\infty))$, $\mathbb{R} = (-\infty, +\infty)$, $\lambda > 0$ is a parameter; $a(t)$, $\tau(t)$, $g(t,\cdot)$ are all ω-periodic functions in t and $\omega > 0$ is a constant.

Li [**5**] employed a fixed point theorem of strict-set-contraction to study the existence of positive periodic solutions of the following periodic neutral Lotka-Volterra system with state dependent delays

$$\begin{aligned}\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = x_i(t)\Bigg[& r_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t) - \sum_{j=1}^{n} b_{ij}(t)x_j(t-\tau_{ij}(t, x_1(t),\dots,x_n(t))) \\ & - \sum_{j=1}^{n} c_{ij}(t)x_j^{'}(t-\sigma_{ij}(t, x_1(t),\dots,x_n(t)))\Bigg].\end{aligned} \tag{1.3}$$

For some other relative works see [**4**]–[**13**] and references cited therein.

In this paper, mainly motivated by [**5, 10, 11**], we use a fixed point theorem of strict-set-contraction to investigate the existence of positive periodic solutions for the impulsive functional differential equation with a parameter

$$\begin{cases} \dot{x}(t) = -a(t)f(t, x(t))x(t) + \lambda g(t, x_t, x(t-\tau(t, x(t)))), \\ \qquad\qquad t \in \mathbb{R}, \text{ and } t \neq t_k, \\ x(t_k^+) - x(t_k^-) = I_k(t_k, x(t_k - \tau(t_k, x(t_k)))), \\ \qquad\qquad k \in \mathbb{Z}, \end{cases} \tag{1.4}$$

where $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}^+ = [0, +\infty)$, $a(t) \in C(\mathbb{R}, \mathbb{R}^+)$ is ω-periodic, $\tau(\cdot,\cdot) \in C(\mathbb{R} \times \mathbb{R}^+, \mathbb{R})$ satisfies $\tau(t+\omega, y)$ for all $t \in \mathbb{R}$, $y \in \mathbb{R}$, $\lambda > 0$ is a parameter and $\omega > 0$ is a constant. $f(\cdot,\cdot) \in C(\mathbb{R} \times \mathbb{R}^+, \mathbb{R}^+)$ satisfies $f(t+\omega, y) = f(t, y)$, $g \in C(\mathbb{R} \times BC^+ \times \mathbb{R}^+, \mathbb{R}^+)$ satisfies $g(t+\omega, x_{t+\omega}, y) = g(t, x_t, y)$ for all $t \in \mathbb{R}$, $x \in BC^+$, $y \in \mathbb{R}^+$, where $BC^+ = \{\eta \in BC\colon\ \eta(t) \in \mathbb{R}^+ \text{ for } t \in \mathbb{R}\}$, BC denotes the Banach space of bounded continuous functions $\eta\colon \mathbb{R} \to \mathbb{R}$ with the norm $\|\eta\| = \sup_{\theta\in\mathbb{R}} |\eta(\theta)|$. If $x \in BC$, then $x_t \in BC$ for any $t \in \mathbb{R}$ is defined by $x_t(\theta) = x(t+\theta)$ for $\theta \in \mathbb{R}$. $I_k \in C(\mathbb{R} \times \mathbb{R}^+, \mathbb{R}^+)$ and there exists a positive integer p such that $t_{k+p} = t_k + \omega$, $I_{k+p}(t_{k+p}, x) = I_k(t_k, x)$, $k \in \mathbb{Z}$. Without loss of generality we also assume that $t_k \neq 0$, for $k = 1, 2, \dots$, and $[0,\omega) \cap \{t_k\colon k \in Z\} = \{t_1, t_2, \dots, t_p\}$.

