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NEIGHBORHOODS OF CERTAIN CLASSES OF ANALYTIC FUNCTIONS DEFINED BY A GENERALIZED DIFFERENTIAL OPERATOR INVOLVING MITTAG-LEFFLER FUNCTION

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Abstract. A generalized differential operator involving Mittag-Leffler function is introduced and two new subclasses are given. Here, we obtain the coefficient estimates and solve the neighborhoods problem.

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1. Introduction

Denote by $\mathcal{A}(n)$ the class of functions consisting of functions f of the form

$$f(z) = z - \sum_{k=n+1}^{\infty} a_k z^k, \qquad (a_k \geq 0, k \in \mathbb{N} \setminus \{1\}, n \in \mathbb{N}) \tag{1}$$

which are analytic in the open unit disc $\mathbb{U} = \{z : z \in \mathbb{C}, |z|<1\}$.

Following the works of Goodman [2], Ruscheweyh [3], and Silverman [4] we define the (n, δ)-neighborhood of a function $f \in \mathcal{A}(n)$ by (see also [1], [5], [6] and [11])

$$N_{n,\delta}(f) = \left\{ g \in \mathcal{A}(n) : g(z) = z - \sum_{k=n+1}^{\infty} b_k z^k and \sum_{k=n+1}^{\infty} k|a_k - b_k| \leq \delta \right\}. \tag{2}$$

In particular, for the identity function $e(z) = z$, we immediately have

$$N_{n,\delta}(e) = \left\{ g \in \mathcal{A}(n) : g(z) = z - \sum_{k=n+1}^{\infty} b_k z^k and \sum_{k=n+1}^{\infty} k|b_k| \leq \delta \right\}. \tag{3}$$

First of all, A function $f \in \mathcal{A}(n)$ is a starlike of complex order $\gamma(\gamma \in \mathbb{C} \setminus \{0\})$, denoted $f \in \mathcal{S}_n^*(\gamma)$ if it satisfies the following condition

$$Re\left\{1+\frac{1}{\gamma}\left[\frac{zf^{'}(z)}{f(z)}-1\right]\right\}>0, \qquad (z \in \mathbb{U}, \gamma \in \mathbb{C} \setminus \{0\}).$$

Moreover, A function $f \in \mathcal{A}(n)$ is a convex of complex order $\gamma(\gamma \in \mathbb{C}\setminus\{0\})$, denoted $f \in \mathcal{C}_n(\gamma)$ if it satisfies the following condition

$$Re\left\{1+\frac{1}{\gamma}\frac{zf^{''}(z)}{f^{'}(z)}\right\}>0, \qquad (z \in \mathbb{U}, \gamma \in \mathbb{C} \setminus \{0\}).$$

The classes $\mathcal{S}_n^*(\gamma)$ and $\mathcal{C}_n(\gamma)$ were studied by [10].

The following defines the familiar Mittag-Leffler function $E_\alpha(z)$ introduced by Mittag-Leffler [7] and [8] and its generalization $E_{\alpha,\beta}(z)$ introduced by Wiman [9].

$$E_\alpha(z) = \sum_{k=o}^{\infty} \frac{z^k}{\Gamma(\alpha+1)},$$

and

$$E_{\alpha,\beta}(z) = \sum_{k=o}^{\infty} \frac{z^k}{\Gamma(\alpha n+\beta)} \qquad (\alpha \geq 0).$$

where $\alpha, \beta \in \mathbb{C}$, $Re(\alpha)>0$ and $Re(\beta)>0$.

As a result, a lot of useful work have been made by many researchers in attempt to explain Mittag-Leffler function and its generalization see for example [12], [13], [14], [17] , and [18].

Let $\mathcal{A}$ be a class of functions f of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{4}$$

which are analytic in the open unit disk $\mathbb{U}$.

