Acta Universitatis Apulensis No. 59/2019
ISSN: 1582-5329 pp. 101-109
http://www.uab.ro/auajournal/ doi: 10.17114/j.aua.2019.59.10

NEW UNIVALENCE CRITERIA FOR AN INTEGRAL OPERATOR WITH MOCANU'S AND ŞERB'S LEMMA

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Abstract. In this paper we consider an integral operator for analytic functions in the open unit disk U and we obtain sufficient conditions for univalence of this integral operator, using Mocanu's and Şerb's Lemma.

2010 *Mathematics Subject Classification*: 30C45.

Keywords: Integral operator; univalence; unit disk.

1. Introduction

Let $\mathcal{A}$ be the class of the functions f which are analytic in the open unit disk $\mathcal{U} = \{z \in \mathbb{C} : \mid z \mid < 1\}$ and $f(0) = f^{'}(0) - 1 = 0$.

We denote by S the subclass of A consisting of functions $f \in A$, which are univalent in $\mathcal{U}$.

We consider the integral operator

$$\mathcal{T}_n(z) = \left\{ \delta \int_0^z t^{\delta-1} \prod_{i=1}^{n} \left[\left(\frac{f_i(t)}{t} \right)^{\alpha_i - 1} \cdot \left(g_i'(t) \right)^{\beta_i} \cdot \left(\frac{h_i(t)}{k_i(t)} \right)^{\gamma_i} \cdot \left(\frac{{h_i}'(t))}{{k_i}'(t)} \right)^{\delta_i} \right] \mathrm{dt} \right\}^{\frac{1}{\delta}}, \tag{1}$$

for $f_i, g_i, h_i, k_i \in \mathcal{A}$ and the complex numbers $\delta, \alpha_i, \beta_i, \gamma_i, \delta_i$, with $\delta \neq 0$, $i = \overline{1, n}$, $n \in \mathbb{N} \setminus \{0\}$.

2. Preliminary results

In order to prove main results we will use the following lemmas.

Lemma 1. [7] *Let γ, δ be complex numbers, $Re\gamma > 0$ and $f \in \mathcal{A}$. If*

$$\frac{1-|z|^{2Re\gamma}}{Re\gamma}\left|\frac{zf''(z))}{f'(z)}\right| \leq 1,$$

for all $z \in \mathcal{U}$, then for any complex number δ, $Re\delta \geq Re\gamma$, the function F_δ defined by

$$F_\delta(z) = \left(\delta \int_0^z t^{\delta-1} f'(t)\, dt\right)^{\frac{1}{\delta}},$$

is regular and univalent in $\mathcal{U}$.

Lemma 2. [5] *Let $M_0 = 1,5936...$ the positive solution of equation*

$$(2 - M)\, e^M = 2. \tag{2}$$

If $f \in \mathcal{A}$ and

$$\left|\frac{f''(z))}{f'(z)}\right| \leq M_0,$$

for $z \in \mathcal{U}$, then

$$\left|\frac{zf'(z))}{f(z)} - 1\right| \leq 1, \qquad (z \in U)$$

The edge M_0 is sharp.

Lemma 3. [3] *Let f be the function regular in the disk $\mathcal{U}_R = \{z \in \mathbb{C} : |z| < R\}$ with $|f(z)| < M$,* M *fixed. If $f(z)$ has in $z = 0$ one zero with multiply $\geq m$, then*

$$|f(z)| \leq \frac{M}{R^m} z^m,$$

the equality for $z \neq 0$ can hold only if

$$f(z) = e^{i\theta} \frac{M}{R^m} z^m,$$

where θ is constant.

3. Main results

Theorem 4. *Let $\gamma, \delta, \alpha_i, \beta_i, \gamma_i, \delta_i$ be complex numbers, $c = Re\gamma > 0$, M_0 the positive solution of the equation (2), $M_0 = 1,5936...$ and $f_i, g_i, h_i, k_i \in \mathcal{A}$, $f_i(z) = z + a_{2i}z^2 + a_{3i}z^3 + ...$, $g_i(z) = z + b_{2i}z^2 + b_{3i}z^3 + ...$, $h_i(z) = z + c_{2i}z^2 + c_{3i}z^3 + ...$, $k_i(z) = z + d_{2i}z^2 + d_{3i}z^3 + ...$, $i = \overline{1,n}$ If*

