Acta Universitatis Apulensis No. 76/2023
ISSN: 1582-5329 pp. 89-98
http://www.uab.ro/auajournal/ doi: 10.17114/j.aua.2023.76.07

SUBORDINATION RESULTS ON THE Q-ANALOGUE OF THE FRACTIONAL Q-DIFFERINTEGRAL OPERATOR

Annapoorna S, Dileep L

Abstract. In this article, we presented the aspects related to applications of q-calculus in geometric function theory. The study concerns the investigation of certain q-analouge differential operators in order to obatin their geometrical properties, which could be developed in further studies. Several interesting properties of the q-analouge of the fractional q - differintegral operator are obatined here by using the differential subordination.

2010 *Mathematics Subject Classification*: 30C45, 30C50.

Keywords: Univalent functions,analytic functions,convex functions and Generalized Integral operator.

1. Introduction

The theory of q-calculus operators are used in describing and solving various problems in applied science such as ordinary fractional calculus, optimal control, q-difference and q-integral equations, as well as geometric function theory of complex analysis. The fractional q-calculus is the q-extension of the ordinary fractional calculus and dates back to early 20-th century [8] and [3].

The geometrical interpretation of q-analysis involves studies of different q-analouge differential operators. The q-analouge of the well-know Ruscheweyh differential operator was defined in [9] and following this idea, the q-analouge of Salagean differential operator was defined in [6]. Those operators provided interesting results when they were used to introduce new sets of univalent functions as seen in [10]-[14].

The differential subordination theory initiated by Miller and Mocanu [11] and [12] is introduced to obtain the main results of this article.

Let $\mathcal{A}_n$ be the set of all analytic and univalent fuctions in the open unit disk $\mathcal{U}$ in the form of

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \ a_k \in \mathbb{C} \tag{1}$$

and note that $\mathcal{A}_1 = \mathcal{A}$. The class of starlike functions is defined as

$$\mathcal{S}^* = \left\{ f \in \mathcal{A} : \ Re \frac{zf'(z)}{f(z)} > 0 \right\}.$$

For any two functions f and g such that

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \quad \text{and} \quad g(z) = z + \sum_{k=2}^{\infty} b_k z^k$$

the Hadamard product or convolution of f and g, denoted by $f * g$, is defined by

$$(f * g)z = z + \sum_{k=2}^{\infty} a_k b_k z^k, \quad z \in \mathcal{U}. \tag{2}$$

A linear multiplier fractional q - differintegral operator [4] is defined as

$$\begin{aligned} \mathcal{D}_{q,\lambda}^{\delta,0} f(z) &= f(z) \\ \mathcal{D}_{q,\lambda}^{\delta,1} f(z) &= (1-\lambda)\Omega_q^{\delta} f(z) + \lambda z \mathcal{D}_q \left(\Omega_q^{\delta} f(z) \right) \\ \mathcal{D}_{q,\lambda}^{\delta,2} f(z) &= \mathcal{D}_{q,\lambda}^{\delta,1} \left(\mathcal{D}_{q,\lambda}^{\delta,1} f(z) \right) \\ &\vdots \end{aligned}$$

$$\mathcal{D}_{q,\lambda}^{\delta,n} f(z) = \mathcal{D}_{q,\lambda}^{\delta,1} \left(\mathcal{D}_{q,\lambda}^{\delta,n-1} f(z) \right) \tag{3}$$

We note that if $f \in \mathcal{A}(n)$ is given by (1) then by (3), we have

$$\mathcal{D}_{q,\lambda}^{\delta,n} f(z) = z + \sum_{k=2}^{\infty} B(k, \delta, \lambda, n, q) a_k z^k \tag{4}$$

where

$$B(k, \delta, \lambda, n, q) = \left(\frac{\Gamma_q(2-\delta)\Gamma_q(k+1)}{\Gamma_q(k+1-\delta)} \left[([k]_q - 1)\lambda + 1 \right] \right)^n. \tag{5}$$

Inspired by the results obtained in [1] using q- analouge of Salagean differential operator, in the next section, we obtain results involing q-analouge of fractional q-differintegral operator using the differential subordination theory.