We define the function $Q_{\alpha,\beta}(z)$ by

$$Q_{\alpha,\beta}(z) = z\Gamma(\beta)E_{\alpha,\beta}(z).$$

Now, for $f \in \mathcal{A}$ we define the following differential operator: $D_\lambda^m(\alpha,\beta)f : \mathcal{A} \longrightarrow \mathcal{A}$ by

$$D_\lambda^0(\alpha,\beta)f(z) = f(z) * Q_{\alpha,\beta}(z), \tag{5}$$

$$D_\lambda^1(\alpha,\beta)f(z) = (1-\lambda)(f(z) * Q_{\alpha,\beta}(z)) + \lambda z(f(z) * Q_{\alpha,\beta}(z))' \tag{6}$$

$$\vdots$$

$$D_\lambda^m(\alpha,\beta)f(z) = D_\lambda^1(D_\lambda^{m-1}(\alpha,\beta)f(z)) \tag{7}$$

If f is given by (4), then from (6) and (7) we see that

$$D_\lambda^m(\alpha,\beta)f(z) = z + \sum_{k=2}^{\infty}[1+(k-1)\lambda]^m \frac{\Gamma(\beta)}{\Gamma(\alpha(k-1)+\beta)} a_k z^k. \tag{8}$$

Note that

- When $\alpha = 0$ and $\beta = 1$ we get Al-Oboudi operator [15].
- When $\alpha = 0$, $\beta = 1$ and $\lambda = 1$ we get Sălăgean operator [16].
- When $m = 0$ we get $\mathbb{E}_{\alpha,\beta}(z)$ [12].

Literature review indicates a large number of related work to operators. This is a direct indication of wide spread and popularity of the generalization of operators (see examples [1], [19] and [20]).

If $f \in \mathcal{A}(n)$ is given by (1) then we have

$$D_\lambda^m(\alpha,\beta)f(z) = z - \sum_{k=n+1}^{\infty}[1+(k-1)\lambda]^m \frac{\Gamma(\beta)}{\Gamma(\alpha(k-1)+\beta)} a_k z^k. \tag{9}$$

Finally, by using the differential operator defined by (9), we define the subclass $\mathcal{S}_{n,\beta}^{m,\alpha}(\gamma,\vartheta,\lambda)$ of $\mathcal{A}(n)$ consisting of functions f which satisfy the inequality

$$\left|\frac{1}{\gamma}\left[\frac{z(D_\lambda^m(\alpha,\beta)f(z))'}{D_\lambda^m(\alpha,\beta)f(z)} - 1\right]\right| < \vartheta \tag{10}$$

$$(z \in \mathbb{U}, \gamma \in \mathbb{C}\setminus\{0\}, \lambda \geq 0, 0 < \vartheta \leq 1).$$

Also, Let, $\mathcal{R}_{n,\beta}^{m,\alpha}(\gamma,\zeta,\vartheta,\lambda)$ denote the subclass of $\mathcal{A}(n)$ consisting of f which satisfy the inequality

$$\left|\frac{1}{\gamma}\left[(1-\zeta)\frac{D_\lambda^m(\alpha,\beta)f(z)}{z} + \zeta(D_\lambda^m(\alpha,\beta)f(z))' - 1\right]\right| < \vartheta, \tag{11}$$

$$(z \in \mathbb{U}, \gamma \in \mathbb{C}\setminus\{0\}, 0 \leq \zeta \leq 1, 0 < \vartheta \leq 1, \lambda \geq 0).$$

In this paper, we obtain the coefficient estimates and results related to neighborhood of the subclasses defined.

2. Coefficient Estimates

Theorem 2.1. Let $f \in \mathcal{A}(n)$. Then $f \in \mathcal{S}_{n,\beta}^{m,\alpha}(\gamma, \vartheta, \lambda)$ if and only if

$$\sum_{k=n+1}^{\infty} (k + \vartheta|\gamma| - 1)[1 + (k-1)\lambda]^m \frac{\Gamma(\beta)}{\Gamma(\alpha(k-1)+\beta)} a_k \leq \vartheta|\gamma| \tag{12}$$

for $0<\vartheta \leq 1, \gamma \in \mathbb{C}\backslash \{0\}, \lambda \geq 0$ and $m \in \mathbb{N}_0$.