$$\left|\frac{f_i''(z)}{f_i'(z)}\right| \le M_0, \quad \left|\frac{g_i''(z)}{g_i'(z)}\right| \le M_0, \quad \left|\frac{h_i''(z)}{h_i'(z)}\right| \le M_0, \quad \left|\frac{k_i''(z)}{k_i'(z)}\right| \le M_0, \tag{3}$$

for all $z \in \mathcal{U}$, $i = \overline{1,n}$ and

$$\frac{1}{c}\sum_{i=1}^{n}|\alpha_i - 1| + \frac{2M_0}{(2c+1)^{\frac{2c+1}{2c}}}\sum_{i=1}^{n}|\beta_i| + \frac{2}{c}\sum_{i=1}^{n}|\gamma_i| + \frac{4M_0}{(2c+1)^{\frac{2c+1}{2c}}}\sum_{i=1}^{n}|\delta_i| \le 1, \tag{4}$$

then for all δ complex numbers, $Re\delta \ge Re\gamma$, the integral operator $\mathcal{T}_n$, given by (1) is in the class $\mathcal{S}$.

Proof. Let us define the function

$$H_n(z) = \int_0^z \prod_{i=1}^{n}\left[\left(\frac{f_i(t)}{t}\right)^{\alpha_i - 1} \cdot \left(g_i'(t)\right)^{\beta_i} \cdot \left(\frac{h_i(t)}{k_i(t)}\right)^{\gamma_i} \cdot \left(\frac{h_i{}'(t)}{k_i{}'(t)}\right)^{\delta_i}\right] \mathrm{dt},$$

for $f_i, g_i, h_i, k_i \in \mathcal{A}$, $i = \overline{1,n}$.

The function H_n is regular in $\mathcal{U}$ and satisfy the following usual normalization conditions $H_n(0) = H_n'(0) - 1 = 0$.

We have

$$\frac{zH_n''(z)}{H_n'(z)} = \sum_{i=1}^{n}\left[(\alpha_i - 1)\left(\frac{zf_i'(z)}{f_i(z)} - 1\right) + \beta_i\frac{zg_i''(z)}{g_i'(z)}\right] +$$

$$+\sum_{i=1}^{n}\left[\gamma_i\left(\frac{zh_i'(z)}{h_i(z)} - \frac{zk_i'(z)}{k_i(z)}\right) + \delta_i\left(\frac{zh_i''(z)}{h_i'(z)} - \frac{zk_i''(z)}{k_i'(z)}\right)\right],$$

for all $z \in \mathcal{U}$.

Therefore

$$\frac{1-|z|^{2c}}{c}\left|\frac{zH_n''(z)}{H_n'(z)}\right| \le \frac{1-|z|^{2c}}{c}\sum_{i=1}^{n}\left[|\alpha_i - 1|\left|\frac{zf_i'(z)}{f_i(z)} - 1\right| + |\beta_i|\left|\frac{zg_i''(z)}{g_i'(z)}\right|\right] +$$

$$+\left[|\gamma_i|\left(\left|\frac{zh_i'(z)}{h_i(z)}-1\right|+\left|\frac{zk_i'(z)}{k_i(z)}-1\right|\right)+|\delta_i|\left(\left|\frac{zh_i''(z)}{h_i'(z)}\right|+\left|\frac{zk_i''(z)}{k_i'(z)}\right|\right)\right], \tag{5}$$

for all $z \in \mathcal{U}$.

Using (3), (4) and Lemma Mocanu and Şerb, from (5) we get

$$\left|\frac{zf_i'(z)}{f_i(z)}-1\right|<1, \quad \left|\frac{zh_i'(z)}{h_i(z)}-1\right|<1, \quad \left|\frac{zk_i'(z)}{k_i(z)}-1\right|<1,$$

for all $z \in \mathcal{U}$, $i=\overline{1,n}$ and hence, we have

$$\frac{1-|z|^{2c}}{c}\left|\frac{zH_n''(z)}{H_n'(z)}\right| \le \frac{1-|z|^{2c}}{c}\sum_{i=1}^{n}|\alpha_i-1|+$$

$$+\frac{1-|z|^{2c}}{c}|z|\,M_0\sum_{i=1}^{n}|\beta_i|+\frac{1-|z|^{2c}}{c}2\sum_{i=1}^{n}|\gamma_i|+\frac{1-|z|^{2c}}{c}|z|\,2M_0\sum_{i=1}^{n}|\delta_i|, \tag{6}$$

