Radiation effects on unsteady boundary layer flow past a stretching plate with suction and heat transfer with convective surface boundary condition

Naseem Ahmad [1] and Kamran Ahmad [2]

[1,2]Department of Mathematics, Faculty of Natural Sciences,
Jamia Millia Islamia, New Delhi-110025, INDIA

[1]nahmad4@jmi.ac.in; [2]qskamrankhan@gmail.com

Abstract: In the present article, we analyse unsteady laminar boundary layer flow of nano-fluids Cu-water and Ag-water past a stretching plate with suction and convective surface boundary condition in the presence of thermal radiation. The flow which has been considered here is of Skiadis type. A closed form solution has been obtained for convective heat transfer under the given conditions. The main aim of our study is to analyse the following:

  (a) effect of radiation parameter on the convective heat transfer,

  (b) the effect of volume fraction of nano-sized particles of Cu in Cu-water and Ag in ag-water, nano-fluids,

  (c) effect of suction parameter on the convective heat transfer, and

  (d) the time dependence of temperature field.

Keywords: Nano-fluids, boundary layer flow, Heat Transfer, Radiation flux, and Nusselt number.

MSC2010: 76N20

---

1   Introduction

The numerous applications of the boundary layer flow past a stretching sheet fascinate the scholars to do researches in several variants. In particular, anonymously the possible applications of boundary layer flow past a stretching sheet are in aerodynamic, extrusion of plastic sheets, formation of boundary layer along liquid film in condensation process, the cooling of metallic plate in a cooling bath, drawing of polymer yarn in textile industry and manufacturing the glass sheet in glass industry. L.J. Crane [11], supposed to be the initiator of the study of boundary layer flow due to a stretching surface in an ambient fluid. He gave the closed form solution to the steady boundary layer flow over stretching sheet which moves in its own plane with a velocity varying linearly with the distance from the slit. Since then several authors have studied various aspects of this problem. Carragher and Crane [12], investigated the heat transfer for the flow over a continuous stretching surface. The temperature field in the flow over a stretching surface subject to a uniform heat flux was studied by Dutta et al. [14], Garubka and Bobba [16], while Elbashbeshy [15], considered the case of a stretching surface with a variable surface [17], studied the heat and mass transfer for the boundary layer flow over a stretching sheet by introducing suction/blowing on the stretching surface. The effects of power law surface temperature and power law surface heat flux on the heat transfer characteristics of a continuous stretching surface

with suction and blowing were investigated by Chen and Char [13]. Magyari and Keller [23], obtained analytical solutions for the case when the sheet is permeable and also for the case when the velocity and temperature of the sheet varies exponentially with the distance along the sheet. Liao and Pop, have studied the problem of a steady boundary layer flow due to a stretching sheet using the Homotopy Analysis Method (HAM) proposed by Liao [22], and obtained analytic solutions of the flow characteristics. There are number of researchers who follow the pioneer classical work of Skadis [11], F.K. Tsou et al. [31] and L.J. Crane [11]. Later, the researchers like N. Ahmad [2], N. Ahmad and K. Ahmad [3, 4], D. Kelly, K. Vijravelu and L. Andrews [20], N. Ahmad and K. Marwah [5], M. Sidhdheshr [30], M. Subhas Abel, P.G. Siddheshwar, M.N Mahantesh [1], and N. Ahmad and M. Mishra [6] have solved unsteady fluid flow past a stretching sheet with various variants.

The study of convective transport of nano-fluids came into force, because of the increasing momentousness of nano-fluids. The circular motion that occurs in a fluids of a non uniform temperature owing to the variation of its density and the action of gravity in the fluid is poor or in other word, the convective heat transfer rate in fluids (water, ethylene, oil, glycol mixture) is poor subsequently the thermal conductivity of these fluids plays an important role on heat transfer coefficient between the medium and the surface. Hence, a number of methods were propounded to make better the thermal conductivity of these fluids by propagating nano-particles materials in the liquids. Chol [10], Introduced the new and innovative technique where he used a mixture of nano-particles and the base fluids so that he developed a advanced heat transfer fluids. Nano-fluid is a liquid suspension of ultra fine particles less than 50 nm. In his experimental observation, he showed that even small change in volumetric fraction of nano-particles influences the thermal conductivity of nano-fluid increased by 10-50% so that a remarkable improvement in the convective heat transfer coefficient takes place. Masusa et al. [24], observed that nano-fluid acted as thermal conductivity enhancer and hence nano-fluids can be used in advanced nuclear system.

In the present article, we discuss the unsteady boundary layer flow of nano-fluid Cu-water and Ag-water over a stretching plate and heat transfer with suction and convective surface boundary condition. A closed form solution has been obtained. Our focus is to read the effect of different parameters like radiation parameter N, suction parameter v_0, and volume fraction φ on the heat transfer.

