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ON APPROXIMATE FUZZY MAPS

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Abstract. The aim of this paper is study of fuzzy (weak, strong) forms of β-irresoluteness and β-closure via the concept of fuzzy gβ-closed sets (gFβ-closed sets) which we call them ap-Fβ-irresolute, ap-Fβ-closed and contra-Fβ-irresolute.

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1. Introduction and preliminaries

The concepts of weak and strong forms of β-irresoluteness and β-closure via the concept of $g\beta$-closed sets are introduced and are called them called ap-β-irresolute, ap-β-closed and contra β-irresolute maps [2]. In this paper, we introduce fuzzy (weak, strong) forms of β-irresoluteness called ap-Fβ-irresoluteness and ap-Fβ-closedness by using gFβ-closed sets and obtain some basic properties of such maps. Also we present a new generalization of contra fuzzy β-irresoluteness. A subset A of a fuzzy topological space X is called fuzzy β-open if $A \subseteq clintcl(A)$, where $cl(A)$ and $int(A)$, the closure and the interior of A respectively. The β-interior of A is the union of all fuzzy β-open sets contained in A and denoted by $\beta int(A)$. The family of all fuzzy β-open sets in X is denoted by $F\beta O(X,T)$. A fuzzy set A is defined by $A = \{(x, M_A((x)) \mid x \in A, M_A(x) \in [0,1]\}$, where $M_A(x)$ is called membership function $M_A(x)$ specifies the grade or degree to which any x in A.

Definition 1.1. *A fuzzy topology is a family T of fuzzy sets in X which satisfies the following conditions*

1- $\emptyset, X \in T$,

2- If $A, B \in T$, then $A \cap B \in T$,

3- If $A_i \in T$ for each $i \in I$, then $\bigcup_{i \in I} A_i \in T$. The pair (X,T) is a fuzzy topological space. Every member of T is called an open fuzzy set, and the complement of an open fuzzy set is called a closed fuzzy set.

Definition 1.2. *Let A be a fuzzy set in X and T a fuzzy topology on X. Then the induced fuzzy topology on A is the family of a fuzzy set of A which are intersections with an open fuzzy sets in X. The induced fuzzy topology is denoted by T_A and the pair (A, T_A) is called a fuzzy subspace of (X, T).*

Definition 1.3. *A fuzzy set A is called a fuzzy pre-open (P-open) set of X if $A \subseteq intcl(A)$.*

Definition 1.4. *A subset F of (X, T) is called generalized fuzzy β-closed (briefly gFβ-closed) if $\beta cl(f) \subseteq O$ whenever $F \subseteq O$ and O is fuzzy β-open in (X, T). A subset B of (X, T) is called generalized fuzzy β-open (briefly gFβ-open) in (X, T) if its complement $B^c = X - B$ is gFβ-closed*

Definition 1.5. *A map $f : (X, T) \rightarrow (Y,$ is called*

1- Fuzzy β-irresolute if for each $V \in F\beta O(Y, \delta)$, $f^{-1}(V) \in F\beta O(X, T)$.

2- Fuzzy pre-β-closed (resp. Fuzzy pre-β-open) if for every fuzzy β-closed (resp. Fuzzy β-open) set B of (X, T) if $f(B)$ is fuzzy β-closed (resp. Fuzzy β-open) in (Y, δ).

3- Constra fuzzy β-closed if $f(U)$ is fuzzy β-open in Y for each fuzzy closed set U of X.

Definition 1.6. *A mapping $f : (X, T) \rightarrow (Y, \delta)$ is called fuzzy contra β-continuous if $f^{-1}(O)$ is fuzzy β-closed in (X, T) for each fuzzy open set O of (Y, δ).*

2. Ap-fuzzy (β-irresolute , β-closed) and contra fuzzy β-irresolute maps

Definition 2.1. *A map $f : (X, T) \rightarrow (Y, \delta)$ is called approximately fuzzy β-irresolute (briefly ap-Fβ-hence irresolute), if $\beta cl(F) \subseteq f^{-1}(O)$, whenever O is a fuzzy β-open subset of (Y, δ), F is a gFβ-closed subset of (X, T) and $F \subseteq f^{-1}(O)$.*

Example 2.2. Let $X = \{a, b\}$, $T = \{\emptyset, A, X\}$, $A = \{(a, 3/4), (b, 1)\}$, $Y = \{x, y\}$ and $\sigma = \{\emptyset, Y\}$. Suppose $f : X \rightarrow Y$, define $f(a) = x$ and $f(b) = y$, hence f is ap-Fβ-irresolute.