Proof. Let $f \in \mathcal{S}_{n,\beta}^{m,\alpha}(\gamma, \vartheta, \lambda)$. Then, we have

$$Re\left\{ \frac{z(D_\lambda^m(\alpha,\beta)f(z))'}{D_\lambda^m(\alpha,\beta)f(z)} - 1 \right\} > -\vartheta|\gamma|$$

Equivalently,

$$Re\left\{ \frac{-\sum_{k=n+1}^{\infty}(k-1)[1+(k-1)\lambda]^m \frac{\Gamma(\beta)}{\Gamma(\alpha(k-1)+\beta)} a_k z^k}{z - \sum_{k=n+1}^{\infty}[1+(k-1)\lambda]^m \frac{\Gamma(\beta)}{\Gamma(\alpha(k-1)+\beta)} a_k z^k} \right\} > -\vartheta|\gamma| \tag{13}$$

since the above inequality is true for all $z \in \mathbb{U}$, choose values of z on the real axis. Upon clearing the denominator in (13) and letting $z \to 1^-$ through real values, we obtain

$$-\sum_{k=n+1}^{\infty}(k-1)[1+(k-1)\lambda]^m \frac{\Gamma(\beta)}{\Gamma(\alpha(k-1)+\beta)} a_k$$

$$\geq -\vartheta|\gamma| \left(1 - \sum_{k=n+1}^{\infty}[1+(k-1)\lambda]^m \frac{\Gamma(\beta)}{\Gamma(\alpha(k-1)+\beta)} a_k \right).$$

Thus, we obtain the desired inequality

$$\sum_{k=n+1}^{\infty} (k + \vartheta|\gamma| - 1)[1 + (k-1)\lambda]^m \frac{\Gamma(\beta)}{\Gamma(\alpha(k-1)+\beta)} a_k \leq \vartheta|\gamma| \tag{14}$$

Conversely, supposed that inequality (12) holds true and $|z| = 1$, we obtain

$$\begin{aligned}\left|\frac{z\left(D_\lambda^m(\alpha,\beta)f(z)\right)'}{D_\lambda^m(\alpha,\beta)f(z)} - 1\right| &= \left|\frac{\sum_{k=n+1}^{\infty}(k-1)[1+(k-1)\lambda]^m \dfrac{\Gamma(\beta)}{\Gamma(\alpha(k-1)+\beta)} a_k z^k}{z - \sum_{k=n+1}^{\infty}[1+(k-1)\lambda]^m \dfrac{\Gamma(\beta)}{\Gamma(\alpha(k-1)+\beta)} a_k z^k}\right| \\ &\leq \frac{\vartheta|\gamma|\left(1 - \sum_{k=n+1}^{\infty}[1+(k-1)\lambda]^m \dfrac{\Gamma(\beta)}{\Gamma(\alpha(k-1)+\beta)} a_k\right)}{1 - \sum_{k=n+1}^{\infty}[1+(k-1)\lambda]^m \dfrac{\Gamma(\beta)}{\Gamma(\alpha(k-1)+\beta)} a_k} \\ &\leq \vartheta|\gamma|\end{aligned}$$

Hence, by the maximum modulus theorem, we have $f \in \mathcal{S}_{n,\beta}^{m,\alpha}(\gamma,\vartheta,\lambda)$.

Similarly, we can prove the following result.

Theorem 2.2. Let $f \in \mathcal{A}(n)$ defined by (1). Then $f \in \mathcal{R}_{n,\beta}^{m,\alpha}(\gamma,\zeta,\vartheta,\lambda)$ if and only if

$$\sum_{k=n+1}^{\infty}[\zeta(k-1)+1][1+(k-1)\lambda]^m \frac{\Gamma(\beta)}{\Gamma(\alpha(k-1)+\beta)} a_k \leq \vartheta|\gamma|, \tag{15}$$

for $(z \in \mathbb{U}, \gamma \in \mathbb{C}\setminus\{0\}, 0 \leq \zeta \leq 1, 0 < \vartheta \leq 1, \lambda \geq 0)$.

Proof. We omit the proofs since it is similar to Theorem 2.1.

3. Inclusion Relation Involving $N_{n,\delta}(e)$

Our first inclusion relation involving $N_{n,\delta}(e)$ is given by following theorems.

