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MHD BOUNDARY-LAYER FLOW OVER A PERMEABLE SHRINKING SURFACE

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Abstract. Steady forced convection boundary layer flow past a permeable shrinking surface in a viscous and electrically conducting fluid is theoretically investigated. Choosing appropriate similarity variables, the partial differential equations are transformed into an ordinary (similarity) differential equation, which is then solved numerically using the function bvp4c from Matlab for different values of the governing parameters. The effects of the two mass suction and shrinking parameters on the reduced skin friction coefficient and the dimensionless velocity profiles are presented graphically and discussed.

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1. Introduction

Due to the numerous applications in industrial manufacturing processes, the problem of the flow due to stretching/shrinking surfaces has attracted the attention of researchers for the past four decades, being a subject of considerable interest in the contemporary literature (Crane [3]; Banks [1]; Grubka and Bobba [8]; Magyari and Keller [10]; Liao and Pop [9], etc.). Some of the application areas are hot rolling, paper production, metal spinning, drawing plastic films, glass blowing, continuous casting of metals and spinning of fibers, etc. Recently, the interest has been extended to the problem of flow and heat transfer over shrinking surfaces. For shrinking problems, the flow is shrunk towards a slot that would cause velocity away from the sheet. Here, the movement of the sheet is in opposite direction to the stretching sheet, therefore the flow induced by a shrinking sheet is, of course, distinct from the stretching flow. The main objective of this paper is to analyze the steady boundary layer flow of a viscous fluid over a shrinking surface with a special velocity form. It is shown that the reduced skin friction or the surface shear stress and the flow velocity are influenced by the mass transfer and the shrinking parameters.

2. Basic equations

Consider the two-dimensional flow of a viscous and electrically conducting fluid over a permeable shrinking surface coinciding with the plane $y = 0$, the flow being confined to $y > 0$, where y is the coordinate measured in the normal direction to the surface of the sheet. It is assumed that the velocity distribution of the shrinking surface is $u_w(x) = \lambda U_w(x)$ where x is the coordinate measured along the shrinking surface and $\lambda < 0$ is the parameter related to the shrinking surface speed. It is also assumed that the mass flux velocity is $v_w(x)$ with $v_w(x) < 0$ for suction and $v_w(x) > 0$ for injection or withdrawal of the fluid, respectively. Further, it assumed that an external variable magnetic field $B(x)$ is applied normal to the plate. Under these conditions along with the Boussinesq approximation, the equations which govern this problem are (see Pop and Ingham [11])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho}u \tag{2}$$

subject to the boundary conditions

$$\begin{array}{ll} v = v_w(x), & u = u_w(x) = \lambda U_w(x) = \lambda a(x+b)^\alpha \quad \text{at} \quad y = 0 \\ u \to 0 & \text{as} \quad y \to \infty \end{array} \tag{3}$$

where u and v are the velocity componets along x and y axes, ν is the kinematic viscosity of the fluid, ρ is the density, σ is the electrical conductivity of the fluid and a, b and α are constants with $a > 0$.

3. Similarity solution

We introduce now the following similarity variables

$$\psi = \sqrt{\frac{\nu}{a(1+\alpha)}}(x+b)^{(\alpha+1)/2}f(\eta), \quad \eta = \sqrt{\frac{a(1+\alpha)}{\nu}}(x+b)^{(\alpha-1)/2}y \tag{4}$$

where $a \neq 0$, $a(1+\alpha) > 0$ and ψ is the stream function, which is defined in the usual way as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. Thus, we have

$$u = a(x+b)^\alpha f'(\eta), \quad v = -\frac{1}{2}\sqrt{a(\alpha+1)\nu}(x+b)^{(\alpha-1)/2}\left[f(\eta) + \frac{\alpha-1}{\alpha+1}\eta f'(\eta)\right] \tag{5}$$

Thus, in order that we have a similarity solution of Eqs. (1) and (2), we take

$$v_w(x) = -\frac{1}{2}\sqrt{a(\alpha+1)\nu}(x+b)^{(\alpha-1)/2}S, \quad B(x) = B_0(x+b)^{(\alpha-1)/2} \tag{6}$$

where B_0 is the constant applied magnetic field and S is the constant parameter of suction ($S > 0$) or injection ($S < 0$), respectively.

