Acta Universitatis Apulensis No. 62/2020
ISSN: 1582-5329 pp. 19-28
http://www.uab.ro/auajournal/ doi: 10.17114/j.aua.2020.62.02

A DIFFERENTIAL SANDWICH THEOREM FOR ANALYTIC FUNCTIONS DEFINED BY AN INTEGRAL OPERATOR

L.I. Cotîrlă and A. Cătaş

Abstract. In this paper we obtain some subordination and superordination results involving a generalized Sălăgean integral operator for certain normalized analytic functions in the open unit disk.

2010 *Mathematics Subject Classification*: 30C80, 30C45.

Keywords: subordination and superordination, sandwich theorem, integral operator.

1. Introduction

We will determine some properties on the admissible functions defined with the generalized Sălăgean integral operator.

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k,\ a_k \geq 0, \tag{1}$$

which are analytic and univalent in the open unit disk $U = \{z : |z| < 1\}$.

If f and g are analytic functions in U, we say that f is subordinate to g in U, written symbolically as $f \prec g$ or $f(z) \prec g(z)$ if there exists a Schwarz function $w(z)$ analytic in U, with $w(0) = 0$ and $|w(z)| < 1$, such that $f(z) = g(w(z))$, $z \in U$. In particular, if the function g is univalent in U, the subordination $f \prec g$ is equivalent to $f(0) = g(0)$ and $f(U) \subset g(U)$, (see [2], [3]).

For the function f given by (1) and $g \in \mathcal{A}$ given by $g(z) = z + \sum_{k=2}^{\infty} b_k z^k$, the Hadamard product (or convolution) of f and g is defined by

$$(f * g)(z) = z + \sum_{k=2}^{\infty} a_k b_k z^k = (g * f)(z).$$

The set of all functions f that are analytic and injective on $\overline{U} - E(f)$, denote by Q where

$$E(f) = \{\zeta \in \partial U : \lim_{z \to \zeta} f(z) = \infty\}$$

and are such that $f'(\zeta) \neq 0$ for $\zeta \in \partial U \backslash E(f)$, (see [4]).

If $\psi : \mathbb{C}^3 \times U \to \mathbb{C}$ and h is univalent in U with $q \in Q$. In [3] Miller and Mocanu consider the problem of determining conditions on admissible functions ψ such that

$$\psi(p(z), zp'(z), z^2 p''(z); z) \prec h(z) \tag{2}$$

implies that $p(z) \prec q(z)$ for all functions $p \in \mathcal{H}[a, n]$ that satisfy the differential subordination (2).

Let $\phi : \mathbb{C}^3 \times U \to \mathbb{C}$ and $h \in \mathcal{H}$ with $q \in \mathcal{H}[a, n]$. In [4] and [5] is studied the dual problem and determined conditions on ϕ such that

$$h(z) \prec \phi(p(z), zp'(z), z^2 p''(z); z) \tag{3}$$

implies $q(z) \prec p(z)$ for all functions $p \in Q$ that satisfy the above subordination. They also found conditions so that the functions q is the largest function with this property, called the best subordinant of the subordination (3) .

Let $\mathcal{H}(U)$ be the class of analytic functions in the open unit disc.

For n a positive integer and $a \in \mathbb{C}$ let

$$\mathcal{H}[a, n] = \{f \in \mathcal{H} : f(z) = a + a_n z^n + \ldots\}.$$

The integral operator I^m of a function f is defined in [7] by

$$I^0 f(z) = f(z),$$

$$I^1 f(z) = I f(z) = \int_0^z f(t) t^{-1} dt,$$

$$\ldots$$

$$I^m f(z) = I\left(I^{m-1} f(z)\right), \; z \in U.$$

For $m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $\lambda > 0$ and $f \in \mathcal{A}$, Patel [6] considered the integral operator I_λ^m defined as follows:

$$I_\lambda^0 f(z) = f(z),$$

$$I_\lambda^1 f(z) = \frac{1}{\lambda} z^{1-\frac{1}{\lambda}} \int_0^z f(t) t^{\frac{1}{\lambda}-2} dt = z + \sum_{k=2}^{\infty} \left[\frac{1}{1 + \lambda(k-1)}\right] a_k z^k,$$

$$I_\lambda^2 f(z) = \frac{1}{\lambda} z^{1-\frac{1}{\lambda}} \int_0^z I_\lambda^1 f(t) t^{\frac{1}{\lambda}-2} dt = z + \sum_{k=2}^{\infty} \left[\frac{1}{1+\lambda(k-1)} \right]^2 a_k z^k,$$