Theorem 3.1. Let

$$\delta = \frac{(n+1)\vartheta|\gamma|\Gamma(\alpha n+\beta)}{(\vartheta|\gamma|+n)[1+n\lambda]^m\Gamma(\beta)}$$

then $\mathcal{S}_{n,\beta}^{m,\alpha}(\gamma,\vartheta,\lambda) \subseteq N_{n,\delta}(e)$

Proof. For $f \in \mathcal{S}_{n,\beta}^{m,\alpha}(\gamma,\vartheta,\lambda)$ and making use of the condition (12), we obtain

$$(\vartheta|\gamma|+n)[1+n\lambda]^m \frac{\Gamma(\beta)}{\Gamma(\alpha n+\beta)} \sum_{k=n+1}^{\infty} a_k \leq \vartheta|\gamma|. \tag{16}$$

so that

$$\sum_{k=n+1}^{\infty} a_k \leq \frac{\Gamma(\alpha n+\beta)\vartheta|\gamma|}{(\vartheta|\gamma|+n)[1+n\lambda]^m\Gamma(\beta)} \tag{17}$$

On the other hand, we also find from (12) and (17) that

$$\begin{aligned}\frac{\Gamma(\beta)}{\Gamma(\alpha n+\beta)}[1+n\lambda]^m\sum_{k=n+1}^{\infty}ka_k &\leq \vartheta|\gamma|+(1-\vartheta|\gamma|)\frac{\Gamma(\beta)}{\Gamma(\alpha n+\beta)}[1+n\lambda]^m\sum_{k=n+1}^{\infty}a_k\\ &\leq \vartheta|\gamma|+\frac{(1-\vartheta|\gamma|)[1+n\lambda]^m\Gamma(\beta)}{\Gamma(\alpha n+\beta)}.\frac{\Gamma(\alpha n+\beta)\vartheta|\gamma|}{(\vartheta|\gamma|+n)[1+n\lambda]^m\Gamma(\beta)}\\ &\leq \frac{(n+1)\vartheta|\gamma|}{\vartheta|\gamma|+n}\end{aligned}$$

Hence,

$$\sum_{k=n+1}^{\infty}ka_k\leq\frac{(n+1)\vartheta|\gamma|\Gamma(\alpha n+\beta)}{(\vartheta|\gamma|+n)[1+n\lambda]^m\Gamma(\beta)}=\delta$$

which in view of (3), proves Theorem 3.1.

Theorem 3.2. Let

$$\delta=\frac{(n+1)\vartheta|\gamma|\Gamma(\alpha n+\beta)}{(n\zeta+1)[1+n\lambda]^m\Gamma(\beta)}$$

then $\mathcal{R}_{n,\beta}^{m,\alpha}(\gamma,\zeta,\vartheta,\lambda)\subseteq N_{n,\delta}(e)$

Proof. For $f\in\mathcal{R}_{n,\beta}^{m,\alpha}(\gamma,\zeta,\vartheta,\lambda)$ and making use of the condition (15), we obtain

$$(n\zeta+1)[1+n\lambda]^m\frac{\Gamma(\beta)}{\Gamma(\alpha n+\beta)}\sum_{k=n+1}^{\infty}a_k\leq\vartheta|\gamma|. \tag{18}$$

so that

$$\sum_{k=n+1}^{\infty}a_k\leq\frac{\Gamma(\alpha n+\beta)\vartheta|\gamma|}{(n\zeta+1)[1+n\lambda]^m\Gamma(\beta)}. \tag{19}$$

Thus, using (15) along with (19), we also obtain

$$\begin{aligned}\zeta[1+n\lambda]^m\frac{\Gamma(\beta)}{\Gamma(\alpha n+\beta)}\sum_{k=n+1}^{\infty}ka_k &\leq \vartheta|\gamma|+(\zeta-1)[1+n\lambda]^m\frac{\Gamma(\beta)}{\Gamma(\alpha n+\beta)}\sum_{k=n+1}^{\infty}a_k\\ &\leq \vartheta|\gamma|+\frac{(\zeta-1)[1+n\lambda]^m\Gamma(\beta)}{\Gamma(\alpha n+\beta)}.\frac{\vartheta|\gamma|\Gamma(\alpha n+\beta)}{(n\zeta+1)[1+n\lambda]^m\Gamma(\beta)}\\ &\leq \frac{\zeta(n+1)\vartheta|\gamma|}{(n\zeta+1)}.\end{aligned}$$

Hence,

$$\sum_{k=n+1}^{\infty}ka_k\leq\frac{(n+1)\vartheta|\gamma|\Gamma(\alpha n+\beta)}{(n\zeta+1)[1+n\lambda]^m\Gamma(\beta)}=\delta$$

which in view of (3), completes the proof of Theorem 3.2.