2. Preliminaries

To prove our main results we are using the following lemmas.

Lemma 2.1: **[12]** Let h be an analytic and convex univalent function in $\mathcal{U}$ with $h(0) = 1$ and $g(z) = 1 + b_1z + b_2z^2 + \cdots$, analytic in $\mathcal{U}$.
If, $g(z) + \frac{z\mathcal{D}_q(g(z))}{c} \prec h(z),\ z \in \mathcal{U},\ c \neq 0$, then

$$g(z) \prec \frac{c}{z^c}\int_0^z t^{c-1}h(t)dt,$$

for $\Re(c) \geq 0$.

Lemma 2.2: **[13]** Let u be any univalent function in $\mathcal{U}$ and $\theta,\ \phi$ be analytic functions in a domain $D \supset q(U)$ with $\phi(w) \neq 0$ for $w \in q(U)$. Consider $Q(z) = z\mathcal{D}_q(u(z))\phi(u(z))$ and $h(z) = \theta(Q(z) + u(z))$ supposing that $Q(z)$ is a starlike univalent function in $\mathcal{U}$ and

$$\Re\left(\frac{z\mathcal{D}_qh(z)}{Q(z)}\right) = \Re\left(\frac{\mathcal{D}_q\theta(u(z))}{\phi(Q(z))}\right) + \left(\frac{z\mathcal{D}_qQ(z)}{Q(z)}\right) > 0,\ z \in \mathcal{U}.$$

If $p(z)$ is an analytic function in $\mathcal{U}$ such that $p(U) \subset D,\ p(0) = q(0)$ and

$$z\mathcal{D}_q(p(z))\phi(p(z)) + \theta(p(z)) \prec z\mathcal{D}_q(u(z))\phi(u(z)) + \theta(u(z)) = h(z),$$

then $p \prec u$, and the best dominant is u.

Lemma 2.3: **[15]** The function , $(1 - z)^\gamma = e^{\gamma log(1-z)},\ \gamma \neq 0$, is univalent in $\mathcal{U}$ if and only if $|\gamma - 1| \leq 1$ or $|\gamma + 1| \leq 1$.

Lemma 2.4: **[16]** Consider the analytic functions $f_i \in \mathcal{U}$ of the form $1 + b_1z + b_2z^2 + \cdots$, that satisfies the inequality $\Re(f_i) > \beta_i,\ 0 \leq \beta_i < 1,\ i = 1, 2$. Then $f_1 * f_2$ is an analytic function in $\mathcal{U}$ of the form $1 + b_1z + b_2z^2 + \cdots$ that satisfies the inequality $\Re(f_1 * f_2) > 1 - 2(1 - \beta_1)(1 - \beta_2)$.

Lemma 2.5: **[17]** Consider the analytic functions $f(z) = 1 + b_1z + b_2z^2 + \cdots$, with property $\Re(f(z)) > \beta,\ 0 \leq \beta < 1$. Then

$$\Re(f(z)) > 2\beta - 1 + \frac{2(1-\beta)}{1+|z|},\ z \in \mathcal{U}.$$

3. Prime Results

Theorem 3.1 If $f \in \mathcal{A}$ and

$$(1-\alpha)\frac{\mathcal{D}^{\delta,n}_{q,\lambda}f(z)}{z} + \alpha\frac{\mathcal{D}^{\delta,n+1}_{q,\lambda}f(z)}{z} \prec \frac{1+Az}{1+Bz}, \tag{6}$$

for $\alpha > 0,\ -1 \leq B < A \leq 1,\ z \neq 0$, then

$$\Re\left\{\left(\frac{\mathcal{D}^{\delta,n}_{q,\lambda}f(z)}{z}\right)^{\frac{1}{n}}\right\} > \left(\frac{1}{\alpha q}\int_0^1 u^{\frac{1}{\alpha q}-1}\frac{1-Au}{1-Bu}du\right)^{\frac{1}{n}},\ n \geq 1, \tag{7}$$

and the result is sharp.

