Acta Universitatis Apulensis No. 31/2012
ISSN: 1582-5329 pp. 179-189

ON SOME CONTACT METRIC STRUCTURES ON HYPERSURFACES IN A KHLERIAN MANIFOLD

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Abstract. We consider almost contact metric hypersurfaces in Khlerian manifolds. Some results detail and make more precise the data on almost contact metric structures on hypersurfaces in Hermitian and special Hermitian manifolds.

2000 *Mathematics Subject Classification*: 53C55, 53D15, 53C25.

1. INTRODUCTION

It is known that the theory of almost contact metric structures is very important in modern differential-geometrical researches. It is due to a number of applications of the theory of almost contact metric structures in mathematical physics (for example, in classical mechanics [5] and in theory of geometrical quantization [13]). Furthermore, we mark out the riches of the internal contents of this theory as well as its close connection of this with other sections of differential geometry (for example, with theory of almost complex and almost Hermitian manifolds).
We recall that an almost contact metric structure on an odd-dimensional manifold N is defined by the system of tensor fields $\{\Phi, \xi, \eta, g\}$ on this manifold, where ξ is a vector field, η is a covector field, Φ is a tensor of the type $(1,\ 1)$ and $g = \langle \cdot, \cdot \rangle$ is a Riemannian metric. Moreover, the following conditions are fulfilled:

$$\eta(\xi) = 1, \Phi(\xi) = 0, \eta \circ \Phi = 0, \Phi^2 = -id + \xi \otimes \eta,$$

$$\langle \Phi X\ , \Phi Y \rangle = \langle X\ , Y \rangle - \eta(X)\eta(Y), X, Y \in \aleph(N),$$

where $\aleph(N)$ is the module of smooth vector fields on N. As an example of an almost contact metric structure we can consider the cosymplectic structure that is characterized by the following condition:

$$\nabla \eta = 0, \quad \nabla \Phi = 0,$$

where $\nabla$is the Levi-Civita connection of the metric. It has been proved that the manifold, admitting the cosymplectic structure, is locally equivalent to the product $M \times R$, where M is a Khlerian manifold [15].
The nearly cosymplectic structure is another important example of an almost contact metric structure. We recall that the nearly cosymplectic structure is defined by the following condition [10], [11]:

$$\nabla_X(\Phi)Y + \nabla_Y(\Phi)X = 0, \nabla_X(\eta)Y + \nabla_Y(\eta)X = 0, \quad X, Y \in \aleph(N).$$

The almost contact metric structures are closely connected to the almost Hermitian structures. For instance, if $(N, \{\Phi, \xi, \eta, g\})$ is an almost contact metric manifold, then an almost Hermitian structure is induced on $N \times R$ [9]. If this almost Hermitian structure is integrable, then the input almost contact metric structure is called normal. As it is known, a normal contact metric structure is called Sasakian [9]. On the other hand, we can characterize the Sasakian structure by the following condition:

$$\nabla_X(\Phi)Y = \langle X, Y \rangle \xi - \eta(Y)X, X, Y \in \aleph(N). \tag{1}$$

For example, Sasakian structures are induced on totally umbilical hypersurfaces in a Khlerian manifold [9]. As it is well known, the Sasakian structures have many remarkable properties and play a fundamental role in contact geometry.
In 1972 Katsuei Kenmotsu has introduced a new class of almost contact metric structures [14], defined by the condition

$$\nabla_X(\Phi)Y = \langle \Phi X, Y \rangle \xi - \eta(Y)\Phi X, X, Y \in \aleph(N). \tag{2}$$

The Kenmotsu manifolds are normal and integrable, but they are not contact, consequently, they can not be Sasakian. V.F. Kirichenko has noted that in spite of the fact that the conditions (1) and (2) are similar, the properties of Kenmotsu manifolds are to some extent antipodal to the Sasakian manifolds properties [16]. Remark that the investigation [16] in this field contains a detailed description of Kenmotsu manifolds as well as a collection of examples of such manifolds.
In the present paper, almost contact metric hypersurfaces in Khlerian manifolds are considered. This note is a continuation of researches of the authors (for example, see [1], [2], [4]). We remark that the class of Khlerian manifolds is the most important class of almost Hermitian manifolds [12]. As it is known, all Cray-Hervella classes of almost Hermitian manifolds (among them nearly Khlerian, almost Khlerian, Hermitian and locally conformal Khlerian manifolds) include the class of Khlerian manifolds.