For convenience, we introduce the notations

$$\delta := \mathrm{e}^{-\int_0^\omega a(t)\mathrm{d}t}, \qquad \sigma := \frac{\delta^{2L_2}(1-\delta^{L_1})}{\delta^{L_1}(1-\delta^{L_2})}, \qquad a^M = \max_{t\in[0,\omega)}\{a(t)\},$$

$$\begin{aligned} g_q^M &= \sup_{\sigma q \le \|u_t\|, \|v\| \le q} \max_{t \in [0,\omega)} \{g(t, u_t, v)\}, \\ I_q^M &= \sup_{\sigma q \le \|v\| \le q} \max_{t \in [0,\omega)} \left\{ \sum_{k=1}^{p} I(t, v) \right\}, \\ g_q^m &= \inf_{\sigma q \le \|u_t\|, \|v\| \le q} \min_{t \in [0,\omega)} \{g(t, u_t, v)\}, \\ I_q^m &= \sup_{\sigma q \le \|v\| \le q} \inf_{t \in [0,\omega)} \left\{ \sum_{k=1}^{p} I(t, v) \right\}. \end{aligned}$$

In the following, we always assume that:

(H_1) $\int_0^\omega a(t)\mathrm{d}t > 0, \quad 0 < \delta := \mathrm{e}^{-\int_0^\omega a(t)\mathrm{d}t} < 1.$

(H_2) For $t \in \mathbb{R},\ u \in \mathbb{R}^+$, there exist positive constants L_1, L_2 such that $L_1 \le f(t,u) \le L_2$.

(H_3) $0 < \sigma := \dfrac{\delta^{2L_2}(1-\delta^{L_1})}{\delta^{L_1}(1-\delta^{L_2})} < 1.$

(H_4) For all $(t,u,v) \in \mathbb{R} \times BC^+ \times \mathbb{R}^+,\ g(t,u_t,v) \ge 0.$

(H_5) $0 < \lambda < \infty$ is a parameter, $\lambda^* = \sup\{\lambda > 0\}$, there exists a positive constant q such that $q \ge B(\lambda^* \omega g_q^M + I_q^M)$.

2. Preliminaries

In order to obtain the existence of a periodic solution of system (1.4), we first make the following preparations:

Let X be a real Banach space and K a closed, nonempty subset of X. Then K is a cone provided

(i) $\alpha u + \beta v \in K$ for all $u, v \in K$ and all α, $\beta \ge 0$;
(ii) $u, -u \in K$ implies $u = 0$.

Let E be a Banach space and K be a cone in E. The semi-order induced by the cone K is denoted by $\le$. That is, $x \le y$ if and only if $y - x \in K$. In addition, for a bounded subset $A \subset E$, let $\alpha_E(A)$ denote the (Kuratowski) measure of non-compactness defined by

$$\alpha_E(A) = \inf\{\gamma > 0 : \text{there is a finite number of subsets } A_i \subset A \text{ such that } A = \bigcup_i A_i \text{ and } \operatorname{diam}(A_i) \le \gamma\},$$

where $\operatorname{diam}(A_i)$ denotes the diameter of the set A_i.

Let E, F be two Banach spaces and $D \subset E$. A continuous and bounded map $\Phi\colon D \to F$ is called k-set contractive if for any bounded set $S \subset D$, we have

$$\alpha_F(\Phi(S)) \le k\alpha_E(S).$$

Φ is called strict-set-contractive if it is k-set-contractive for some $0 \le k < 1$.

The following lemma is useful for the proof of our main results of this paper.

Lemma 2.1 ([**1, 3, 5**]). *Let K be a cone in the real Banach space X and $K_{r,R} = \{x \in K\colon\ r \le\| x \|\le R\}$ with $R > r > 0$. Suppose that $\Phi\colon K_{r,R} \to K$ is strict-set-contractive such that one of the following two conditions is satisfied:*

(i) $\Phi x \nleq x, \quad \forall x \in K, \|x\| = r$ *and* $\Phi x \ngeq x, \forall x \in K, \quad \|x\| = R.$

(ii) $\Phi x \ngeq x, \quad \forall x \in K, \|x\| = r$ *and* $\Phi x \nleq x, \forall x \in K, \quad \|x\| = R.$

Then Φ has at least one fixed point in $K_{r,R}$.