Proof: Let $p(z) = \dfrac{\mathcal{D}^{\delta,n}_{q,\lambda}f(z)}{z} = 1 + b_1 z + b_2 z^2 + \cdots$ for $f \in \mathcal{A}$. Applying the logarithmic q-differentiation, we obatin

$$\mathcal{D}_q(p(z)) = \mathcal{D}_q\left\{\frac{\mathcal{D}^{\delta,n}_{q,\lambda}f(z)}{z}\right\} = \frac{\mathcal{D}^{\delta,n+1}_{q,\lambda}f(z) - \mathcal{D}^{\delta,n}_{q,\lambda}f(z)}{qz^2}.$$

Consider

$$\frac{z\mathcal{D}_q(p(z))}{p(z)} = \frac{z^2}{\mathcal{D}^{\delta,n}_{q,\lambda}f(z)}\left\{\frac{\mathcal{D}^{\delta,n+1}_{q,\lambda}f(z) - \mathcal{D}^{\delta,n}_{q,\lambda}f(z)}{qz^2}\right\} = \frac{1}{q}\left[\frac{\mathcal{D}^{\delta,n+1}_{q,\lambda}f(z)}{\mathcal{D}^{\delta,n}_{q,\lambda}f(z)} - 1\right].$$

$$\frac{qz\mathcal{D}_q(p(z))}{p(z)} = \frac{\mathcal{D}^{\delta,n+1}_{q,\lambda}f(z)}{\mathcal{D}^{\delta,n}_{q,\lambda}f(z)} - 1$$

$$qz\mathcal{D}_q(p(z)) + p(z) = \frac{\mathcal{D}^{\delta,n+1}_{q,\lambda}f(z)}{z},$$

and

$$\begin{aligned}(1-\alpha)\frac{\mathcal{D}^{\delta,n}_{q,\lambda}f(z)}{z} + \alpha\frac{\mathcal{D}^{\delta,n+1}_{q,\lambda}f(z)}{z} &= (1-\alpha)p(z) + \alpha\left[qz\mathcal{D}_q(p(z)) + p(z)\right] \\ &= p(z) + \alpha qz\mathcal{D}_q(p(z)).\end{aligned}$$

The differential subordination (6), can be written as,

$$p(z) + \alpha qz\mathcal{D}_q(p(z)) \prec \frac{1+Az}{1+Bz}.$$

Applying Lemma 2.1, we find

$$p(z) \prec \frac{1}{\alpha q}z^{\frac{-1}{\alpha q}}\int_0^z t^{\frac{1}{\alpha q}-1}\frac{1+At}{1+Bt}dt,$$

or by using subordination concept,

$$\frac{\mathcal{D}^{\delta,n}_{q,\lambda}f(z)}{z} = \frac{1}{\alpha q}\int_0^1 u^{\frac{1}{\alpha q}-1}\frac{1+Auw(z)}{1+Buw(z)}du.$$

Taking into account that $-1 \leq B < A \leq 1$, we obatin

$$\Re\left\{\frac{\mathcal{D}_{q,\lambda}^{\delta,n} f(z)}{z}\right\} > \frac{1}{\alpha q}\int_0^1 u^{\frac{1}{\alpha q}-1}\frac{1-Au}{1-Bu}du,$$

using the inequality $\Re(w)^{\frac{1}{n}} \geq (\Re(w))^{\frac{1}{n}}$, for $\Re(w) > 0$ and $n \geq 1$. To prove the sharpness of (7), we define $f \in \mathcal{A}$ by

$$\frac{\mathcal{D}_{q,\lambda}^{\delta,n} f(z)}{z} = \frac{1}{\alpha q}\int_0^1 u^{\frac{1}{\alpha q}-1}\frac{1+Auz}{1+Buz}du.$$