for all $z \in \mathcal{U}$

Since

$$\max_{|z|\le 1}\frac{\left(1-|z|^{2c}\right)|z|}{c}=\frac{2}{(2c+1)^{\frac{2c+1}{2c}}}, \tag{7}$$

from (6) and (7) we obtain

$$\frac{1-|z|^{2c}}{c}\left|\frac{zH_n''(z)}{H_n'(z)}\right| \le$$

$$\frac{1}{c}\sum_{i=1}^{n}|\alpha_i-1|+\frac{2M_0}{(2c+1)^{\frac{2c+1}{2c}}}\sum_{i=1}^{n}|\beta_i|+\frac{2}{c}\sum_{i=1}^{n}|\gamma_i|\frac{4M_0}{(2c+1)^{\frac{2c+1}{2c}}}\sum_{i=1}^{n}|\delta_i|, \tag{8}$$

for all $z \in \mathcal{U}$, $i=\overline{1,n}$.

Using(6), from (8) we have

$$\frac{1-|z|^{2c}}{c}\left|\frac{zH_n''(z)}{H_n'(z)}\right| \le 1. \tag{9}$$

Now, from (9), by Lemma 2.1, it results that the integral operator $\mathcal{T}_n$, given by (1) is in the class $\mathcal{S}$.

Letting $\delta = 1$ in Theorem 3.1, we have

Corollary 5. *Let $\gamma, \alpha_i, \beta_i, \gamma_i, \delta_i$ be complex numbers, $0 < Re\gamma \leq 1$, $c = Re\gamma$, M_0 the positive solution of the equation (3), $M_0 = 1,5936...$ and $f_i, g_i, h_i, k_i \in \mathcal{A}$, $f_i(z) = z + a_{2i}z^2 + a_{3i}z^3 + ...$, $g_i(z) = z + b_{2i}z^2 + b_{3i}z^3 + ...$, $h_i(z) = z + c_{2i}z^2 + c_{3i}z^3 + ...$, $k_i(z) = z + d_{2i}z^2 + d_{3i}z^3 + ...$, $i = \overline{1,n}$.*

If

$$\left|\frac{f_i''(z)}{f_i'(z)}\right| \leq M_0, \quad \left|\frac{g_i''(z)}{g_i'(z)}\right| \leq M_0, \quad \left|\frac{h_i''(z)}{h_i'(z)}\right| \leq M_0, \quad \left|\frac{k_i''(z)}{k_i'(z)}\right| \leq M_0,$$

for all $z \in \mathcal{U}$, $i = \overline{1,n}$ and

$$\frac{1}{c}\sum_{i=1}^{n}|\alpha_i - 1| + \frac{2M_0}{(2c+1)^{\frac{2c+1}{2c}}}\sum_{i=1}^{n}|\beta_i| + \frac{2}{c}\sum_{i=1}^{n}|\gamma_i| + \frac{4M_0}{(2c+1)^{\frac{2c+1}{2c}}}\sum_{i=1}^{n}|\delta_i| \leq 1,$$

then the integral operator $\mathcal{F}_n$ defined by

$$\mathcal{F}_n(z) = \int_0^z \prod_{i=1}^{n}\left[\left(\frac{f_i(t)}{t}\right)^{\alpha_i - 1} \cdot \left(g_i'(t)\right)^{\beta_i} \cdot \left(\frac{h_i(t)}{k_i(t)}\right)^{\gamma_i} \cdot \left(\frac{h_i{}'(t))}{k_i{}'(t)}\right)^{\delta_i}\right] dt, \tag{10}$$

is in the class $\mathcal{S}$.

Letting $\delta = 1$ and $\delta_1 = \delta_2 = ... = \delta_n = 0$ in Theorem 3.1, we have

Corollary 6. *Let $\gamma, \alpha_i, \beta_i, \gamma_i$ be complex numbers, $0 < Re\gamma \leq 1$, $c = Re\gamma$, M_0 the positive solution of the equation (2), $M_0 = 1,5936...$ and $f_i, g_i, h_i, k_i \in \mathcal{A}$, $f_i(z) = z + a_{2i}z^2 + a_{3i}z^3 + ...$, $g_i(z) = z + b_{2i}z^2 + b_{3i}z^3 + ...$, $h_i(z) = z + c_{2i}z^2 + c_{3i}z^3 + ...$, $k_i(z) = z + d_{2i}z^2 + d_{3i}z^3 + ...$, $i = \overline{1,n}$.*

