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ALEPH-FUNCTION AND EQUATION OF INTEGRAL BLOOD PRESSURE

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Abstract. The aim of the present note is to establish an equation of Internal Blood Pressure pertaining to the Aleph-function. A few interesting special cases have also been recorded.

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1. Introduction

The Aleph ($\aleph$)-function, introduced by Südland *et al.* [3], however the notation and complete definition is presented here in the following manner in terms of the Mellin-Barnes type integrals [see also 4]

$$\aleph[z] = \aleph^{m,n}_{p_i,q_i,\tau_i;r}[z] = \aleph^{m,n}_{p_i,q_i,\tau_i;r}\left[z \left| \begin{matrix} (a_j,A_j)_{1,n}, [\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m}, [\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r} \end{matrix} \right.\right]$$

$$= \frac{1}{2\pi\omega} \int_L \Omega^{m,n}_{p_i,q_i,\tau_i;r}(s)\, z^{-s}\, ds. \tag{1}$$

For all $z \neq 0$, where $\omega = \sqrt{-1}$ and

$$\Omega^{m,n}_{p_i,q_i,\tau_i;r}(s) = \frac{\prod_{j=1}^{m} \Gamma(b_j + B_j s) \prod_{j=1}^{n} \Gamma(1 - a_j - A_j s)}{\sum_{i=1}^{r} \tau_i \prod_{j=n+1}^{p_i} \Gamma(a_{ji} + A_{ji} s) \prod_{j=m+1}^{q_i} \Gamma(1 - b_{ji} - B_{ji} s)}, \tag{2}$$

the integration path $L = L_{i\gamma}\infty$, $\gamma \in R$ extends from $\gamma - i\infty$ to $\gamma + i\infty$, and is such that the poles, assumed to be simple, of $\Gamma(1 - a_j - A_j s)$, $j = 1,...,n$ do not concide with the poles of $\Gamma(b_j + B_j s)$, $j = 1,..., m$ the parameter p_i, q_i are non-negative integers satisfying $0 \leq n \leq p_i$, $1 \leq m \leq q_i$, $\tau_i > 0$ for$i = 1, ..., r$. The parameter

$A_j, B_j, A_{ji}, B_{ji}, > 0$ and $a_j, b_j, a_{ji}, b_{ji} \in C$. The empty product in (2) is interpreted as unity. The existence conditions for the defining integral (1) are given below:

$$\varphi_\ell > 0,\ |\arg\ (z)| < \frac{\pi}{2}\varphi_\ell, \quad \ell = 1,...,r; \tag{3}$$

$$\varphi_\ell \geq 0,\ |\arg(z)| < \frac{\pi}{2}\varphi_\ell \text{ and } R\{\xi_\ell\} + 1 < 0, \tag{4}$$

where

$$\varphi_\ell = \sum_{j=1}^{n} A_j + \sum_{j=1}^{m} B_j - \tau_\ell \left(\sum_{j=n+1}^{p_\ell} A_{j\ell} + \sum_{j=m+1}^{q_\ell} B_{j\ell} \right) \tag{5}$$

$$\xi_\ell = \sum_{j=1}^{m} b_j - \sum_{j=1}^{n} a_j + \tau_\ell \left(\sum_{j=m+1}^{q_\ell} b_{j\ell} - \sum_{j=n+1}^{p_\ell} a_{j\ell} \right) + \frac{1}{2}(p_\ell - q_\ell), \quad \ell = 1,...,r \tag{6}$$

For detailed account of the Aleph (ℵ)-function see [3] and [4].