2.PRELIMINARIES

We consider an almost Hermitian manifold M^{2n}, i.e. a $2n$-dimensional manifold with a Riemannian metric $g = \langle \cdot, \cdot \rangle$ and an almost complex structure J. Moreover, the following condition must hold:

$$\langle JX,\ JY \rangle = \langle X,\ Y \rangle\,, \quad X, Y \in \aleph(M^{2n}),$$

where $\aleph(M^{2n})$ is the module of smooth vector fields on M^{2n}All considered manifolds, tensor fields and similar objects are assumed to be of the class C^{∞}. We recall that the fundamental (or Khlerian) form of an almost Hermitian manifold is determined by

$$F(X,\ Y) = \langle X,\ JY \rangle\,, \quad X, Y \in \aleph(M^{2n}).$$

Let $\left(M^{2n}, \{\ J,\ g = \langle \cdot, \cdot \rangle\}\right)$ be an arbitrary almost Hermitian manifold. We fix a point $p \in M^{2n}$. As $T_p(M^{2n})$ we denote the tangent space at the point p, $\{J_p,\ g_p = \langle \cdot, \cdot \rangle\}$ is the almost Hermitian structure at the point p induced by the structure $\{J,\ g = \langle \cdot, \cdot \rangle\}$. The frames adapted to the structure (or the A-frames) look as follows $(p,\ \varepsilon_1, \ldots, \varepsilon_n,\ \varepsilon_{\hat{1}}, \ldots, \varepsilon_{\hat{n}})$, where ε_a are the eigenvectors corresponded to the eigenvalue $i = \sqrt{-1}$, and $\varepsilon_{\hat{a}}$ are the eigenvectors corresponded to the eigenvalue $-i$ [3]. Here the index a ranges from 1 to n, and we state $\hat{a} = a + n$.
The matrix of the operator of the almost complex structure written in an A-frame looks as follows:

$$\left(J_j^k\right) = \left(\begin{array}{c|c} iI_n & 0 \\ \hline 0 & -iI_n \end{array}\right),$$

where I_n is the identity matrix; $k, j = 1, \ldots, 2n$. By direct computing, it is easy to obtain that the matrices of the metric g and of the fundamental form F in an A-frame look as follows, respectively:

$$(g_{kj}) = \left(\begin{array}{c|c} 0 & I_n \\ \hline I_n & 0 \end{array}\right), (F_{kj}) = \left(\begin{array}{c|c} 0 & iI_n \\ \hline -iI_n & 0 \end{array}\right).$$

An almost Hermitian manifold is called Khlerian, if

$$\nabla F \ = \ 0.$$

The first group of the Cartan structural equations of an almost Hermitian manifold (or an AH-manifold) written in an A-frame looks as follows [3]:

$d\,\omega^a \;=\; \omega^a_b \wedge \omega^b \;+\; B^{ab}_c\,\omega^c \wedge \omega_b \;+\; B^{abc}\,\omega_b \wedge \omega_c\;;$ $and d\,\omega_a \;=$ $-\omega^b_a \wedge \omega_b \;+\; B^c_{ab}\,\omega_c \wedge \omega^b \;+\; B_{abc}\,\omega^b \wedge \omega^c\;,$

where $\left\{B^{ab}_c\right\}$,$\{B^c_{ab}\}$ are the components of the Kirichenko virtual tensors and $\{\ B^{abc}\ \}$,$\{\ B_{abc}\ \}$ are the components of the Kirichenko structural tensors of M^{2n}[3], $a, b, c = 1, ..., n.$

3.THE MAIN RESULTS

Theorem 1. The first group of the Cartan structural equations of an almost contact metric structure on a hypersurface N in a Khlerian manifold M^{2n} is the following:

$$d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta + i\sigma^\alpha_\beta \omega^\beta \wedge \omega + i\sigma^{\alpha\beta}\omega_\beta \wedge \omega,$$

$$d\omega_\alpha = -\omega^\beta_\alpha \wedge \omega_\beta - i\sigma^\beta_\alpha \omega_\beta \wedge \omega - i\sigma_{\alpha\beta}\omega^\beta \wedge \omega,$$

$$d\omega = -2i\sigma^\alpha_\beta \omega^\beta \wedge \omega_\alpha + i\sigma_{n\beta}\,\omega \wedge \omega^\beta + i\sigma^\beta_n \omega \wedge \omega_\beta.$$