and (in general)

$$I_\lambda^m f(z) = \frac{1}{\lambda} z^{1-\frac{1}{\lambda}} \int_0^z I_\lambda^{m-1} f(t) t^{\frac{1}{\lambda}-2} dt = z + \sum_{k=2}^{\infty} \left[\frac{1}{1+\lambda(k-1)} \right]^m a_k z^k =$$

$$= \underbrace{I_\lambda^1 \left(\frac{z}{1-z} \right) * I_\lambda^1 \left(\frac{z}{1-z} \right) * \ldots * I_\lambda^1 \left(\frac{z}{1-z} \right)}_{m-times} * f(z). \tag{4}$$

Then, from (4) we can easily deduce that

$$\lambda z \left(I_\lambda^m f(z) \right)' = I_\lambda^{m-1} f(z) - (1-\lambda) I_\lambda^m f(z), \ \lambda > 0, m \in \mathbb{N}.$$

We note that $I_1^m f(z) = I^m f(z)$, where I^m is Sălăgean integral operator [7].

2. Preliminaries

In our present investigation we shall need the following results.

Theorem 1. *[3] Let the function q be univalent in U and let θ, φ be analytic in a domain D containing $q(U)$ with $\varphi(w) \neq 0$, where $w \in q(U)$. Set*

$$Q(z) = zq'(z)\varphi(q(z)) \quad \text{and} \quad h(z) = \theta(q(z)) + Q(z).$$

Suppose that either

i) h is convex or

ii) Q is starlike.

In adition, assume that

$$iii) \ \mathrm{Re} \left\{ \frac{zh'(z)}{Q(z)} \right\} > 0.$$

If p is analytic with $p(0) = q(0)$, $p(U) \subset D$ and

$$\theta(p(z)) + zp'(z) \cdot \varphi(p(z)) \prec \theta(q(z)) + zq'(z)\varphi(q(z)) = h(z)$$

then $p(z) \prec q(z)$ and q is the best dominant.

By taking $\theta(w) := w$ and $\varphi(w) = \gamma$ in Theorem 2, we get

Corollary 2. *Let q be univalent in U, $\gamma \in \mathbb{C}^*$ and suppose*

$$\mathrm{Re}\left\{1 + \frac{zq''(z)}{q'(z)}\right\} > \max\left\{0, -\mathrm{Re}\left(\frac{1}{\gamma}\right)\right\}.$$

If p is analytic in U with $p(0) = q(0)$ and

$$p(z) + \gamma z p'(z) \prec q(z) + \gamma z q'(z)$$

then $p(z) \prec q(z)$ and q is the best dominant.

Theorem 3. *[5] Let θ and φ be analytic in a domain D and let the function q be univalent in U, with $q(0) = a$, $q(U) \subset D$. Set*

$$\begin{aligned} Q(z) &= zq'(z)\varphi(q(z)) \\ h(z) &= \theta(q(z)) + Q(z) \end{aligned}$$

and suppose that
1. $\mathrm{Re}\left\{\dfrac{\theta'(q(z))}{\varphi(q(z))}\right\} > 0$ *for $z \in U$ and*
2. $Q(z)$ is starlike in U.
If $p \in \mathcal{H}[q(0), 1] \cap Q$ with $p(U) \subset D$ and $\theta(p(z)) + zp'(z)\varphi(p(z))$ is univalent in U and

$$\theta(q(z)) + zq'(z)\varphi(q(z)) \prec \theta(p(z)) + zp'(z)\varphi(p(z))$$

then $q(z) \prec p(z)$ and q is the best subordinant.

By taking $\theta(w) := w$ and $\varphi(w) = \gamma$ in Theorem 3, we get

Corollary 4. *[1] Let q be convex in U, $q(0) = a$ and $\gamma \in \mathbb{C}$, $\mathrm{Re}\,(\gamma) > 0$. If $p \in \mathcal{H}[a, 1] \cap Q$ and $p(z) + \gamma z p'(z)$ is univalent in U, then*

$$q(z) + \gamma z q'(z) \prec p(z) + \gamma z p'(z)$$

implies $q(z) \prec p(z)$ and q is the best subordinant.