2. Main Result

Our main result of the present paper is the equation of Internal Blood Pressure in terms of Aleph (ℵ)-function contained in the following main theorem:

Main Theorem. *With φ_ℓ and ξ_ℓ given by (5) and (6), let P be the Internal Blood Pressure in Blood vessel having volume V, at any time and P_1 and V_1 be the partial change in internal pressure and volume, with following conditions*

(i) $V > V_1, P > P_1$,

(ii) $\varphi_\ell > 0,\ |\arg\ (z)| < \frac{\pi}{2}\varphi_\ell, \quad \ell = 1,...,r$,

(iii) $\varphi_\ell \geq 0,\ |\arg(z)| < \frac{\pi}{2}\varphi_\ell \,\text{and} R\{\xi_\ell\} + 1 < 0$,

then

$$\aleph^{m+1,n}_{p_i+1,q_i+1,\tau_i;r}\left[z \left| \begin{matrix} (a_j,A_j)_{1,n}, [\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r} (V,V_1) \\ (1+V,V_1), (b_j,B_j)_{1,m}, [\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r} \end{matrix} \right.\right]$$
$$= h\ \aleph^{m+1,n}_{p_i+1,q_i+1,\tau_i;r}\left[z \left| \begin{matrix} (a_j,A_j)_{1,n}, [\tau_i,(a_{ji},A_{ji})]_{n+1,p_i,r} (P,P_1) \\ (1+P,P_1), (b_j,B_j)_{1,m}, [\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r} \end{matrix} \right.\right]$$
$$+ k\ \aleph^{m,n}_{p_i,q_i,\tau_i;r}\left[z \left| \begin{matrix} (a_j,A_j)_{1,n}, [\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r} \\ (b_j,B_j)_{1,m}, [\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r} \end{matrix} \right.\right] \tag{7}$$

Where h is proportional constant.

Proof. Let P be the Internal Blood Pressure in Blood vessel having volume V, at any time. If P_1 and V_1 be the partial change in Internal Pressure and Volume respectively, then Internal Blood Pressure is given by the following equation [9, p.77]:

$$V \alpha P \tag{8}$$

from which we get the following differential equation

$$\frac{\mathrm{dV}}{\mathrm{dP}} = \mathrm{h}; \quad \mathrm{V} \to 0, \mathrm{P} \to 0 \tag{9}$$

where h is proportional constant.
Integrating (9), we have

$$\mathrm{V} = \mathrm{hP} + \mathrm{k} \text{ or } \frac{\Gamma(1+\mathrm{V})}{\Gamma(\mathrm{V})} = \mathrm{h}\frac{\Gamma(1+\mathrm{P})}{\Gamma(\mathrm{P})} + \mathrm{k}, \tag{10}$$

where k is integral constant.
Replacing $P = P + P_1 s$ and $V = V + V_1 s$ (since as volume increases Internal Blood Pressure will also increase) in (10) and multiplying both sides by $\frac{1}{2\pi\omega}\Omega^{\mathrm{m,n}}_{\mathrm{p_i,q_i,\tau_i;r}}(\mathrm{s})\mathrm{z}^{-\mathrm{s}}$, further integrating with respect to s in the direction of contour from $\gamma - i\infty$ to $\gamma + i\infty$ and with the help of (1), we get (7).

3. Special Cases

As the Aleph-function is the most generalized special function, numerous special cases with potentially useful transcendental functions, for sake of brevity, some interesting special cases of main theorem are given below:
(i) If we take $\tau_1 = \tau_2 = ... = \tau_{\mathrm{r}} = 1$ in (7), then the Aleph-function reduces to an I-function [8] and we get equation of Internal Blood Pressure in terms of I-function.
(ii) If we set $\tau_1 = \tau_2 = ... = \tau_{\mathrm{r}} = 1$ and $\mathrm{r} = 1$ in (7), then the Aleph-function reduces to Fox's H-function, we have a known result recently obtained by Srivastava [6,p.184, (9.6.4)].
(iii) Letting $r = 1$ and $\tau_1 = \tau_2 = ... = \tau_{\mathrm{r}} = 1$ in equation (7), we get a known result due to Chaurasia [7] when $a_i = b_j = 1 (i = 1, ..., n; j = m + 1, ..., q)$.

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