Definition 2.3. *A map $f : (X, T) \rightarrow (Y, \delta)$ is called approximately fuzzy β-closed (briefly ap-Fβ-closed), if $f(B) \subseteq int(A)$, whenever A is a fuzzy gβ-open subset of (Y, δ), B is a fuzzy β-closed subset of (X, T) and $f(B) \subseteq A$.*

Example 2.4. Let $X = \{a, b\}$ and $T = \{\emptyset, A, X\}$, $A = \{(a, 3/4), (b, 1)\}$ and $f : X \to X$. Define $f(a) = b$ and $f(b) = a$, hence f is ap-Fβ-closed.

Theorem 2.5.
(1) A map $f : (X, T) \to (Y, \delta)$ is ap-Fβ-irresolute if $f^{-1}(O)$ is fuzzy β-closed in (X, T) for every $O \in F\beta O(Y, \delta)$.
(2) A map $f : (X, T) \to (Y, \delta)$ is ap-Fβ-closed if $f(B) \in F\beta O(Y, \delta)$ for every fuzzy β-closed subset B of (X, T).

Proof. (1) Let $F \subseteq f^{-1}(O)$, where $O \in F\beta O(Y, \delta)$ and F is a gFβ-closed subset of (X, T), we get that $\beta cl(F) \subseteq cl(f^{-1}(O)) = f^{-1}(O)$. Thus f is ap-Fβ-irresolute.
(2) Let $f(B) \subseteq A$, where B is a fuzzy β-closed subset of (X, T) and A is a gF-β-open subset of (Y, δ). Therefore $\beta int(f(B)) \subseteq int(A)$, now $f(B) \in F\beta O(Y, \delta)$, we get that $f(B) \subseteq \beta int(A)$. Thus f is ap-Fβ-closed.

Corollary 2.6.
1- Every Fβ-irresolute mapping is ap-Fβ-irresolute.
2- Every fuzzy pre-β-closed mapping is ap-Fβ-closed.

By the following example we show that the converse of above theorem is not true

Example 2.7. Let $X = \{-1, 1\}$ and $T = \{\emptyset, A, X\}$, where $A = \{(-1, 1)\}$. Define $f(-1) = 1$, $f(1) = -1$. Since the image of every fuzzy β-closed set is fuzzy β-open, then f is ap-Fβ-closed (similarly, since the inverse image of every fuzzy β-open set is fuzzy β-closed, then f is ap-Fβ-irresolute). However $A^c = \{(1, 1)\}$ is fuzzy β-closed (X, T). (resp. $A = \{(-1, 1)\}$ is fuzzy β-open but $f(A^c)$ is not fuzzy β-closed), (resp. $f^{-1}(A)$) is not fuzzy β-open) in (X, T). Therefore f is not fuzzy pre-β-closed (resp. f is not fuzzy β-irresolute).

In the following results, the converse of (1) and (2) in Theorem 2.5 are true under the certain conditions.

Theorem 2.8. *Let $f : (X, T) \to (Y, \delta)$ be a mapping*
1- All subsets of (X, T) are fuzzy clopen and f is ap-Fβ-irresolute, then $f^{-1}(O)$ is Fβ-closed in (X, T) for any $O \in F\beta O(Y, \delta)$.
2- All subsets of (Y, δ) be fuzzy clopen and f is ap-Fβ-closed, then $f(B) \in F\beta O(Y, \delta)$ for every fuzzy β-closed subset B of X.

Proof. Let all subsets of (X, T) be fuzzy clopen and f be ap-Fβ-irresolute. Now, let

$A \subseteq X$ be such that $A \subseteq Q$ where $Q \in F\beta O(X,T)$, $\beta cl(A) \subseteq \beta cl(Q) \subseteq Q$. Hence A is gFβ-closed, therefore all subsets of X are gFβ-closed subsets of X.