2   Mathematical Formulation

Considering two dimensional boundary layer flow over a stretching sheet, we assume a coordinate system where x-axis is along the stretching sheet and y-axis is normal to the surface of the sheet in positive direction. The Figure-1 shows the geometry of the problem where the continuous stretching surface is governed by $U(x) = \frac{bx}{1-at}$, where a and b are constants and $t < \frac{1}{a}$.

The problems are as follows:

I. **Boundary Layer Flow problem:**

The governing equations for steady boundary layer flow of nano-fluids Cu-water and Ag-water past a stretching plate are:

(a) Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$
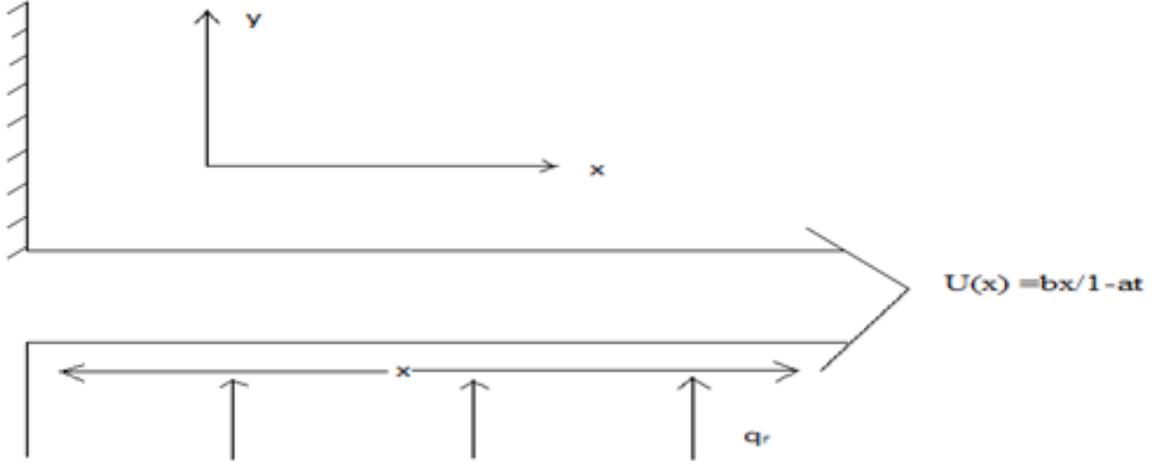
Figure 1:

(b) Momentum Equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

where u and v are the velocity components along x and y axes, respectively. μ_{nf} and ρ_{nf} are dynamic viscosity and density of nano-fluids, respectively.

The appropriate boundary conditions for flow problem are:

$$u(x,0) = U(x) = \frac{bx}{1-at}, \quad v(x,0) = -v_0, \tag{3}$$

and

$$y \to \infty, \quad u = 0, \tag{4}$$

where v_0 is the initial strength of the suction.

Now, we introduce dimensionless variables as follows:

$$\bar{x} = \frac{x}{h}, \quad \bar{y} = \frac{y}{h}, \quad \bar{u} = \frac{uh}{\nu_{nf}},$$

and

$$\bar{v} = \frac{vh}{\nu_{nf}}, \quad \bar{t} = \frac{t\nu_{nf}}{h^2},$$

where h is characteristic length, ν_{nf} is the kinematic viscosity of the nano-fluids. Using the dimensionless variables in the equations (1), (2), (3), 4 and following N. Ahmad and Ranivs [7], we get the velocity distribution as follows:

$$u = \frac{b_0}{1-a_0 t}xe^{-ry}, \quad v = -\frac{b_0}{r(1-a_0 t)}\left(1-e^{-ry}\right) - c_0 v_0, \tag{5}$$

where

$$r = \frac{c_0 v_0 + \sqrt{c_0^2 v_0^2 + \frac{4(a_0+b_0)}{1-a_0 t}}}{2},$$

$$b_0 = \frac{bh^2}{\nu_{nf}}, \quad a_0 = \frac{ah^2}{\nu_{nf}}, \quad c_0 = \frac{h}{\nu_{nf}}.$$