If

$$\left|\frac{f_i''(z)}{f_i'(z)}\right| \leq M_0, \quad \left|\frac{g_i''(z)}{g_i'(z)}\right| \leq M_0, \quad \left|\frac{h_i''(z)}{h_i'(z)}\right| \leq M_0, \quad \left|\frac{k_i''(z)}{k_i'(z)}\right| \leq M_0,$$

for all $z \in \mathcal{U}$, $i = \overline{1,n}$ and

$$\frac{1}{c}\sum_{i=1}^{n}|\alpha_i - 1| + \frac{2M_0}{(2c+1)^{\frac{2c+1}{2c}}}\sum_{i=1}^{n}|\beta_i| + \frac{2}{c}\sum_{i=1}^{n}|\gamma_i| \leq 1,$$

then the integral operator $\mathcal{S}_n$ defined by

$$\mathcal{S}_n(z) = \int_0^z \prod_{i=1}^{n}\left[\left(\frac{f_i(t)}{t}\right)^{\alpha_i - 1} \cdot \left(g_i'(t)\right)^{\beta_i} \cdot \left(\frac{h_i(t)}{k_i(t)}\right)^{\gamma_i}\right] dt, \tag{11}$$

is in the class $\mathcal{S}$.

Letting $\delta = 1$ and $\beta_1 = \beta_2 = ... = \beta_n = 0$ in Theorem 3.1, we obtain

Corollary 7. *Let $\gamma, \alpha_i, \gamma_i, \delta_i$ be complex numbers, $0 < Re\gamma \le 1$, $c = Re\gamma$, $i = \overline{1,n}$, M_0 the positive solution of the equation (2), $M_0 = 1,5936...$ and $f_i, h_i, k_i \in \mathcal{A}$, $f_i(z) = z + a_{2i}z^2 + a_{3i}z^3 + ...$, $h_i(z) = z + c_{2i}z^2 + c_{3i}z^3 + ...$, $k_i(z) = z + d_{2i}z^2 + d_{3i}z^3 + ...$, $i = \overline{1,n}$.*

If

$$\left|\frac{f_i''(z)}{f_i'(z)}\right| \le M_0, \quad \left|\frac{h_i''(z)}{h_i'(z)}\right| \le M_0, \quad \left|\frac{k_i''(z)}{k_i'(z)}\right| \le M_0,$$

for all $z \in \mathcal{U}$, $i = \overline{1,n}$ and

$$\frac{1}{c}\sum_{i=1}^{n}|\alpha_i - 1| + \frac{2}{c}\sum_{i=1}^{n}|\gamma_i| + \frac{4M_0}{(2c+1)^{\frac{2c+1}{2c}}}\sum_{i=1}^{n}|\delta_i| \le 1,$$

then the integral operator $\mathcal{X}_n$ defined by

$$\mathcal{X}_n(z) = \int_0^z \prod_{i=1}^{n}\left[\left(\frac{f_i(t)}{t}\right)^{\alpha_i - 1} \cdot \left(\frac{h_i(t)}{k_i(t)}\right)^{\gamma_i} \cdot \left(\frac{{h_i}'(t))}{{k_i}'(t)}\right)^{\delta_i}\right] dt\ , \qquad (12)$$

is in the class $\mathcal{S}$.

Letting $\delta = 1$ and $\alpha_1 = \alpha_2 = ... = \alpha_n = 0$ in Theorem 3.1, we have

Corollary 8. *Let $\gamma, \beta_i, \gamma_i, \delta_i$ be complex numbers, $0 < Re\gamma \le 1$, $c = Re\gamma$, M_0 the positive solution of the equation (2), $M_0 = 1,5936...$ and $g_i, h_i, k_i \in \mathcal{A}$, $g_i(z) = z + b_{2i}z^2 + b_{3i}z^3 + ...$, $h_i(z) = z + c_{2i}z^2 + c_{3i}z^3 + ...$, $k_i(z) = z + d_{2i}z^2 + d_{3i}z^3 + ...$, $i = \overline{1,n}$.*