1- Let $O \in F\beta O(Y,\delta)$. We get that $f^{-1}(O) \subseteq X$, $f^{-1}(O)$ is gFβ-closed $f^{-1}(O) \subseteq f^{-1}(O)$ and f is ap-Fβ-irresolute, $\beta cl(f^{-1}(O)) \subseteq f^{-1}(O)$ also $f^{-1}(O) \subseteq \beta cl(f^{-1}(O))$, then we get that $f^{-1}(O) = \beta cl(f^{-1}(O))$, $f^{-1}(O)$ is fuzzy β-closed in X.

2- By above all subset of (Y,δ) is gFβ-open and let B be fuzzy β-closed in X. Therefore $f(B)$ is gFβ subset of Y, $f(B) \subseteq f(B)$ and f is ap-Fβ-closed, hence $f(B) \subseteq \beta int(f(B))$, therefore $f(B)$ is fuzzy β-open.

Corollary 2.9. *Let $f : (X,T) \to (Y,\delta)$ be a mapping*

1- Let all subsets of (X,T) be clopen, then f is ap-Fβ-irresolute iff f is fuzzy β-irresolute.

2- Let all subsets of (Y,δ) be clopen, then f is ap-Fβ-closed iff f is fuzzy pre-β-closed.

Definition 2.10. *A mapping $f : (X,T) \to (Y,\delta)$ is called fuzzy contra β-irresolute if $f^{-1}(O)$ is fuzzy β-closed in (X,T) for each $O \in F\beta O(Y,\delta)$.*

Definition 2.11. *A mapping $f : (X,T) \to (Y,\delta)$ is called fuzzy contra pre β-closed if $f(O)$ is fuzzy β-open in (Y,δ) for each fuzzy β-closed O of X.*

Remark 2.12. In fact, fuzzy contra β-irresoluteness and fuzzy β-irresoluteness are not independent notions. Example 2.8 shows that the fuzzy contra β-irresoluteness does not imply fuzzy β-irresoluteness. While the converse is shown in the following example.

Example 2.13. Let $X = \{a,b,c\}$ and $T = \{\emptyset, A, B, X\}$ where $A = \{(a,1)\}$ and $B = \{(a,1),(b,1)\}$. Define $f(x) = x$, $\forall x \in X$, f fuzzy β-irresolute but not fuzzy contra β-irresoluteness

Proposition 2.14. *Every fuzzy contra β-irresolute is fuzzy contra β-continuous.*

The converse of Proposition 2.14 is not true.

Example 2.15. Let $X = \{a,b,c\}$, $T = \{\emptyset, A, B, C\}$ where $A = \{(a,1)\}$, $B = \{(b,1)\}$, $C = \{(a,1),(b,1)\}$ and $Y = \{p,q\}$, $\delta = \{\emptyset, P, Y\}$ where $P = \{(p,1)\}$ and $f : (X,T) \to (Y,\delta)$. Defined by $f(a) = p, f(b) = f(c) = q$. Then f is fuzzy contra β-continuous, but f is not fuzzy contra β-irresolute.

Definition 2.16. *A mapping $f:(X,T)\rightarrow(Y,\delta)$ is called fuzzy perfectly contra β-irresolute if the inverse image of every fuzzy β-open set in Y is fuzzy β-clopen in X.*

Proposition 2.17.

1- Let $f:(X,T)\rightarrow(Y,\delta)$ and $g:(Y,\delta)\rightarrow(Z,\gamma)$ be two fuzzy perfectly contra β-irresolute, then $g\circ f$ is fuzzy perfectly contra β-irresolute.

2- Let $f:(X,T)\rightarrow(Y,\delta)$ be fuzzy contra β-irresolute and $g:(Y,\delta)\rightarrow(Z,\gamma)$ be fuzzy β-irresolute, then $g\circ f$ is fuzzy contra β-irresolute.

Theorem 2.18. *Every fuzzy perfectly contra β-irresolute is fuzzy contra β-irresolute and fuzzy β-irresolute.*

Remark 2.19. The converse of 2.18 is not true. In Example 2.8 is fuzzy contra β-irresolute which is not fuzzy perfectly contra β-irresolute and in Example 2.14 which is fuzzy β-irresolute, but is not fuzzy perfectly contra β-irresolute.

Theorem 2.20. *Let f:$(X,T)\beta(Y,\delta)$ be a mapping. The following conditions are equivalent:*

1- f is fuzzy perfectly contra β-irresolute.