In order to apply Lemma 2.1 to system (1.4), we set

$$\begin{aligned} PC(R) = \{x(t)\colon \mathbb{R} \to \mathbb{R},\ \ x|_{(t_k,\, t_{k+1})} \in C(t_k, t_{k+1}),\\ \exists\ x(t_k^-) = x(t_k),\ x(t_k^+),\ k \in Z\}. \end{aligned}$$

Consider the Banach space

$$X = \{x(t) : x(t) \in PC(R),\, x(t+\omega) = x(t)\}$$

with the form defined by $\|x\| = \max_{t\in[0,\omega]}\{|x(t)| : x \in X\}$.

Let the map Φ be defined by

$$\begin{aligned} (\Phi x)(t) = \lambda \int_t^{t+\omega} G(t,s) g(s, x_s, x(s-\tau(s,x(s))))\mathrm{d}s \\ + \sum_{k:t_k\in[t,t+\omega)} G(t,t_k) I_k(t_k, x(t-\tau(t_k, x(t_k)))), \end{aligned} \tag{2.1}$$

where $x \in K,\ t \in \mathbb{R}$ and

$$G(t,s) = \frac{\mathrm{e}^{\int_t^s a(u)f(u,x(u))\mathrm{d}u}}{\mathrm{e}^{\int_0^\omega a(u)f(u,x(u))\mathrm{d}u} - 1}.$$

It is easy to see that $G(t+\omega,\, s+\omega) = G(t,s)$.

Define the cone K in X by

$$K = \{x \in X : x(t) \ge \sigma \parallel x \parallel\}, \tag{2.2}$$

where

$$0 < \sigma = A/B < 1$$

and

$$A := \min\{G(t,s) : 0 \le t \le s \le \omega\} = \frac{1}{\delta^{-L_2} - 1} > 0,$$

$$B := \max\{G(t,s) : 0 \le t \le s \le \omega\} = \frac{\delta^{-L_2}}{\delta^{-L_1} - 1} > 0.$$

It is not difficult to verify that K is a cone in X.

In the following, we will give some lemmas concerning K and Φ defined by (2.1) and (2.2), respectively.

Lemma 2.2. *Assume that (H_1), (H_3) hold, then $\Phi\colon K \to K$ is well defined.*

Proof. For any $x \in K$, it is clear that $\Phi x \in PC(R)$. In view of (2.1), for $t \in \mathbb{R}$, we obtain

$$\begin{aligned}
&(\Phi x)(t+\omega)\\
&= \lambda \int_{t+\omega}^{t+2\omega} G(t+\omega, s) g(s, x_s, x(s-\tau(s,x(s)))\mathrm{d}s\\
&\quad + \sum_{k:t_k\in[t+\omega,t+2\omega)} G(t+\omega,t_k) I_k(t_k, x(t_k-\tau(t_k,x(t_k))))\\
&= \lambda \int_{t}^{t+\omega} G(t+\omega, u+\omega) g(u+\omega, x_{u+\omega}, x(u+\omega-\tau(u+\omega, x(u+\omega)))\mathrm{d}u\\
&\quad + \sum_{j:t_j\in[t,t+\omega)} G(t+\omega, t_j+\omega) I_j(t_j+\omega, x(t_j+\omega-\tau(t_j+\omega, x(t_j+\omega)))\\
&= \lambda \int_{t}^{t+\omega} G(t,u) g(u, x_u, x(u-\tau(u,x(u)))\mathrm{d}u\\
&\quad + \sum_{j:t_j\in[t,t+\omega)} G(t,t_j) I_j(t_j, x(t_j-\tau(t_j,x(t_j))))\\
&= (\Phi x)(t)
\end{aligned}$$