For this function, we obatin

$$(1-\alpha)\frac{\mathcal{D}_{q,\lambda}^{\delta,n} f(z)}{z} + \alpha\frac{\mathcal{D}_{q,\lambda}^{\delta,n+1} f(z)}{z} = \frac{1+Az}{1+Bz}$$

and

$$\frac{\mathcal{D}_{q,\lambda}^{\delta,n} f(z)}{z} \to \frac{1}{\alpha q}\int_0^1 u^{\frac{1}{\alpha q}-1}\frac{1-Au}{1-Bu}du \text{ as } z \to 1.$$

Which completes the proof.

Corollary 3.2 If $f \in \mathcal{A}$ and

$$(1-\alpha)\frac{\mathcal{D}_{q,\lambda}^{\delta,n} f(z)}{z} + \alpha\frac{\mathcal{D}_{q,\lambda}^{\delta,n+1} f(z)}{z} \prec \frac{1+(2\beta-1)z}{1+z}, \tag{8}$$

for $0 \leq \beta < 1$, $\alpha > 0$, then

$$\Re\left\{\left(\frac{\mathcal{D}_{q,\lambda}^{\delta,n} f(z)}{z}\right)^{\frac{1}{n}}\right\} > \left((2\beta-1) + \frac{2(1-\beta)}{\alpha q}\int_0^1 \frac{u^{\frac{1}{\alpha q}-1}}{1+u}du\right)^{\frac{1}{n}}, \ n \geq 1.$$

Proof: Similler to the proof of Theorem 3.1, for $p(z) = \frac{\mathcal{D}_{q,\lambda}^{\delta,n} f(z)}{z}$, the differential subordination (8) passes into

$$p(z) + \alpha q z \mathcal{D}_q(p(z)) \prec \frac{1+(2\beta-1)z}{1+z}.$$

Therefore,

$$\Re\left\{\left(\frac{\mathcal{D}_{q,\lambda}^{\delta,n} f(z)}{z}\right)^{\frac{1}{n}}\right\} > \left(\frac{1}{\alpha q}\int_0^1 u^{\frac{1}{\alpha q}-1}\frac{1+(2\beta-1)u}{1+u}du\right)^{\frac{1}{n}}$$

$$= \left(\frac{1}{\alpha q} \int_0^1 u^{\frac{1}{\alpha q}-1} \left((2\beta - 1) + \frac{2(1-\beta)}{1+u} \right) du \right)^{\frac{1}{n}}$$

$$= \left((2\beta - 1) + \frac{2(1-\beta)}{\alpha q} \int_0^1 \frac{u^{\frac{1}{\alpha q}-1}}{1+u} du \right)^{\frac{1}{n}}.$$

Theorem 3.3 Let $0 \leq \rho < 1$, and $\gamma \in \mathbf{C} \backslash \{0\}$ such that

$$\left| \frac{2(1-\rho)\gamma}{q} - 1 \right| \leq 1 \qquad \text{or} \qquad \left| \frac{2(1-\rho)\gamma}{q} + 1 \right| \leq 1.$$

If $f \in \mathcal{A}$ satisfies the condition

$$\Re \left(\frac{\mathcal{D}_{q,\lambda}^{\delta,n+1} f(z)}{\mathcal{D}_{q,\lambda}^{\delta,n} f(z)} \right) > \rho, \ z \in \mathcal{U},$$

then

$$\left(\frac{\mathcal{D}_{q,\lambda}^{\delta,n} f(z)}{z} \right)^{\gamma} \prec \frac{1}{(1-z)^{\frac{2\gamma(1-\rho)}{q}}}, \ z \in \mathcal{U},$$

and the best dominant is $\dfrac{1}{(1-z)^{\frac{2\gamma(1-\rho)}{q}}}$.