II. **Heat Transfer Problem:**

The energy equation with convective surface boundary condition is given by

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p{}_{nf}}\frac{\partial q_r}{\partial y} \tag{6}$$

with following relevant boundary conditions:

$$y = 0, \quad -k_{nf}\frac{\partial T}{\partial y} = h_f(T_p - T_\infty), \tag{7}$$

and

$$y \to \infty, \quad T \to T_\infty, \tag{8}$$

where k_{nf} is the thermal conductivity of the nano-fluid, α_{nf} is thermal diffusivity of the nano-fluid, q_r is the radiative heat flux, T_p is temperature of the plate and T_∞ is ambient fluid temperature, i.e., the temperature of the fluid far away from the plate, h_f is heat transfer coefficient. Referring Rosseland, S and Siegel R, Howell JR [28, 18], the radiative heat flux may be considered as follows:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{9}$$

where σ^* and k^* are the Stefan-Bltzmann constant and the mean absorption coefficient, respectively. Here we use the approximation as it is being used by Battler [8], Pal [27], Mukhopadhya and layek [26], Ishak [19] and very recently by N. Ahmad and Ravins [7] as

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \tag{10}$$

Using the above equations (9) and (10) together with dimensionless variables in the equation 6, we have

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{v_{nf}}\left(\alpha_{nf} + \frac{16\sigma^* T_\infty^3}{3K^*(\rho c_p)_{nf}}\frac{\partial^2 T}{\partial y^2}\right), \tag{11}$$

where bar has been suspended for our convenience.

We now define the dimensionless temperature T by $\theta(\eta) = \frac{T - T_\infty}{T_p - T_\infty}$ and assume that $\eta = ry$. Substituting u and v from equation (5) into (11), we get

$$\theta'' + \left\{\frac{K_0 b_0 (Pr)_{nf}}{r^2(1 - a_0 t)}(1 - e^{-\eta}) + \frac{(Pr)_{nf} K_0 c_0 v_0}{r}\right\}\theta' = 0, \tag{12}$$

and the boundary conditions (7) and (8) reduce to the following:

$$\theta'(0) = -\frac{2h_f}{k_{nf}r} \quad \text{and} \quad \theta \to 0, \quad \text{as} \quad \eta \to \infty, \tag{13}$$

where $(Pr)_{nf} = \frac{v_{nf}}{\alpha_{nf}}$ is the Prandlt number of nano-fluid, $K_0 = \frac{3N}{3N+4}$ with $N = \frac{k_{nf}k^*}{4\sigma^* T_\infty^3}$, the radiation parameter.

A solution of the equation (12) together with boundary conditions (13) is

$$\theta(\eta) = \gamma\beta^{-\alpha}\gamma(\alpha, \beta e^{-\eta}), \tag{14}$$

where

$$\gamma = \frac{2h_f}{k_{nf}r}e^{\left(\frac{4(Pr)_{nf}K_0 b_0}{(1-a_0 t)r^2}\right)}, \quad \beta = e^{\left(-\frac{4(Pr)_{nf}K_0 b_0}{(1-a_0 t)r^2}\right)},$$

$$\alpha = e^{\left(\frac{4(Pr)_{nf}K_0 b_0}{(1-a_0 t)r^2} + \frac{2(Pr)_{nf}K_0 c_0 v_0}{r}\right)},$$

and $\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt$ is the incomplete gamma function.

The effective density of nano-fluid is given by

$$\rho_{nf} = (1 - \varphi)\rho_f + \varphi\rho_s, \tag{15}$$

where φ is the volume of solid nano-particles in carrier fluid. Thus, the Thermal diffusivity of the nano-fluid becomes

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, \tag{16}$$

where the heat capacitance of the nano-fluid is taken as follows:

$$(\rho c_p)_{nf} = (1 - \varphi)\rho c_{p_f} + \varphi\rho c_{p_s}. \tag{17}$$

Due to Brinkman [9], the effective dynamic viscosity of the nano-fluid becomes

$$\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{2.5}}. \tag{18}$$

Now, the thermal conductivity calculated by Maxwell [25] out to be as

$$k_{nf} = k_f \left\{ \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + \varphi(k_f - k_s)} \right\}. \tag{19}$$

3   Skin Friction & Nusselt Number

In this section, we define both skin friction and Nusselt number. Then we calculate Nusselt number for Cu-water and Ag-water for different values of volume fraction φ (see Table-1).

Skin Friction: The wall shear stress at the stretching plate is given by

$$\tau_p = -\mu_{nf} \frac{\partial u}{\partial y}_{y=0} = \frac{\mu_f b_0 x r}{2(1 - \varphi)^{2.5}(1 - a_0 t)}.$$

Thus, the Skin friction becomes

$$C_f = \frac{\tau_p}{\rho_f u^2 h} = \frac{r R_e}{2(1 - \varphi)^{2.5}}, \tag{20}$$

where $R_e = \frac{v_f}{uh}$ is Reynolds number.