Substituting (4) into Eq. (2), the following ordinary differential equation results

$$f''' + \frac{1}{2}ff'' - \beta f'^2 - Mf' = 0 \tag{7}$$

and the boundary conditions (3) become

$$f(0) = S, \quad f'(0) = \lambda, \quad f'(\eta) \to 0 \quad \text{as} \quad \eta \to \infty \tag{8}$$

where β is a dimensionless constant parameter and M is the magnetic field parameter, which are defined by

$$\beta = \frac{\alpha}{1+\alpha}, \quad M = \frac{\sigma B_0^2}{\rho a(1+\alpha)} \tag{9}$$

It is worth mentioning that for $\lambda = 1$, $\beta = 0$, $M = 0$ and $S = 0$, Eq. (7) becomes identical with Eq. (6) from the paper by Sakiadis [12].

The physical quantity of interest is the skin friction coefficient C_f, which is defined as

$$C_f = \frac{\tau_w}{\rho U_w^2(x)} \tag{10}$$

where ρ is the density of the fluid and τ_w is the skin friction or shear stress along the shrinking surface, which is given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{11}$$

where μ is the dynamic viscosity of the fluid. Using (5) and (10), we get

$$Re_x^{1/2}C_f = \sqrt{1-\alpha}f''(0) \tag{12}$$

where $Re_x = U_w(x)(x+b)/\nu$ is the local Reynolds number.

4. Results and discussion

The ordinary differential equation (7) subject to the boundary conditions (8) has been solved numerically using the function bvp4c from Matlab for different values of the parameters λ, β, M and S. The relative tolerance was set to 10^{-7}. In this method, a suitable finite value of $\eta \to \infty$, namely $\eta = \eta_\infty = 20$ has been chosen. We start with an initial guess satisfing the boundary conditions (8) and reveal the behavior of the solution. The technique called continuation (Shampine et al. [13]) has been then used. Table 1 shows the comparison of the values of $-f''(0)$ for $\lambda = 1$, $\beta = 0$, $M = 0$ and $S = 0$ with those reported by Sakiadis [12] for several values of the similarity variable η . We can see that there is an excellent agreement between these results, so that we are confident that the present numerical method works very efficiently.

η	$-f''(0)$ Present study	$-f''(0)$ Sakiadis [12]
0	0.44375	0.44375
0.2	0.43946	0.43946
0.3	0.43431	0.43431
0.4	0.42736	0.42736
0.5	0.41878	0.41878
0.9	0.37212	0.37212
1	0.35831	0.35831

Table 1. Comparison of the values of $-f''(0)$ for several values of the similarity variable η with the results of Sakiadis [12], when $\lambda = 1$, $\beta = 0$, $M = 0$ and $S = 0$.

Figures 1 to 3 present the dimensionless velocity profiles $f'(\eta)$ for several values of the parameters β, M and S in the case of $\lambda = -1$ (the shrinking sheet). It can be seen from these figures that the far field boundary condition $f'(\eta) \to 0$ as $\eta \to \infty$ is satisfied asymptotically. Therefore, it is supported the validity of the numerical results obtained by us.

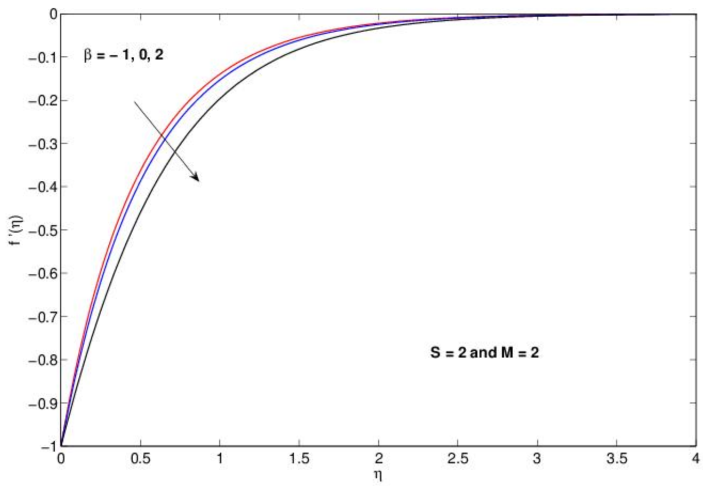

Figure 1: Dimensionless velocity profiles $f'(\eta)$ for several values of β.