Proof: Taking $p(z) = \left(\dfrac{\mathcal{D}_{q,\lambda}^{\delta,n} f(z)}{z} \right)^{\gamma}$ and applying logarithmic q-differentiation, we obtain

$$\mathcal{D}_q(p(z)) = \gamma \left(\frac{\mathcal{D}_{q,\lambda}^{\delta,n} f(z)}{z} \right)^{\gamma - 1} \frac{\mathcal{D}_{q,\lambda}^{\delta,n+1} f(z) - \mathcal{D}_{q,\lambda}^{\delta,n} f(z)}{qz^2}$$

and

$$\frac{z\mathcal{D}_q(p(z))}{p(z)} = \frac{\gamma}{q} \left[\frac{\mathcal{D}_{q,\lambda}^{\delta,n+1} f(z)}{\mathcal{D}_{q,\lambda}^{\delta,n} f(z)} - 1 \right],$$

we obtain

$$\frac{\mathcal{D}_{q,\lambda}^{\delta,n+1} f(z)}{\mathcal{D}_{q,\lambda}^{\delta,n} f(z)} = 1 + \frac{q}{\gamma} \frac{z\mathcal{D}_q(p(z))}{p(z)}.$$

Relation $\Re \left(\dfrac{\mathcal{D}_{q,\lambda}^{\delta,n+1} f(z)}{\mathcal{D}_{q,\lambda}^{\delta,n} f(z)} \right) > \rho$ can be written as

$$\frac{\mathcal{D}_{q,\lambda}^{\delta,n+1} f(z)}{\mathcal{D}_{q,\lambda}^{\delta,n} f(z)} \prec \frac{1 + (1-2\rho)z}{1-z}, \ z \in \mathcal{U}$$

which is equivalent with

$$1+\frac{qz\mathcal{D}_q(p(z))}{\gamma p(z)} \prec \frac{1+(1-2\rho)z}{1-z}.$$

Assuming

$$u(z)=\frac{1}{(1-z)^{\frac{2\gamma(1-\rho)}{q}}},\ \phi(w)=\frac{q}{\gamma w},\ \theta(w)=1,$$

we find that $u(z)$ is univalent from Lemma 2.3. It is easy to show that u θ and ϕ meet the conditions from Lemma 2.2. The functions $Q(z)=z\mathcal{D}_q(u(z))\phi(u(z))=\dfrac{2(1-\rho)z}{1-z}$ is starlike univalent in $\mathcal{U}$ and $h(z)=\theta(Q(z)+u(z))=\dfrac{1+(1-2\rho)z}{1-z}$. Applying Lemma 2.2, we can complete the proof.

Theorem 3.4 Let $\alpha<1,\ -1\leq B_i<A_i\leq 1$ and $i=1,2$. If $f_i\in\mathcal{A}$ serve the differential subordination

$$(1-\alpha)\frac{\mathcal{D}^{\delta,n}_{q,\lambda}f_i(z)}{z}+\alpha\frac{\mathcal{D}^{\delta,n+1}_{q,\lambda}f_i(z)}{z} \prec \frac{1+A_iz}{1+B_iz},\ i=1,\ 2. \tag{9}$$

then

$$(1-\alpha)\frac{\mathcal{D}^{\delta,n}_{q,\lambda}(f_1(z)*f_2(z))}{z}+\alpha\frac{\mathcal{D}^{\delta,n+1}_{q,\lambda}(f_1(z)*f_2(z))}{z} \prec \frac{1+(1-2\gamma)z}{1+z},$$

where $*$ means the convolution product of f_1 and f_2 and

$$\gamma=1-\frac{4(A_1-B_1)(A_2-B_2)}{(1-B_1)(1-B_2)}\left(1-\frac{1}{\alpha q}\int_0^1\frac{u^{\frac{1}{\alpha q}-1}}{1+u}du\right).$$