Nusselt Number: The coefficient of convectional heat transfer is called Nusselt number Nu and it is defined and calculated as

$$Nu = -\frac{\frac{\partial T}{\partial y}_{y=0}}{T_p - T_\infty} = \frac{h_f}{k_{nf}}. \tag{21}$$

φ	Nu(Cu - water)	Nu(Ag - water)
0.0	21.37030995	21.37030995
0.1	16.0481002	16.04678311
0.2	12.24162966	12.23960335

Table 1: Nusselt number for Cu-water and Ag-water for different values of volume fraction φ
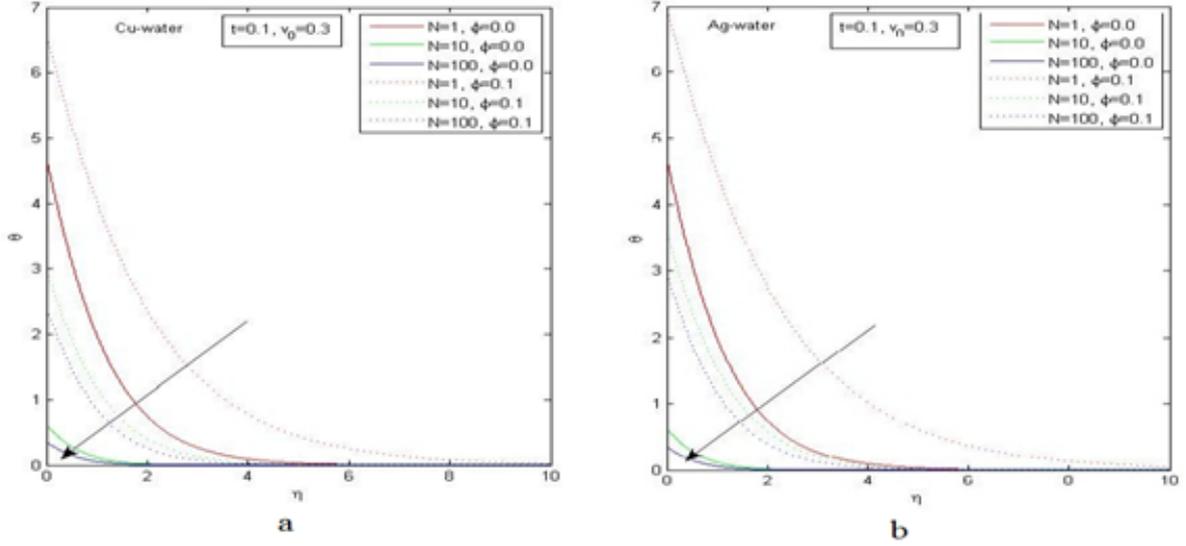
Figure 2:

4   Discussion and Results

The nano-fluids Cu-water and Ag-water have been considered for unsteady boundary layer flow past a stretching plate with suction and heat transfer with convective surface boundary condition to read the radiation effect. The exact solution to this problem has been obtained. Skin friction and the Nusselt number have also been derived. The effect of Radiation parameter N, suction parameter v_0, time t dependence and the volume fraction φ of nano-sized particles have been studied on temperature field through graphs. We summarize the results in the following paragraphs:

I. In Figure 2, we have shown the temperature profile θ for volume fraction $\varphi = 0$ and $\varphi = 0.1$ keeping time $t = 0.1$ and suction on the stretching plate $v_0 = 0.3$, fixed. We see that as N increases, temperature field decreases in both the cases $\varphi = 0.0$ and $\varphi = 0.1$. Further, we notice that temperature field is maximum for radiation $N = 1$, i.e., the intensity of radiation contributes to increase the temperature profile but when the radiation intensity increases, the rate of heat transfer increases, in turn temperature profile decreases.