That is, $(\Phi x)(t+\omega) = (\Phi x)(t)$, $t \in \mathbb{R}$. So $\Phi x \in X$. For any $x \in K$, we have

$$\|\Phi x| \leq \lambda B \int_0^\omega g(s, x_s, x(s-\tau(s,x(s)))\mathrm{d}s + B\sum_{k=1}^{p} I_k(t_k, x(t_k-\tau(t_k,x(t_k))))$$

and

$$(\Phi x)(t) \geq \quad \lambda A \int_0^\omega g(s, x_s, x(s-\tau(s,x(s)))\mathrm{d}s + A\sum_{k=1}^{p} I_k(t_k, x(t_k-\tau(t_k,x(t_k)))).$$

So we have

$$(\Phi x)(t) \geq \frac{A}{B}\|\Phi x\| = \sigma\|\Phi x\|$$

i.e. $\Phi \in K$. This completes the proof of Lemma 2.2. □

Lemma 2.3. *Assume that (H_1)–(H_5) hold and $g_R^M < \infty$, then $\Phi\colon K \bigcap \overline{\Omega}_R \to K$ is strict-set-contractive, where $\Omega_R = \{x \in X : \|x\| < R\}$.*

Proof. It is easy to see that Φ is continuous and bounded. Now we prove that $\alpha_X(\Phi(S)) \le k\alpha_X(S)$ for any bounded set $S \subset \overline{\Omega}_R$ and $0 < k < 1$.

Let $\eta = \alpha_X(S)$. Then, for any positive number $\epsilon < \eta$, there is a finite family of subsets $\{S_i\}$ satisfying $S = \bigcup_i S_i$ with $\mathrm{diam}(S_i) \le \eta + \varepsilon$. As S and S_i are precompact in X, it follows that there is a finite family of subsets S_{ij} of S_i such that $S_i = \bigcup_j S_{ij}$ and $\|x - y\| \le \varepsilon$ for $x, y \in S_{ij}$.

In addition, from (H$_5$), it follows that there exists a positive constant $\lambda^* = \sup\{\lambda > 0\}$ such that $0 < \lambda \le \lambda^*$ for any $y \in S$ and $t \in [0, \omega]$. We have

$$\begin{aligned}
|(\Phi x)(t)| &= \Bigg|\lambda \int_t^{t+\omega} G(t,s)g(s, x_s, x(s - \tau(s, x(s)))\mathrm{d}s \\
&\quad + \sum_{k:t_k \in [t,t+\omega)} G(t,t_k)I_k(t_k, x(t - \tau(t_k, x(t_k))))\Bigg| \\
&\le \left|\lambda \int_t^{t+\omega} Bg_R^M \mathrm{d}s + BI_R^M\right| \\
&\le B(\lambda\omega g_R^M + I_R^M) \le B(\lambda^*\omega g_R^M + I_R^M) := H
\end{aligned}$$

and

$$\begin{aligned}
|(\Phi x)'(t)| &= |-a(t)f(t, (\Phi x)(t))(\Phi x)(t) + \lambda g(t, x_t, x(t - \tau(t, x(t)))| \\
&\le a^M L_2 H + \lambda g_R^M \le a^M L_2 H + \lambda^* g_R^M.
\end{aligned}$$

Applying the Arzela-Ascoli theorem, we know that $\Phi(S)$ is precompact in $PC(R)$. Then, there is a finite family of subsets $\{S_{ijl}\}$ of S_{ij} such that $Sij = \bigcup_l S_{ijl}$ and $\|(\Phi x) - (\Phi y)\| \le \varepsilon$ for any $x, y \in S_{ijl}$.
As ε is arbitrary small, it follows that

$$\alpha_X(\Phi(S)) \le k\alpha_X(S).$$

Therefore, Φ is strict-set-contractive. The proof of Lemma 2.3 is complete. □

3. Main result

In this section, we state and prove the following result.