Proof: Let $h_i(z)=(1-\alpha)\dfrac{\mathcal{D}^{\delta,n}_{q,\lambda}f_i(z)}{z}+\alpha\dfrac{\mathcal{D}^{\delta,n+1}_{q,\lambda}f_i(z)}{z}$.
The differential subordination (9) can be written as $\Re(h_i(z))>\dfrac{1-A_i}{1-B_i},\ i=1,2.$
By Theorem 3.1, we obatin

$$\mathcal{D}^{\delta,n}_{q,\lambda}f_i(z)=\frac{1}{\alpha q}\int_0^1 t^{\frac{1}{\alpha q}-1}h_i(t)dt,\ i=1,2,$$

and

$$\mathcal{D}^{\delta,n}_{q,\lambda}(f_1*f_2)z=\frac{1}{\alpha q}z^{1-\frac{1}{\alpha q}}\int_0^1 t^{\frac{1}{\alpha q}-1}h_0(t)dt,$$

with

$$h_0(z) = (1-\alpha)\frac{\mathcal{D}_{q,\lambda}^{\delta,n} f_1(z) * f_2(z)}{z} + \alpha\frac{\mathcal{D}_{q,\lambda}^{\delta,n+1} f_1(z) * f_2(z)}{z} = \frac{1}{\alpha q} z^{1-\frac{1}{\alpha q}} \int_0^1 t^{\frac{1}{\alpha q}-1}(h_1 * h_2)(t)dt.$$

Applying Lemma 2.4, we obatin $h_1 * h_2$ is a function analytic in $\mathcal{U}$ written as $1+b_1z+b_2z^2+\cdots$ that satisfies the inequality $\Re(h_1*h_2) > 1-2(1-\beta_1)(1-\beta_2) = \beta$. By Lemma 2.5, we obatin

$$\begin{aligned}
\Re(h_0(z)) &= \frac{1}{\alpha q}\int_0^1 u^{\frac{1}{\alpha q}-1}\Re(h_1 * h_2)(uz)du \\
&\geq \frac{1}{\alpha q}\int_0^1 u^{\frac{1}{\alpha q}-1}\left(2\beta - 1 + \frac{2(1-\beta)}{1+u|z|}\right)du \\
&> \frac{1}{\alpha q}\int_0^1 u^{\frac{1}{\alpha q}-1}\left(2\beta - 1 + \frac{2(1-\beta)}{1+u}\right)du \\
&= \left(\frac{2\beta-1}{\alpha q}(\alpha q)(u)^{\frac{1}{\alpha q}}\right)_0^1 + \frac{2(1-\beta)}{\alpha q}\int_0^1 \frac{u^{\frac{1}{\alpha q}-1}}{1+u}du \\
&= 2\beta - 1 + \frac{2(1-\beta)}{\alpha q}\int_0^1 \frac{u^{\frac{1}{\alpha q}-1}}{1+u}du
\end{aligned}$$

we have

$$1 - \frac{4(A_1-B_1)(A_2-B_2)}{(1-B_1)(1-B_2)} + \frac{4(A_1-B_1)(A_2-B_2)}{(1-B_1)(1-B_2)}\frac{1}{\alpha q}\int_0^1 \frac{u^{\frac{1}{\alpha q}-1}}{1+u}du$$

$$= 1 - \frac{4(A_1-B_1)(A_2-B_2)}{(1-B_1)(1-B_2)}\left(1 - \frac{1}{\alpha q}\int_0^1 \frac{u^{\frac{1}{\alpha q}-1}}{1+u}du\right) = \gamma,$$

as the assertion of Theorem 3.4, holds.

Conclusion

Here, in our present investigation, we have successfully introduced a differential subordination results by using fractional q-differintegral operator. Many properties and characteristics of this newly-defined function have been studied. The results obtained during this research could be further used for writting sandwich type results in the dual theory of differenatial subordination.

Acknowledgement

The authors are grateful to the referees of this article for their valuable comments and advice.

References

[1] Alina Alb Lupas, Subordinations results on the q-analouge of the salagean differential operator,*Symmetry,* 2022,14,1744.