II. In Figure 3, we study the trend of temperature field θ keeping radiation and suction parameter fixed as $N = 10$ and $v_0 = 0.1$. For volume fraction $\varphi = 0.0$ and $\varphi = 0.1$, we observe that as time increases, the temperature field increases within the boundary layer.
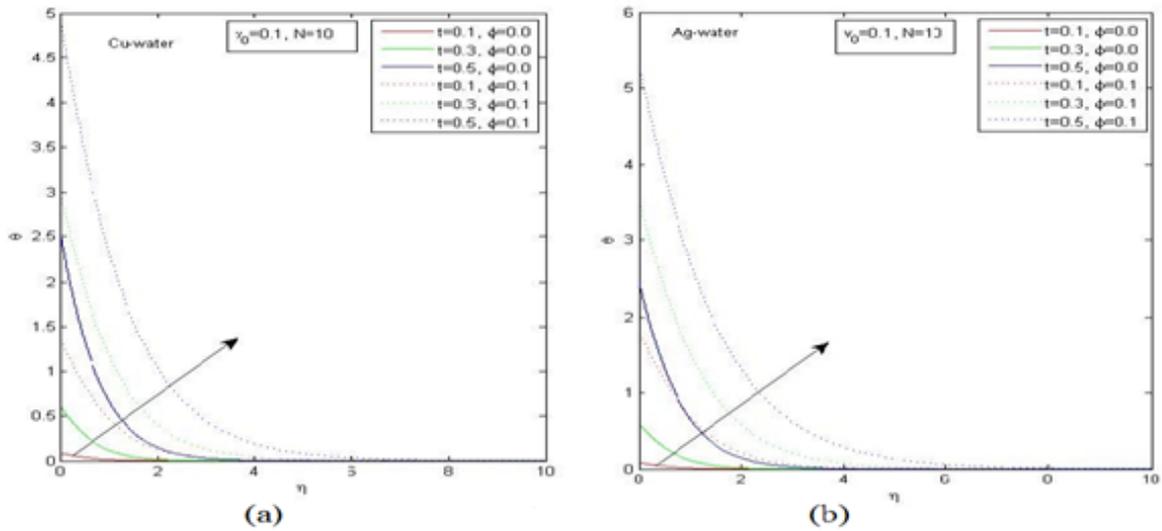
Figure 3:

It has been observed too that for $\varphi = 0.1$ the value of temperature field is more than respective values for temperature field for $\varphi = 0.0$. Hence, the presence of nano-particles changes thermal conductivity of nano-fluid.
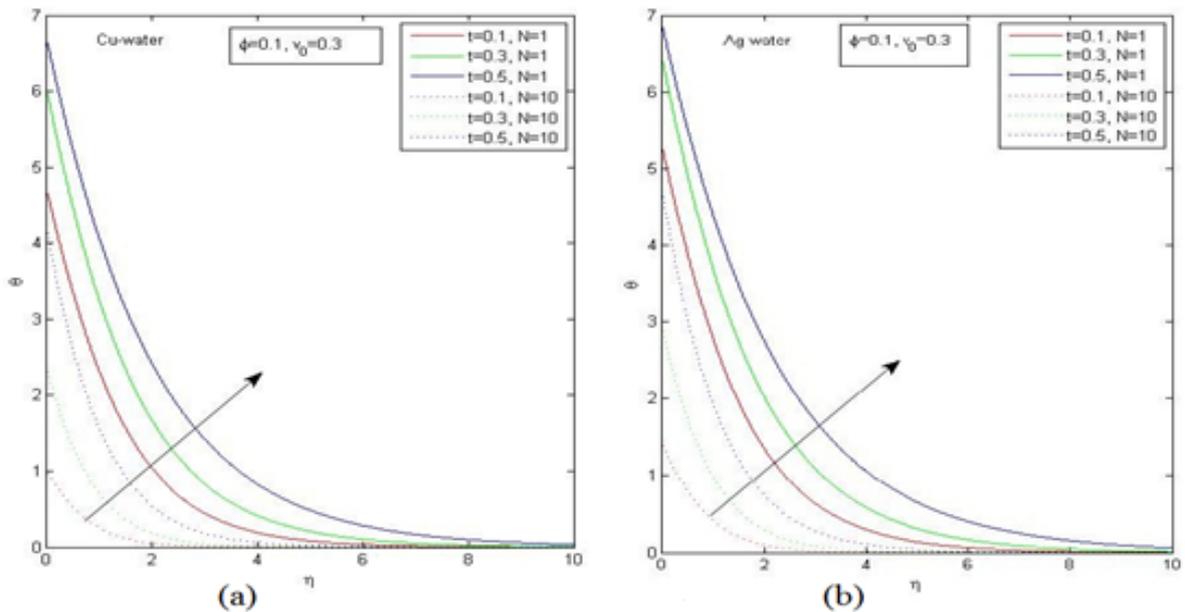
Figure 4:

III. The temperature profile θ in Figure 4 have been drawn for fixed volume fraction $\varphi = 0.1$ and the suction parameter $v_0 = 0.3$. In both the cases for radiation $N = 1$ and $N = 10$, the temperature field increases when t increases. Temperature field for $N = 10$, is lower than the value of temperature field for $N = 1$ due to the faster heat transfer rate for $N = 10$.
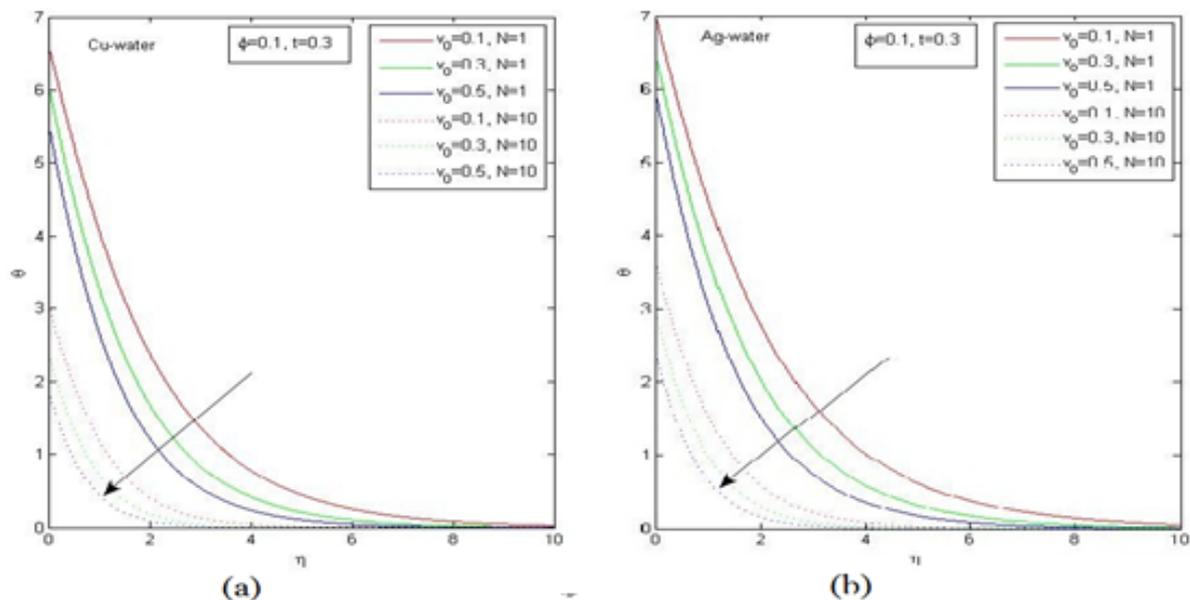
Figure 5:

IV. In Figure 5, we take $\varphi = 0.1$ and $t = 0.3$ fixed so that the effect of suction parameter may be read for two values of radiation parameter N. It is noticed that as suction parameter increases, the value of temperature profile θ decreases. This phenomenon supports the cooling process in heating process.
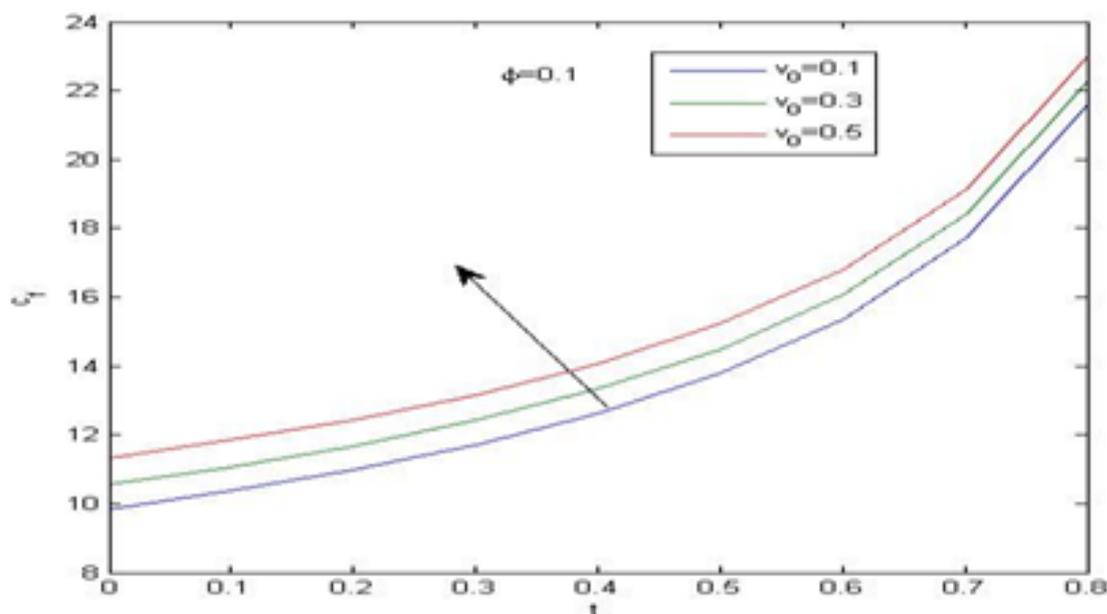
Figure 6: Skin friction C_f versus time t for different values of suction parameter v_0 with fixed volume friction $\varphi = 0.1$.