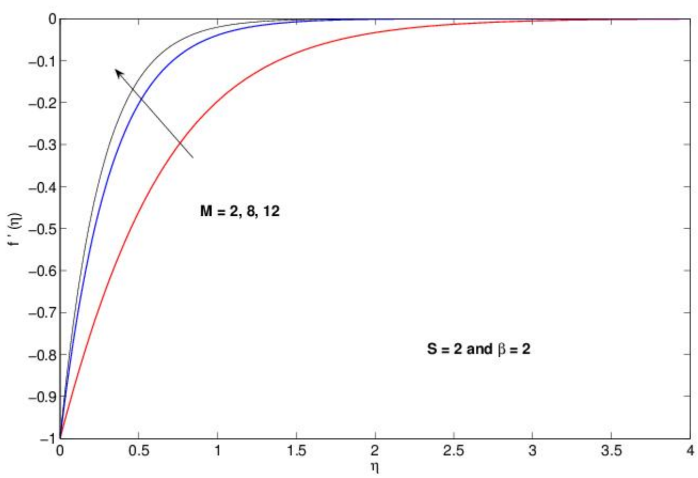

Figure 2: Dimensionless velocity profiles $f'(\eta)$ for several values of M.

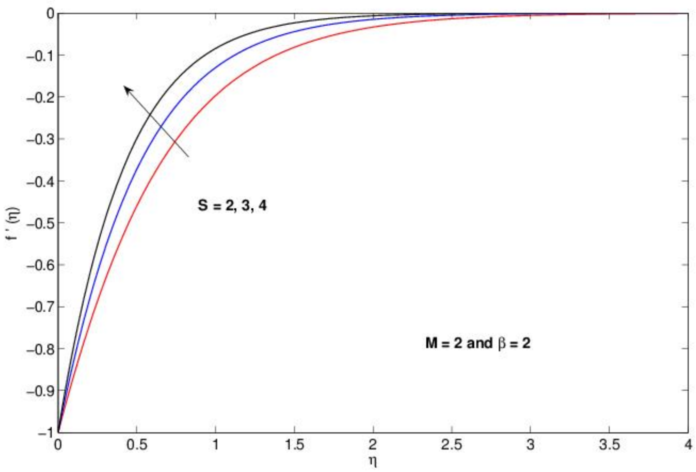

Figure 3: Dimensionless velocity profiles $f'(\eta)$ for several values of S.

In order to see the effects of the parameters S, M and β on the reduced skin friction coefficient $f''(0)$, we believe that instead of figures we can give a multiple linear regression denoted by Sfr. In this way one can easily see the effects of these parameters S, M and β on $f''(0)$. When $\lambda = -1$ (shrinking surface), we consider a regression of the form

$$Sfr_{est} = a_0 + a_1 S + a_2 M + a_3 \beta \tag{13}$$

where Sfr_{est} is the response variable while S, M and β are independent variables. The following range values of the parameters are considered in the numerical experiments: $S = 2, 3, 4$, $M = 2, 8, 12$ and $\beta = -1, 0, 2$. Hence, we used 27, 3-uple of the form (S, M, β), with the corresponding values of $f''(0)$. Thus, we obtain the following form of the multiple linear regression function Sfr_{est}, with the coefficients obtained by using the function *regress* from Matlab:

$$Sfr_{est} = 0.8903 + 0.3339S + 0.2055M - 0.1346\beta \tag{14}$$

The coefficient of multiple determination is $R^2 = 0.98$ and the maximum relative error defined by $\varepsilon = |(Sfr_{est} - f''(0))/f''(0)|$ is $\varepsilon = 0.1856$. We observe from (14) that an increase in the parameters S and M leads to an increase in the value of Sfr_{est}, while a decrease in the parameter β leads to an increase of Sfr_{est}. This regression can be repeated for other values of $\lambda < 0$ (shrinking surface), but for the sake of space it will not be presented here.

If we wish to have a more accurate formula, a quadratic regression can be performed. Thus, in the case when $\lambda = -1$ (shrinking surface), instead of Eq. (14) we obtain the following formula:

$$\begin{aligned} Sfr_{est} &= 0.8716 + 0.2120S + 0.29908M - 0.2265\beta + 0.0206S^2 - 0.0069M^2 - \\ &-0.0113\beta^2 + 0.0119M\beta - 0.000000004341SM + 0.0052S\beta \end{aligned} \tag{15}$$

where the coefficient of multiple determination is $R^2 = 0.99$ and the maximum relative error is $\varepsilon = 0.0594$. As we can see from the regresion Eq. (15), there is a relatively large interaction between M and β, and almost no interaction between S and M and S and β.

5. Conclusions

This paper investigates the effect of the magnetic field on the steady boundary-layer flow past a permeable shrinking surface. Using appropiate similarity transformations, the partial differential equations are transformed into an ordinary (similarity) differential equation, that is then solved numerically. Comparison with known results from the open literature is also done. A multiple linear regression and a quadratic regression are also performed. It is found that the governing parameters substantially affect the flow. In our opinion, the results are new and original.

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