3. Main results

Theorem 5. *Let q be univalent in U, with $q(0)=1$, $\alpha \in \mathbb{C}^*$, $\delta > 0$ and suppose*

$$\mathrm{Re}\left\{1+\frac{zq''(z)}{q'(z)}\right\} > \max\left\{0, -\mathrm{Re}\,\frac{\delta}{\alpha}\right\}.$$

If $f \in \mathcal{A}$ satisfies the subordination

$$\left(1-\frac{\alpha}{\lambda}\right)\left(\frac{I_\lambda^{m+1}\left(f(z)\right)}{z}\right)^\sigma + \frac{\alpha}{\lambda}\left(\frac{I_\lambda^{m+1}\left(f(z)\right)}{z}\right)^\sigma \cdot \frac{I_\lambda^{m}\left(f(z)\right)}{I_\lambda^{m+1}\left(f(z)\right)} \prec q(z)+\frac{\alpha}{\delta}zq'(z) \quad (5)$$

then

$$\left(\frac{I_\lambda^{m+1}\left(f(z)\right)}{z}\right)^\sigma \prec q(z)$$

and q is the best dominant.

Proof. We define the function

$$p(z) = \left(\frac{I_\lambda^{m+1}\left(f(z)\right)}{z}\right)^\sigma, \; z \in U.$$

By calculating the logarithmic derivative of p, we obtain

$$\frac{zp'(z)}{p(z)} = \delta\left(\frac{z\left(I_\lambda^{m+1}\left(f(z)\right)\right)'}{I_\lambda^{m+1}\left(f(z)\right)} - 1\right). \quad (6)$$

Because

$$\lambda z\left(I_\lambda^{m+1}f(z)\right)' = I_\lambda^{m}f(z) - (1-\lambda)\,I_\lambda^{m+1}f(z), \quad (7)$$

ecuation (6) becomes

$$\frac{zp'(z)}{p(z)} = \frac{\delta}{\lambda}\left(\frac{I_\lambda^{m}\left(f(z)\right)}{I_\lambda^{m+1}\left(f(z)\right)} - 1\right)$$

and therefore

$$\frac{zp'(z)}{\delta} = \frac{1}{\lambda}\left(\frac{I_\lambda^{m+1}\left(f(z)\right)}{z}\right)^\sigma\left(\frac{I_\lambda^{m}\left(f(z)\right)}{I_\lambda^{m+1}\left(f(z)\right)} - 1\right).$$

The subordination (5) from the hypothesis becomes

$$p(z)+\frac{\alpha}{\delta}zp'(z) \prec q(z)+\frac{\alpha}{\delta}zq'(z).$$

We apply now Corrolary 4 with $\gamma = \frac{\alpha}{\delta}$ to obtain the conclusion of our theorem.

If we consider $m = 0$ in Theorem 5 we obtain the following result.

Corollary 6. *Let q be univalent in U, with $q(0) = 1$, $\alpha \in \mathbb{C}^*$, $\delta > 0$ and suppose*

$$\text{Re}\left\{1 + \frac{zq''(z)}{q'(z)}\right\} > \max\left\{0, -\text{Re}\,\frac{\delta}{\alpha}\right\}.$$

If $f \in \mathcal{A}$ satisfies the subordination

$$\left(1 - \frac{\alpha}{\lambda}\right)\left(\frac{I_\lambda^1(f(z))}{z}\right)^\sigma + \frac{\alpha}{\lambda}\left(\frac{I_\lambda^1(f(z))}{z}\right)^\sigma \cdot \frac{f(z)}{I_\lambda^1(f(z))} \prec q(z) + \frac{\alpha}{\delta}zq'(z) \tag{8}$$

then

$$\left(\frac{I_\lambda^1(f(z))}{z}\right)^\sigma \prec q(z)$$

and q is the best dominant.

If $\lambda = 1$ in Theorem 5 we get the following corollary.

Corollary 7. *Let q be univalent in U, with $q(0) = 1$, $\alpha \in \mathbb{C}^*$, $\delta > 0$ and suppose*

$$\text{Re}\left\{1 + \frac{zq''(z)}{q'(z)}\right\} > \max\left\{0, -\text{Re}\,\frac{\delta}{\alpha}\right\}.$$

If $f \in \mathcal{A}$ satisfies the subordination

$$(1 - \alpha)\left(\frac{I^{m+1}(f(z))}{z}\right)^\sigma + \alpha\left(\frac{I^{m+1}(f(z))}{z}\right)^\sigma \cdot \frac{I^m(f(z))}{I^{m+1}(f(z))} \prec q(z) + \frac{\alpha}{\delta}zq'(z)$$

then

$$\left(\frac{I^{m+1}(f(z))}{z}\right)^\sigma \prec q(z)$$

and q is the best dominant.