V. Figure 6 is the graph of skin friction C_f versus time t for different values of suction parameter v_0. We observe that as time progresses, skin friction increases. Also as v_0

increases, the skin friction increases because the magnitude of shear stress increases due to increase in suction parameter
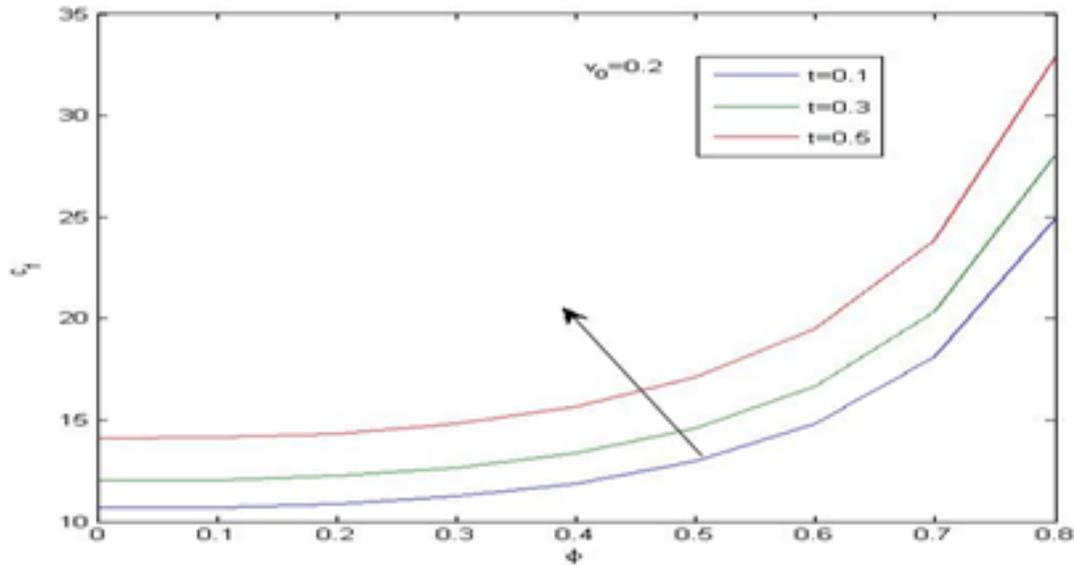
Figure 7: Skin fraction C_f versus volume fraction φ of nano-particle for different time t with suction parameter $v_0 = 0.2$.

VI. The Figure 7 shows the dependence of skin friction on volume fraction parameter φ. As the volume fraction φ increases, the solid particles ratio increases in the nano-fluid. Hence skin friction C_f increases. Also suction parameter v_0 increases, the skin friction increases. Therefore suction parameter v_0 and volume fraction φ both act as the enhancer of skin friction C_f.

VII. The Nusselt number is independent of time. While k_{nf} is function of volume fraction parameter. So, as φ increases, Nusselt number (Nu) decreases for Cu-water and Ag-water nano-fluids (see Table-1).

5   Conclusion

We conclude the following:

(a) A closed form solution has been obtained to the unsteady boundary layer flow of nano-fluids Cu-water and ag-water past a stretching plate and heat transfer with suction and convective surface boundary condition.

(b) With the help of above graphs, we discuss the effect of various parameters on temperature profile.

(c) As t, the time and v_0, suction on the surface of stretching plate increases, the skin friction increases.

(d) Looking at Table-1, we observe that the Nusselt number is independent of time but it is influenced by volume fraction parameter φ.

References

[1] M.S. Abel, P.G. Siddheshwar, M.M. Nandeppanavar, *Heat transfer in a viscoelastic boundary layer flow over a stretching sheet with viscous dissipation and non-uniform heat source*, International Journal of Heat and Mass Transfer 50, 5 (2007), 960-966.

[2] N. Ahmad, *On temperature distribution in a no-participating medium with radiation boundary condition*, International Journal of Heat and Technology 14, 1 (1995), 19-28.

[3] N. Ahmad, K. Ahmad, *Radiation Effects on Boundary Layer flow of Nanofluids Cu-water and Ag-water Past a Stretching Plate with Convective Boundary Condition*, International Journal of dynamics of Fluids 13, 1 (2017) 1-12.

[4] N. Ahmad, K. Ahmad, *Effects on Unsteady Boundary Layer flow of Nanofluids Cu-water and Ag-water Past a Stretching Plate with Convective Boundary-Condition*, Global Journal of Pure and Applied Mathematics 13, 7 (2017) 3237-3247.

[5] N. Ahmad, K. Marwah, *Visco-elastic Boundary Layer Flow Past a Stretching Plate with Suction and Heat Transfer with Variable Conductivity*, Indian Journal of Engineering and Material Sciences (IJMS) 7 (2000), 54-56.