Proof. Let us use the Cartan structural equations of an almost contact metric structure on a hypersurface in an almost Hermitian manifold [4]:

$$d\,\omega^a \;=\; \omega^a_b \wedge \omega^b \;+\; B^{ab}_c\,\omega^c \wedge \omega_b \;+\; B^{abc}\,\omega_b \wedge \omega_c \;+\; (\sqrt{2}\,B^{an}_b +$$

$$+\; i\sigma^a_b)\,\omega^b \wedge \omega \;+\; (-\sqrt{2}\,\tilde{B}^{nab} \;-\; \frac{1}{\sqrt{2}}B^{ab}_n \;-\; \frac{1}{\sqrt{2}}\tilde{B}^{abn} \;+\; i\,\sigma^{an})\,\omega_b \wedge \omega\;;$$

$$d\,\omega_a \;=\; -\omega^b_a \wedge \omega_b \;+\; B^c_{ab}\,\omega_c \wedge \omega^b \;+\; B_{abc}\,\omega^b \wedge \omega^c \;+\; (\sqrt{2}\,B^b_{an} -\; i\sigma^b_a)\,\omega_b \wedge \omega \;+$$

$$+\,(-\sqrt{2}\,\tilde{B}_{nab} - \frac{1}{\sqrt{2}}\tilde{B}_{abn} - \frac{1}{\sqrt{2}}B^n_{ab} -\; i\,\sigma_{ab})\,\omega^b \wedge \omega\;; \tag{3}$$

$$d\,\omega \;=\; \sqrt{2}\,B_{nab}\,\omega^a \wedge \omega^b + \sqrt{2}\,B^{nab}\,\omega_a \wedge \omega_b + \left(\sqrt{2}\,B^{na}_b \;-\; \sqrt{2}\,B^a_{nb} \;- 2i\,\sigma^a_b\right)\;\omega^b \wedge \omega_a +$$

$$+ \left(\tilde{B}_{nbn} + B^{n}_{nb} + i\,\sigma_{nb}\right) \omega \wedge \omega^b + \left(\tilde{B}^{nbn} + B^{nb}_{n} - i\,\sigma^{b}_{n}\right) \omega \wedge \omega_b \,.$$

Here and further the indices α, β, γ range from 1 to $n-1$.

Taking into account that an almost Hermitian structure is Hermitian (i.e. an integrable structure) if and only if its Kirichenko structural tensors vanish [3]

$$B^{abc} = 0\,,\; B_{abc} = 0\,,$$

we can get from 3:

$$d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta + B^{\alpha\beta}_{\gamma}\omega^\gamma \wedge \omega_\beta + \left(\sqrt{2}B^{\alpha n}_{\beta} + i\sigma^\alpha_\beta\right)\omega^\beta \wedge \omega + \left(-\frac{1}{\sqrt{2}}B^{\alpha\beta}_{n} + i\sigma^{\alpha\beta}\right)\omega_\beta \wedge \omega,$$

$$d\omega_\alpha = -\omega^\beta_\alpha \wedge \omega_\beta + B^{\gamma}_{\alpha\beta}\omega_\gamma \wedge \omega^\beta + \left(\sqrt{2}B^{\beta}_{\alpha n} - i\sigma^\beta_\alpha\right)\omega_\beta \wedge \omega + \left(-\frac{1}{\sqrt{2}}B^{n}_{\alpha\beta} - i\sigma_{\alpha\beta}\right)\omega^\beta \wedge \omega,$$

$$d\omega = \left(\sqrt{2}B^{n\alpha}_{\beta} - \sqrt{2}B^{\alpha}_{n\beta} - 2i\sigma^\alpha_\beta\right)\omega^\beta \wedge \omega_\alpha + \left(B^{n}_{n\beta} + i\sigma_{n\beta}\right)\omega \wedge \omega^\beta + \left(B^{n\beta}_{n} - i\sigma^\beta_n\right)\omega \wedge \omega_\beta.$$

Now, we use M. Banaru-Kirichenko conditions for a Hermitian structure to be Khlerian [8]:

$$B^{ab}_{c} = 0\,,\; B^{c}_{ab} = 0\,.$$

So, we obtain

$$d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta + i\sigma^\alpha_\beta\omega^\beta \wedge \omega + i\sigma^{\alpha\beta}\omega_\beta \wedge \omega,$$

$$d\omega_\alpha = -\omega^\beta_\alpha \wedge \omega_\beta - i\sigma^\beta_\alpha\omega_\beta \wedge \omega - i\sigma_{\alpha\beta}\omega^\beta \wedge \omega, \tag{4}$$

$$d\omega = -2i\sigma^\alpha_\beta\omega^\beta \wedge \omega_\alpha + i\sigma_{n\beta}\,\omega \wedge \omega^\beta + i\sigma^\beta_n\omega \wedge \omega_\beta.$$