2- f is fuzzy contra β-irresolute and fuzzy β-irresolute.

Theorem 2.21. *If a mapping $f:(X,T)\rightarrow(Y,\delta)$ is fuzzy β-irresolute and ap-Fβ-closed, then $f^{-1}(A)$ is gFβ-closed (resp. gFβ-open) whenever A is gFβ-closed (resp. gFβ-open) subset of (Y,δ).*

Proof. Let A be gFβ-closed subset of (Y,δ). Let $f^{-1}(A)\subseteq O$ where $O\in F\beta O(X,T)$. Taking complements we obtain $O^c\subseteq f^{-1}(A^c)$ or $f(O^c)\subseteq A^c$. Since f is gFβ-closed, then $f(O^c)\subseteq \beta int(A^c)=(\beta cl(A))^c$. It follows that $O^c\subseteq(f^{-1}(\beta cl(A))^c)$ and hence $f^{-1}(\beta cl(A))\subseteq O$. Since f is fuzzy β-irresolute $f^{-1}(\beta cl(A))$ is fuzzy β-closed. We have $\beta cl(f^{-1}(A))\subseteq\beta cl(f^{-1}(\beta cl(A)))=f^{-1}(\beta cl(A))\subseteq O$. Therefore $f^{-1}(A)$ is gFβ-closed.

Theorem 2.22. *If $f:(X,T)\rightarrow(Y,\delta)$ is ap Fβ-irresolute and fuzzy pre-β-closed, then for every gFβ-closed F of (X,T), $f(F)$ is a gFβ-closed subset of (Y,δ).*

Proof. Let F be gFβ-closed subset of (X,T) and $f(F)\subseteq O$ where $O\in F\beta O(Y,\delta)$. Then $F\subseteq f^{-1}(O)$, f is ap-Fβ-irresolute, $\beta cl(F)\subseteq f^{-1}(O)$ and $f(\beta cl(F))\subseteq O$. Therefore $\beta cl(f(F))\subseteq\beta cl(\beta cl(f(F)))=\beta cl(f(F))\subseteq O$.

Theorem 2.23. *Let $f : (X,T) \to (Y,\delta)$ and $g : (Y,\delta) \to (Z,\gamma)$ be two mappings. Then*

1- $g \circ f$ is ap-Fβ-closed if f is fuzzy pre-β-closed and g is ap-Fβ-closed.

2- $g \circ f$ is ap-Fβ-closed if f is ap-Fβ-closed and g is is fuzzy pre-β-open, g^{-1} preserves gFβ-open sets.

3- $g \circ f$ is ap-Fβ-irresolute if f is ap-Fβ-irresolute and g is Fβ-irresolute.

Theorem 2.24.

1- If $f : (X,T) \to (Y,\delta)$ is ap Fβ-closed and A is fuzzy β-closed set of (X,T), then the restriction $f_A : (A,T_A) \to (Y,\delta)$ is ap-Fβ-closed.

2- If $f : (X,T) \to (Y,\delta)$ is ap Fβ-irresolute and A is fuzzy open set, gFβ-closed subset of (X,T), then the restriction $f_A : (A,T_A) \to (Y,\delta)$ is ap-Fβ-irresolute.

Proof.

1- Let H be fuzzy β-closed subset of (A,T_A) and O is a gFβ-open subset of (Y,δ) for which $f_A(H) \subseteq O$. H is fuzzy β-closed set of (X,T), since A is a fuzzy β-closed set of (X,T). Then $f_A(H) = f(H) \subseteq O$ we have $f_A(H) \subseteq \beta int(O)$. Thus f_A is an ap-Fβ-closed mapping.

2- Let F be a gFβ-closed subset relative to A and G is an fuzzy β-open subset of (Y,δ) for which $F \subseteq f_A^{-1}(G) = f^{-1}(G) \cap A$, F is gFβ-closed subset of (X,T), since f is ap Fβ-irresolute, then $\beta cl(F) \subseteq f^{-1}(G)$. We get that $\beta cl(F) \cap A \subseteq f^{-1}(G) \cap A$. Now $\beta cl(F) \cap A = \beta cl_A(F) \subseteq f^{-1}(G)$. Thus f_A is ap-Fβ-irresolute.

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