[6] N. Ahmad, M. Mishra, *Unsteady boundary layer flow and heat transfer over a stretching sheet*, In Proceedings of 28th UIT Heat Transfer Congress, Brescia (2010), 37-40.

[7] N. Ahmad, Ravins, *Unsteady visco-elastic boundary layer flow past a stretching plate and heat transfer*, Russian journal of mathematics research series A 4 (2016), 46-55.

[8] R.C. Bataller, *Similarity solutions for boundary layer flow and heat transfer of a FENE-P fluid with thermal radiation*, Physics Letters A 372, 14 (2008), 2431-2439.

[9] H.C. Brinkman, *The viscosity of concentrated suspensions and solutions*, The Journal of Chemical Physics 20, 4 (1952), 571-571.

[10] S.U.S. Chol, *Enhancing thermal conductivity of fluids with nanoparticles*, ASME-Publications-Fed 231 (1995), 99-106.

[11] L.J. Crane, *Flow past a stretching plate*, Zeitschrift für angewandte Mathematik und Physik (ZAMP) 21, 4 (1970), 645-647.

[12] P. Carragher, L.J. Crane, *Heat transfer on a continuous stretching sheet*, ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik 62, 10 (1982), 564-565.

[13] M.I. Char, *Heat transfer of a continuous, stretching surface with suction or blowing*, Journal of Mathematical Analysis and Applications 135, 2 (1988), 568-580.

[14] B.K. Dutta, P. Roy, A.S. Gupta, *Temperature field in flow over a stretching sheet with uniform heat flux*, International Communications in Heat and Mass Transfer 12, 1 (1985), 89-94.

[15] E.M. Elbashbeshy, *Heat transfer over a stretching surface with variable surface heat flux*, Journal of Physics D: Applied Physics 31, 16 (1998), 1951.

[16] L.J. Grubka, K.M. Bobba, *Heat transfer characteristics of a continuous, stretching surface with variable temperature*, Journal of Heat Transfer 107, 1 (1985), 248-250.

[17] P.S. Gupta, A.S. Gupta, *Heat and mass transfer on a stretching sheet with suction or blowing*, The Canadian Journal of Chemical Engineering 55, 6 (1977), 744-746.

[18] J.R. Howell, M.P. Menguc, R. Siegel, *Thermal radiation heat transfer*, CRC press (2010).

[19] A. Ishak, *Thermal boundary layer flow over a stretching sheet in a micropolar fluid with radiation effect*, Meccanica 45, 3 (2010), 367-373.

[20] D. Kelly, K. Vajravelu, L. Andrews, *Analysis of heat and mass transfer of a viscoelastic, electrically conducting fluid past a continuous stretching sheet*, Nonlinear Analysis: Theory, Methods & Applications 36, 6 (1999), 767-784.

[21] C.R. Lin, C.K. Chen, *Exact solution of heat transfer from a stretching surface with variable heat flux*, Heat and mass transfer 33, 5 (1998), 477-480.

[22] S. Liao, *Beyond perturbation: introduction to the homotopy analysis method*, CRC press (2003).

[23] E. Magyari, B. Keller, *Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface*, Journal of Physics D: Applied Physics 32, 5 (1999), 577.

[24] H. Masuda, A. Ebata, K. Teramae, *Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles. Dispersion of Al2O3, SiO2 and TiO2 ultra-fine particles*, NETSU BUSSEI 7, 4 (1993), 227-233.

[25] J.C. Maxwell, *A treatise on electricity and magnetism*, Clarendon press (1881).

[26] S. Mukhopadhyay, G.C. Layek, *Radiation effect on forced convective flow and heat transfer over a porous plate in a porous medium*, Meccanica 44, 5 (2009), 587-597.

[27] D. Pal, *Heat and mass transfer in stagnation-point flow towards a stretching surface in the presence of buoyancy force and thermal radiation*, Meccanica 44, 2 (2009), 145-158.

[28] S. Rosseland, *Theoretical astrophysics*, Oxford, Clarendon press (1936).

[29] B. Sakiadis, *Boundary layer behaviour on continuous solid surfaces: I and II*, AIChE., 7 (1961).

[30] P.G. Siddheshwar, U.S. Mahabaleswar, *Effects of radiation and heat source on MHD flow of a viscoelastic liquid and heat transfer over a stretching sheet*, International Journal of Non-Linear Mechanics, 40, 6 (2005), 807-820.

[31] F.K. Tsou, E.M. Sparrow, R.J. Goldstein, *Flow and heat transfer in the boundary layer on a continuous moving surface*, International Journal of Heat and Mass Transfer 10, 2 (1967), 219-235.