4. Neighborhood for the classes $\mathcal{S}_{n,\beta}^{m,\alpha,(\eta)}(\gamma,\vartheta,\lambda)$ and $\mathcal{R}_{n,\beta}^{m,\alpha,(\eta)}(\gamma,\zeta,\vartheta,\lambda)$

In this section, we determine the neighborhood for each of the classes $\mathcal{S}_{n,\beta}^{m,\alpha,(\eta)}(\gamma,\vartheta,\lambda)$ and $\mathcal{R}_{n,\beta}^{m,\alpha,(\eta)}(\gamma,\zeta,\vartheta,\lambda)$, which we define as follows:

Definition 4.1. A function $f \in \mathcal{A}(n)$ is said to be in the class $\mathcal{S}_{n,\beta}^{m,\alpha,(\eta)}(\gamma,\vartheta,\lambda)$ if there exists a function $h \in \mathcal{S}_{n,\beta}^{m,\alpha}(\gamma,\vartheta,\lambda)$ such that

$$\left|\frac{f(z)}{h(z)} - 1\right| < 1-\eta. \qquad (z \in \mathbb{U}, 0 \leq \eta < 1) \tag{20}$$

Also, a function $f \in \mathcal{R}_{n,\beta}^{m,\alpha,(\eta)}(\gamma,\zeta,\vartheta,\lambda)$ if there exists a function $h \in \mathcal{R}_{n,\beta}^{m,\alpha}(\gamma,\zeta,\vartheta,\lambda)$ such that the inequality (20) holds true.

Theorem 4.2. If $h \in \mathcal{S}_{n,\beta}^{m,\alpha}(\gamma,\vartheta,\lambda)$ and

$$\eta = 1 - \frac{(\vartheta|\gamma|+n)\delta[1+n\lambda]^m\Gamma(\beta)}{(n+1)\left[(\vartheta|\gamma|+n)[1+n\lambda]^m\Gamma(\beta) - \vartheta|\gamma|\Gamma(\alpha n+\beta)\right]}$$

then $N_{n,\delta}(h) \subseteq \mathcal{S}_{n,\beta}^{m,\alpha,(\eta)}(\gamma,\vartheta,\lambda)$

Proof. Let $f \in N_{n,\delta}(h)$ we then find from (2) that

$$\sum_{k=n+1}^{\infty} k\,|a_k - b_k| \leq \delta,$$

which easily implies the coefficient inequality

$$\sum_{k=n+1}^{\infty} |a_k - b_k| \leq \frac{\delta}{n+1} \qquad (n \in \mathbb{N}).$$

Since $h \in \mathcal{S}_{n,\beta}^{m,\alpha}(\gamma,\vartheta,\lambda)$, we have from equation (17) that

$$\sum_{k=n+1}^{\infty} b_k \leq \frac{\Gamma(\alpha n+\beta)\vartheta|\gamma|}{(\vartheta|\gamma|+n)[1+n\lambda]^m\Gamma(\beta)}$$

and so

$$\begin{aligned}\left|\frac{f(z)}{h(z)} - 1\right| &< \frac{\sum_{k=n+1}^{\infty}|a_k-b_k|}{1-\sum_{k=n+1}^{\infty} b_k} \\ &\leq \frac{\delta}{n+1}.\frac{(\vartheta|\gamma|+n)[1+n\lambda]^m\Gamma(\beta)}{(\vartheta|\gamma|+n)[1+n\lambda]^m\Gamma(\beta)-\vartheta|\gamma|\Gamma(\alpha n+\beta)} \\ &= 1-\eta.\end{aligned}$$

This completes the proof of Theorem 4.2.

Similarly, we can prove the following result:

Theorem 4.3. If $h \in \mathcal{R}_{n,\beta}^{m,\alpha}(\gamma, \zeta, \vartheta, \lambda)$ and

$$\eta = 1 - \frac{(n\zeta + 1)\delta[1 + n\lambda]^m \Gamma(\beta)}{(n+1)\left[(n\zeta + 1)[1 + n\lambda]^m \Gamma(\beta) - \vartheta|\gamma|\Gamma(\alpha n + \beta)\right]}$$

then $N_{n,\delta}(h) \subseteq \mathcal{R}_{n,\beta}^{m,\alpha,(\eta)}(\gamma, \zeta, \vartheta, \lambda)$

Proof. We omit the proofs since it is similar to Theorem 4.2.

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