If we take $m = 0$ and $\lambda = 1$ in Theorem 5 then we obtain the next result.

Corollary 8. *Let q be univalent in U, with $q(0) = 1$, $\alpha \in \mathbb{C}^*$, $\delta > 0$ and suppose*

$$\text{Re}\left\{1 + \frac{zq''(z)}{q'(z)}\right\} > \max\left\{0, -\text{Re}\,\frac{\delta}{\alpha}\right\}.$$

If $f \in \mathcal{A}$ satisfies the subordination

$$(1 - \alpha)\left(\frac{I^1(f(z))}{z}\right)^\sigma + \alpha\left(\frac{I^1(f(z))}{z}\right)^\sigma \cdot \frac{f(z)}{I^1(f(z))} \prec q(z) + \frac{\alpha}{\delta}zq'(z)$$

then

$$\left(\frac{I^1\left(f(z)\right)}{z}\right)^{\sigma} \prec q(z)$$

and q is the best dominant.

We consider a particular convex function $q(z) = \frac{1+Az}{1+Bz}$ to give the following application of Theorem 5.

Corollary 9. *Let $A, B, \alpha \in \mathbb{C}$, $A \neq B$ be such that $|B| \leq 1$, $\Re\alpha > 0$ and let $\delta > 0$. If $f \in \mathcal{A}$ satisfies the subordination*

$$\left(1-\frac{\alpha}{\lambda}\right)\left(\frac{I_{\lambda}^{m+1}\left(f(z)\right)}{z}\right)^{\sigma}+\frac{\alpha}{\lambda}\left(\frac{I_{\lambda}^{m+1}\left(f(z)\right)}{z}\right)^{\sigma}\cdot\frac{I_{\lambda}^{m}\left(f(z)\right)}{I_{\lambda}^{m+1}\left(f(z)\right)} \prec \frac{1+Az}{1+Bz}+\frac{\alpha}{\delta}\frac{(A-B)\,z}{(1+Bz)^2}$$

then

$$\left(\frac{I_{\lambda}^{m+1}\left(f(z)\right)}{z}\right)^{\sigma} \prec \frac{1+Az}{1+Bz}$$

and $q(z) = \frac{1+Az}{1+Bz}$ is the best dominant.

Theorem 10. *Let q be convex in U, with $q(0) = 1$, $\alpha \in \mathbb{C}$ with $\Re\alpha > 0$, $\delta > 0$. If $f \in \mathcal{A}$ such that*

$$\left(\frac{I_{\lambda}^{m+1}\left(f(z)\right)}{z}\right)^{\delta} \in \mathcal{H}[q(0), 1] \cap Q,$$

$\left(1-\frac{\alpha}{\lambda}\right)\left(\frac{I_{\lambda}^{m+1}(f(z))}{z}\right)^{\sigma}+\frac{\alpha}{\lambda}\left(\frac{I_{\lambda}^{m+1}(f(z))}{z}\right)^{\sigma}\cdot\frac{I_{\lambda}^{m}(f(z))}{I_{\lambda}^{m+1}(f(z))}$ *is univalent in U and satisfies the subordination*

$$q(z)+\frac{\alpha}{\delta}zq'(z) \prec \left(1-\frac{\alpha}{\lambda}\right)\left(\frac{I_{\lambda}^{m+1}\left(f(z)\right)}{z}\right)^{\sigma}+\frac{\alpha}{\lambda}\left(\frac{I_{\lambda}^{m+1}\left(f(z)\right)}{z}\right)^{\sigma}\cdot\frac{I_{\lambda}^{m}\left(f(z)\right)}{I_{\lambda}^{m+1}\left(f(z)\right)}, \quad (9)$$

then $q(z) \prec \left(\frac{I_{\lambda}^{m+1}(f(z))}{z}\right)^{\sigma}$ and q is the best subordinant.

Proof. Let

$$p(z) = \left(\frac{I_{\lambda}^{m+1}\left(f(z)\right)}{z}\right)^{\sigma}, \ z \in U.$$

If we proceed as in the proof of Theorem 5, the subordination (9) becomes

$$q(z) + \frac{\alpha\lambda}{\delta}zq'(z) \prec p(z) + \frac{\alpha\lambda}{\delta}zp'(z).$$

Applying Corollary 4 with $\gamma = \frac{\alpha\lambda}{\delta}$ the proof is completed.

If we consider $m = 0$ in Theorem 10 we obtain the following result.