If

$$\left|\frac{g_i''(z)}{g_i'(z)}\right| \le M_0, \quad \left|\frac{h_i''(z)}{h_i'(z)}\right| \le M_0, \quad \left|\frac{k_i''(z)}{k_i'(z)}\right| \le M_0,$$

for all $z \in \mathcal{U}$, $i = \overline{1,n}$ and

$$\frac{2M_0}{(2c+1)^{\frac{2c+1}{2c}}}\sum_{i=1}^{n}|\beta_i| + \frac{2}{c}\sum_{i=1}^{n}|\gamma_i| + \frac{4M_0}{(2c+1)^{\frac{2c+1}{2c}}}\sum_{i=1}^{n}|\delta_i| \le 1,$$

then the integral operator $\mathcal{D}_n$ defined by

$$\mathcal{D}_n(z) = \int_0^z \prod_{i=1}^{n}\left[\left(g_i'(t)\right)^{\beta_i} \cdot \left(\frac{h_i(t)}{k_i(t)}\right)^{\gamma_i} \cdot \left(\frac{{h_i}'(t))}{{k_i}'(t)}\right)^{\delta_i}\right] dt, \qquad (13)$$

is in the class $\mathcal{S}$.

Letting $\delta = 1$ and $\gamma_1 = \gamma_2 = ... = \gamma_n = 0$ in Theorem 3.1, we have

Corollary 9. *Let $\gamma, \alpha_i, \beta_i, \delta_i$ be complex numbers, $0 < Re\gamma \le 1$, $c = Re\gamma$, M_0 the positive solution of the equation (2), $M_0 = 1,5936...$ and $f_i, g_i, h_i, k_i \in \mathcal{A}$, $f_i(z) = z + a_{2i}z^2 + a_{3i}z^3 + ...$, $g_i(z) = z + b_{2i}z^2 + b_{3i}z^3 + ...$, $h_i(z) = z + c_{2i}z^2 + c_{3i}z^3 + ...$, $k_i(z) = z + d_{2i}z^2 + d_{3i}z^3 + ...$, $i = \overline{1,n}$.*

If

$$\left|\frac{f_i''(z)}{f_i'(z)}\right| \le M_0, \quad \left|\frac{g_i''(z)}{g_i'(z)}\right| \le M_0, \quad \left|\frac{h_i''(z)}{h_i'(z)}\right| \le M_0, \quad \left|\frac{k_i''(z)}{k_i'(z)}\right| \le M_0,$$

for all $z \in \mathcal{U}$, $i = \overline{1,n}$ and

$$\frac{1}{c}\sum_{i=1}^{n}|\alpha_i - 1| + \frac{2M_0}{(2c+1)^{\frac{2c+1}{2c}}}\sum_{i=1}^{n}|\beta_i| + \frac{4M_0}{(2c+1)^{\frac{2c+1}{2c}}}\sum_{i=1}^{n}|\delta_i| \le 1,$$

then the integral operator $\mathcal{Y}_n$ defined by

$$\mathcal{Y}_n(z) = \int_0^z \prod_{i=1}^{n}\left[\left(\frac{f_i(t)}{t}\right)^{\alpha_i - 1} \cdot \left(g_i'(t)\right)^{\beta_i} \cdot \left(\frac{h_i{}'(t))}{k_i{}'(t)}\right)^{\delta_i}\right] dt\ , \tag{14}$$

is in the class $\mathcal{S}$.

Letting $n = 1$, $\delta = \gamma = \alpha$ and $\alpha_i - 1 = \beta_i = \gamma_i$ in Theorem 3.1, we obtain

Corollary 10. *Let α be complex number, $a = Re\alpha > 0$, M_0 the positive solution of the equation (2), $M_0 = 1,5936...$ and $f, g, h, k \in \mathcal{A}$, $f(z) = z + a_2z^2 + a_3z^3 + ...$, $g(z) = z + b_2z^2 + b_3z^3 + ...$, $h(z) = z + c_2z^2 + c_3z^3 + ...$, $k(z) = z + d_2z^2 + d_3z^3 + ...$*