Theorem 3.1. *Assume that (H$_1$)–(H$_5$) hold, then there exists $\lambda^* > 0$ such that* (1.4) *has at least a positive ω-periodic solution associated with some $\lambda \in (0, \lambda^*]$.*

Proof. From (H$_5$), it is clear that there exists a positive constant $\lambda^* = \sup\{\lambda > 0\}$ such that $0 < \lambda \le \lambda^*$. Let $R := R_\lambda = B(\lambda\omega g_R^M + I_R^M)$ and $0 < r := r_\lambda < A(\lambda\omega g_R^m + I_R^m)$, where $\lambda \in (0, \lambda^*]$. Obviously, $0 < r_\lambda < R_\lambda$ for the same λ in $(0, \lambda^*]$. From Lemmas 2.2 and 2.3, we know that Φ is strict-set-contractive on $K_{r,R}$. Now, we shall prove that condition (i) of Lemma 2.1 holds.

First, we prove that $\Phi x \nleq x$ for all $x \in K$, $\|x\| = r$. Otherwise, there exists $x \in K$, $\|x\| = r$ such that $\Phi x \leq x$. So $|x| > 0$ and $x - \Phi x \in K$ which implies that

$$x(t) - (\Phi x)(t) \geq \sigma \|x - \Phi x\| \geq 0 \qquad \text{for any } \ t \in [0, \omega] \tag{3.1}$$

Moreover, for $t \in [0, \omega]$, we have

$$\begin{aligned}(\Phi x)(t) &= \lambda \int_t^{t+\omega} G(t,s) g(s, x_s, x(s - \tau(s, x(s))) \mathrm{d}s \\ &\quad + \sum_{k: t_k \in [t, t+\omega)} G(t, t_k) I_k(t_k, x(t - \tau(t_k, x(t_k)))), \\ &> \lambda \int_t^{t+\omega} A g_R^m \mathrm{d}s + A \Sigma_{k=1}^p I_k(t_k, x(t_k - \tau(t_k, x(t_k)))) \\ &\geq A(\lambda \omega g_R^m + I_R^m) > r.\end{aligned} \tag{3.2}$$

In view of (3.1) and (3.2), we have

$$\|x\| \geq \|\Phi x\| > r = \|x\|,$$

which is a contradiction.

Finally, we prove that $\Phi x \ngeq x$ for all $x \in K$, $\|x\| = R$ also holds. In this case, we only need to prove that

$$\Phi x \not> x, \quad x \in K, \quad \|x\| = R.$$

Suppose, for the sake of contradiction, that there exists $x \in K$ and $\|x\| = R$ such that $x < \Phi x$. Thus $\Phi x - x \in K \setminus \{0\}$. Furthermore, for any $t \in [0, \omega]$, we have

$$(\Phi x)(t) - x(t) \geq \sigma \|x - \Phi x\| > 0. \tag{3.3}$$

In addition, for any $t \in [0, \omega]$, we find

$$\begin{aligned}(\Phi x)(t) &= \lambda \int_t^{t+\omega} G(t,s) g(s, x_s, x(s - \tau(s, x(s))) \mathrm{d}s \\ &\quad + \sum_{k: t_k \in [t, t+\omega)} G(t, t_k) I_k(t_k, x(t - \tau(t_k, x(t_k)))), \\ &\leq \lambda \int_t^{t+\omega} B g_R^M \mathrm{d}s + B \Sigma_{k=1}^p I_k(t_k, x(t_k - \tau(t_k, x(t_k)))) \\ &\leq B(\lambda \omega g_R^M + I_R^M) = R.\end{aligned} \tag{3.4}$$

From (3.3) and (3.4), we obtain

$$\|x\| < \|\Phi x\| \leq R = \|x\|,$$

which is a contradiction. Therefore, conditions (i) and (ii) hold. By Lemma 2.1, we see that Φ has at least one nonzero fixed point in K. Therefore, system (1.4) has at least one positive ω-periodic solution associated with some $\lambda \in (0, \lambda^*]$. The proof of Theorem 3.1 is complete. □

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