Theorem 2. The almost contact metric structure on a hypersurface N in a Khlerian manifold M^{2n} is Kenmotsu if and only if the matrix of the second fundamental form of the immersion of N intoM^{2n} looks as follows:

$$(\sigma_{ps}) = \left(\begin{array}{c|c|c} 0 & \begin{matrix}0\\ \dots\\ 0\end{matrix} & 0 \\ \hline 0\dots 0 & \lambda & 0\dots 0 \\ \hline 0 & \begin{matrix}0\\ \dots\\ 0\end{matrix} & 0 \end{array}\right). \qquad (5)$$

Proof. Let us use the Cartan structural equations of a Kenmotsu structure [16]:

$$d\omega^{\alpha} = \omega^{\alpha}_{\beta} \wedge \omega^{\beta} + \omega \wedge \omega^{\alpha},$$

$$d\omega_{\alpha} = -\omega^{\beta}_{\alpha} \wedge \omega_{\beta} + \omega \wedge \omega_{\alpha},$$

$$d\omega = 0.$$

From these structural equations and 4 we conclude the conditions, whose simultaneous fulfillment is a criterion for the structure on N to be Kenmotsu:

$$1) i\sigma^{\alpha}_{\beta} = -\delta^{\alpha}_{\beta},\ \ 2) i\sigma^{\alpha\beta} = 0, \qquad (6)$$

$$3) 2i\sigma^{\alpha}_{\beta} = 0,\ \ 4) i\sigma^{\beta}_{n} = 0$$

and the formulae obtained by complex conjugation (no need to write them explicitly).
From 6 and 6 we have:

$$\sigma^{\alpha}_{\beta} = 0.$$

From 6 and 6 we have:

$$\sigma^{\alpha\beta} = 0, \sigma^{\beta}_{n} = 0.$$

So, besides σ_{nn} , the rest of components of the second fundamental form of the immersion of N into M^{2n} vanish. That is why the matrix of the second fundamental form of a Kenmotsu. hypersurface in a Khlerian manifold looks as 5,

Theorem 3. The following propositions on a Kenmotsu hypersurface N in a Khlerian manifold M^{2n} are equivalent:

1. N is a totally umbilical hypersurface;
2. N is totally geodesic hypersurface;
3. N is a minimal hypersurface.

Proof. Now, let us use a criterion of the minimality of an arbitrary hypersurface [17]:

$$g^{ps}\sigma_{ps} = 0, \quad p, s = 1, ..., 2n-1.$$

Knowing how the matrix of the contravariant metric tensor on N looks [2], [7]:

$$(g^{ps}) = \left(\begin{array}{c|c|c} 0 & \begin{matrix} 0 \\ \dots \\ 0 \end{matrix} & I_{n-1} \\ \hline 0 \dots 0 & 1 & 0 \dots 0 \\ \hline I_{n-1} & \begin{matrix} 0 \\ \dots \\ 0 \end{matrix} & 0 \end{array} \right), \tag{7}$$

we obtain:

$$g^{ps}\sigma_{ps} = g^{\alpha\beta}\sigma_{\alpha\beta} + g^{\hat\alpha\hat\beta}\sigma_{\hat\alpha\hat\beta} + g^{\hat\alpha\beta}\sigma_{\hat\alpha\beta} + g^{\alpha\hat\beta}\sigma_{\alpha\hat\beta} + g^{nn}\sigma_{nn} = g^{\hat\alpha\beta}\sigma_{\hat\alpha\beta} + g^{\alpha\hat\beta}\sigma_{\alpha\hat\beta} + g^{nn}\sigma_{nn}.$$

By force of Theorem B we get:

$$g^{ps}\sigma_{ps} = g^{nn}\sigma_{nn} = 1 \cdot \lambda = \lambda.$$

That is why

$$g^{ps}\sigma_{ps} = 0 \quad \Leftrightarrow \quad \lambda = 0.$$

As the condition $\lambda = 0$ holds if and only if the matrix (σ_{ps}) vanish, we conclude that this condition is a criterion for N to be a totally geodesic hypersurface in M^{2n}. Taking into account how the matrices (σ_{ps}) and (g^{ps}) look, no doubt that the fulfillment of the equality

$$\sigma_{ps} = \mu \, g_{ps}, \mu - const$$

is possible if and only if $\lambda = 0$.
So, the propositions (I), (II) and (III) are equivalent.