If

$$\left|\frac{f''(z)}{f'(z)}\right| \le M_0, \quad \left|\frac{g''(z)}{g'(z)}\right| \le M_0, \quad \left|\frac{h''(z)}{h'(z)}\right| \le M_0, \quad \left|\frac{k''(z)}{k'(z)}\right| \le M_0,$$

for all $z \in \mathcal{U}$, and

$$\frac{\alpha - 1}{a} + \frac{2\beta M_0}{(2a+1)^{\frac{2a+1}{2a}}} + \frac{2\gamma}{a} + \frac{4\delta M_0}{(2a+1)^{\frac{2a+1}{2a}}} \le 1,$$

then the integral operator $\mathcal{T}$ defined by

$$\mathcal{T}(z) = \left[\alpha\int_0^z t^{\alpha-1}\left(f(t) \cdot g'(t) \cdot \frac{h(t)}{k(t)} \cdot \frac{h'(t))}{k'(t)}\right)^{\alpha-1} dt\right]^{\frac{1}{\alpha}}, \tag{15}$$

is in the class $\mathcal{S}$.

Letting $M_0 = M$ from (3) in Theorem 3.1, we obtain

Corollary 11. *Let* $\gamma, \delta, \alpha_i, \beta_i, \gamma_i, \delta_i$ *be complex numbers,* $c = Re\gamma > 0$*,* M *a positive number and* M_0 *the positive solution of the equation (2),* $M_0 = 1,5936...$ *and* $f_i, g_i, h_i, k_i \in \mathcal{A}$*,* $f_i(z) = z + a_{2i}z^2 + a_{3i}z^3 + ...$*,* $g_i(z) = z + b_{2i}z^2 + b_{3i}z^3 + ...$*,* $h_i(z) = z + c_{2i}z^2 + c_{3i}z^3 + ...$*,* $k_i(z) = z + d_{2i}z^2 + d_{3i}z^3 + ...$*,* $i = \overline{1,n}$

If

$$\left|\frac{f_i''(z)}{f_i'(z)}\right| \leq M_0, \quad \left|\frac{g_i''(z)}{g_i'(z)}\right| \leq M, \quad \left|\frac{h_i''(z)}{h_i'(z)}\right| \leq M_0, \quad \left|\frac{k_i''(z)}{k_i'(z)}\right| \leq M_0,$$

for all $z \in \mathcal{U}$*,* $i = \overline{1,n}$ *and*

$$\frac{1}{c}\sum_{i=1}^{n}|\alpha_i - 1| + \frac{2M}{c}\sum_{i=1}^{n}|\beta_i| + \frac{2}{c}\sum_{i=1}^{n}|\gamma_i| + \frac{4M_0}{(2c+1)^{\frac{2c+1}{2c}}}\sum_{i=1}^{n}|\delta_i| \leq 1,$$

then $f_i, g_i, h_i, k_i \in \mathcal{S}$*,* $i = \overline{1,n}$ *and for all* δ *complex numbers,* $Re\delta \geq Re\gamma$*, the integral operator* $\mathcal{T}_n$*, given by (1) is in the class* $\mathcal{S}$*.*

References

[1] D. Breaz, N. Breaz, *Two Integral Operators.* Studia Univ."Babes-Bolyai", Cluj-Napoca, Mathematica, 47(2002), no. 3, pg. 13-21.

[2] D. Breaz, N. Breaz, H. M. Srivastava, *An extension of the univalent condition for a family of integral operators.* Appl. Math. Lett., 22(2009), no. 3, 41-44.

[3] O. Mayer, *The Functions Theory of the One Variable Complex* . Acad. Ed., Bucuresti, Romania, 1981, 101-117.

[4] P. T. Mocanu, T. Bulboaca, G. S. Salagean, *Teoria geometrica a functiilor univalente.* Casa Cartii de Stiinta, Cluj Napoca (1999), 77-81.

[5] P. T. Mocanu, I. Şerb, *A sharp simple criterion for a subclass of starlike functions.* Complex variables, 32(1997), 161-168.

[6] S. Ozaki, M. Nunokawa, *The Schwarzian derivative and univalent functions.* Proceedings of the American Mathematical Society, Mathematics, 33(1972), 392-394.

[7] N. N. Pascu, *An a univalence criterion II.* Itinerant Seminar on Functional Equations, Approximation and Convexity, Cluj Napoca 1985, 153-154.

[8] V. Pescar, *New univalence criteria for some integral operators.* Studia Univ."Babes-Bolyai", Cluj-Napoca, Mathematica, 59(2014), no. 2, 185-192.

[9] V. Pescar, N. Breaz, *Mocanu and Şerb type univalence criteria for some general integral operators.* Acta Universitatis Apulensis, 44(2015), 1-8.

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