[2] Aldweby H and Darus H, Some subordination results on q-analogue Ruscheweyh differential operator, *Abstr. Appl. Anal.,* 2014, 958563.

[3] G. E. Andrews, R. Askey and R. Roy,Special Functions *Encyclopedia of Mathematics and its Applications, Cambridge University Press, Cambridge,* 71, 1999. MR1688958.

[4] Annapoorna S and Dileep L, Application of Linear Multiplier Fractional q-Differintegral Operator on analytic functions, *, Int. J. Math. Trends and Tech.,* ISSN: 2231-5373, Vol 69, Issue 2, 155-159. Feb 2023.

[5] El-Deeb S M and Bulboaca T, differential sandwich-type results for symmetric functions connected with a q-analouge integral operator, *Mathematics,* 2019, 7,1185,

[6] Govindaraj M and Sivasubramanian S, On a class of analytic functions related to conic domains involving q-calculus *Anal. Math.,* 2017, 43, 475-487.

[7] Hadi S A and Darus M, Differenatial subordination and superordination for a q-derivative operator connected with the q-exponential function, *Int. J. Nonlinear Anal. Appl.,* 2022, 13, 2795-2806.

[8] Jackson F H, q-Difference Equations,*Amer. J. Math.,* 32(4) (1910), 305-314. MR1506108.

[9] Kanas S and Raducanu, D., Some class of analytic functions related to conic domians, *Math. Slovaca,* 2014, 64, 1183-1196.

[10] Khan S, Hussain S, Zaighum M A and Darus M, A subclass of uniformly convex functions and corresponding subclass of starlike function with fixed coeeficient associated with q- analouge of Rushceweyh operator, *Math. Slovaca,* 2019, 69, 825-832.

[11] Miller S S and Mocanu P T, Second order differential inqualitites in the complex plane, *J. Math. Anal. Appl.,* 1978, 65, 293-305.

[12] Miller S S and Mocanu P T, Differential subordinations and univalent functions, *Mich. Math. J.,* 1981, 28, 157-171.

[13] Miller S S and Mocanu P T, On some classes of first order differential subordinations, *Mich. Math. J.,* 1985, 32, 185-195.

[14] Naeem M, Hussain S, Mahmood T, Khan S and Darus M, A new subclass of analytic functions defined by using Salagean q-differential operator, *Mathematics,* 2019,7, 458.

[15] Robertson M S, Certain classes of starlike functions,*Mich. Math. J.,* 1985, 32, 135-140.

[16] Rao G S and Saravanan R, Some results concerning best uniform co-approximation, *J. Inqual. Pure Appl. Math.,* 2002, 3, 24.

[17] Rao G S and Chandrashekaran K R, Characterization of elements of best co-aaproximation in normed linear spaces, Pure appl. Math. Sci., 1987, 26, 139-147.

[18] Salagean G S, Subclasses of univalent functions, *Lect. Notes Math.,* 1983, 1013, 362-372.

[19] Srivastava H M, Univalent Functions, fractional calculus and associated generalized hypergeometric functions, *Chichester, UK,John Wiley and Sons: New york, USA,* 1989, pp329-354.

[20] Srivastava H M, Operators of basic calculus and fractional q-calculus and their applications in geometric function theory of complex analysis, *Iran J. Sci. Tech. Trans. A Sci.,* 2020, 44, 327-344.

[21] Zainab S, Raza M, Xin Q, Jabeen M, Malik S N and Riaz S, On q-starlike functions defined by q-Ruscheweyh differential operator in symmetric conic domain, *Symmetry,* 2021, 13, 1947.

Annapoorna S
Department of Mathematics
Vidyavardhaka College of Engineering
Gokolum 3rd stage
Mysore - 570002
India
email:*anu.megalamane@gmail.com*

L Dileep
Department of Mathematics
Vidyavardhaka College of Engineering
Gokolum 3rd stage
Mysore - 570002
India
email:*dileepL84@vvce.ac.in*