As it is well-known (see, for example, [18] or [19]), when we give a Riemannian manifold and its submanifold, the rank of the determined second fundamental form is called the type number. Considering the matrix of the second fundamental form of a Kenmotsu hypersurface in a Khlerian manifold, it is easy to see that

$$T = rank\ (\sigma_{ps}) \leq 1.$$

That is why we can state the additional result of this paper:
Theorem 4. The type number T of a Kenmotsu hypersurface in a Khlerian manifold is at most one.
We note that the case $T = 0$ corresponds to a minimal totally geodesic Kenmotsu hypersurface in a Khlerian manifold as well as the condition $T = 1$ holds precisely when such a hypersurface is not minimal.

Now, we consider a Sasakian hypersurface N in a Khlerian manifoldM^{2n}. Let us use the Cartan structural equations of a Sasakian structure [20]:

$$d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta + i\delta^\alpha_\beta\, \omega \wedge \omega^\beta,$$

$$d\omega_\alpha = -\omega^\beta_\alpha \wedge \omega_\beta - i\delta^\beta_\alpha\, \omega \wedge \omega_\beta,$$

$$d\omega = -2i\omega^\alpha \wedge \omega_\alpha.$$

From these structural equations and 4 we conclude the conditions, whose simultaneous fulfillment is a criterion for the structure on N to be Sasakian:

$$1)\sigma^\alpha_\beta = -\delta^\alpha_\beta, \quad 2)\sigma^{\alpha\beta} = 0, \quad 3)i\sigma^\beta_n = 0 \tag{8}$$

and the formulae obtained by complex conjugation (no need to write them explicitly).
From 8 we get:

$$\sigma^\alpha_\beta = 0.$$

From 8 we get:

$$\sigma^\beta_n = 0.$$

So, besides σ_{nn} , the rest of components of the second fundamental form of the immersion of N into M^{2n} vanish. We obtain that the matrix of the second fundamental form of a Sasakian hypersurface in a Khlerian manifold looks as 5. We have proved the following.

Theorem 5. The almost contact metric structure on a hypersurface N in a Khlerian manifold M^{2n} is Sasakian if and only if the matrix of the second fundamental form of the immersion of N into M^{2n} looks as 5.
Using again the criterion of the minimality of a hypersurface [17] $g^{ps}\sigma_{ps} = 0, \quad p, s = 1, ..., 2n-1$, we conclude $g^{ps}\sigma_{ps} = 0 \quad \Leftrightarrow \quad \sigma_{nn} = 0 \quad \Leftrightarrow \quad \lambda = 0$. So, the equality $\lambda = 0$ is the necessary and sufficient condition of the minimality for a Sasakian hypersurface in a Khlerian manifold. Therefore we can state the following result.
Theorem 6. *T*he following propositions on a Sasakian hypersurface N in a Khlerian manifold $M^{2n}$are equivalent:

1. N is a totally umbilical hypersurface;
2. N is totally geodesic hypersurface;
3. N is a minimal hypersurface.

Similarly, considering the matrix of the second fundamental form (σ_{ps}) of the immersion of a Sasakian hypersurface in a Khlerian manifold (see (6)), we observe that $T = rank\ (\sigma_{ps}) \leq 1$. So, we get the following Theorem.
Theorem 7. *T*he type number T of a Kenmotsu hypersurface in a Khlerian manifold is at most one.
Remark. As it is evident from the proof of Theorems 3 and 6, the condition of the minimality of a Kenmotsu or Sasakian hypersurface in a Khlerian manifold is equivalent to $\sigma_{nn} = \lambda = 0$. However, $\sigma_{nn} = 0$ means that

$$\sigma(\xi, \xi) = 0. \tag{9}$$

We note that this condition 9 is a criterion for a minimality of Kenmotsu or Sasakian hypersurface in a W_3-manifold [2], [6].
Returning to mentioned V.F. Kirichenko remark [16] about of the contrast of the Kenmotsu and Sasakian manifolds properties, we respectfully object to this statement. Namely, we mark out that the Theorems 5,6 and 7 are perfect analogs for Theorems 2,3 and 4 , respectively. The similarity of the conditions 1 and 2 that determine the Sasakian and Kenmotsu structure, respectively, in certain cases implies to some extent the similarity of some properties of these structures. The above mentioned results for almost contact metric hypersurfaces in Khlerian manifolds are a confirmation to this statement.

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