Corollary 11. *Let q be convex in U, with $q(0) = 1$, $\alpha \in \mathbb{C}$ with $\Re\alpha > 0$, $\delta > 0$. If $f \in \mathcal{A}$ such that*

$$\left(\frac{I_\lambda^1(f(z))}{z}\right)^\delta \in \mathcal{H}[q(0), 1] \cap Q,$$

$\left(1-\frac{\alpha}{\lambda}\right)\left(\frac{I_\lambda^1(f(z))}{z}\right)^\sigma + \frac{\alpha}{\lambda}\left(\frac{I_\lambda^1(f(z))}{z}\right)^\sigma \cdot \frac{f(z)}{I_\lambda^1(f(z))}$ *is univalent in U and satisfies the subordination*

$$q(z) + \frac{\alpha}{\delta} z q'(z) \prec \left(1-\frac{\alpha}{\lambda}\right)\left(\frac{I_\lambda^1(f(z))}{z}\right)^\sigma + \frac{\alpha}{\lambda}\left(\frac{I_\lambda^1(f(z))}{z}\right)^\sigma \cdot \frac{f(z)}{I_\lambda^1(f(z))},$$

then $q(z) \prec \left(\frac{I_\lambda^1(f(z))}{z}\right)^\sigma$ and q is the best subordinant.

If $\lambda = 1$ in Theorem 10 we obtain the following corollary.

Corollary 12. *Let q be convex in U, with $q(0) = 1$, $\alpha \in \mathbb{C}$ with $\Re\alpha > 0$, $\delta > 0$. If $f \in \mathcal{A}$ such that*

$$\left(\frac{I^{m+1}(f(z))}{z}\right)^\delta \in \mathcal{H}[q(0), 1] \cap Q,$$

$(1-\alpha)\left(\frac{I^{m+1}(f(z))}{z}\right)^\sigma + \alpha\left(\frac{I^{m+1}(f(z))}{z}\right)^\sigma \cdot \frac{I^m(f(z))}{I^{m+1}(f(z))}$ *is univalent in U and satisfies the subordination*

$$q(z) + \frac{\alpha}{\delta} z q'(z) \prec (1-\alpha)\left(\frac{I^{m+1}(f(z))}{z}\right)^\sigma + \alpha\left(\frac{I^{m+1}(f(z))}{z}\right)^\sigma \cdot \frac{I^m(f(z))}{I^{m+1}(f(z))},$$

then $q(z) \prec \left(\frac{I^{m+1}(f(z))}{z}\right)^\sigma$ and q is the best subordinant.

Concluding the results of differential subordination and superordination we state the following sandwich theorem.

Theorem 13. *Let q_1, q_2 be convex in U, with $q_1(0) = q_2(0) = 1$, $\alpha \in \mathbb{C}$ with $\Re\alpha > 0$, $\delta > 0$. If $f \in \mathcal{A}$ such that*

$$\left(\frac{I_\lambda^{m+1}(f(z))}{z}\right)^\delta \in \mathcal{H}[q(0), 1] \cap Q,$$

$\left(1-\frac{\alpha}{\lambda}\right)\left(\frac{I_\lambda^{m+1}(f(z))}{z}\right)^\sigma + \frac{\alpha}{\lambda}\left(\frac{I_\lambda^{m+1}(f(z))}{z}\right)^\sigma \cdot \frac{I_\lambda^m(f(z))}{I_\lambda^{m+1}(f(z))}$ *is univalent in U and satisfies*

$$q_1(z) + \frac{\alpha}{\delta} z q_1'(z) \prec \left(1-\frac{\alpha}{\lambda}\right)\left(\frac{I_\lambda^{m+1}(f(z))}{z}\right)^\sigma + \frac{\alpha}{\lambda}\left(\frac{I_\lambda^{m+1}(f(z))}{z}\right)^\sigma \cdot \frac{I_\lambda^m(f(z))}{I_\lambda^{m+1}(f(z))} \prec q_2(z) + \frac{\alpha}{\delta} z q_2'(z)$$

then $q_1(z) \prec \left(\frac{I_\lambda^{m+1}(f(z))}{z}\right)^\sigma \prec q_2(z)$ *and* q_1, q_2 *are the best subordinant and the best dominant respectively .*

If $m = 0$ in Theorem 13 we obtain the following result.

Corollary 14. *Let* q_1, q_2 *be convex in* U, *with* $q_1(0) = q_2(0) = 1$, $\alpha \in \mathbb{C}$ *with* $\Re\alpha > 0$, $\delta > 0$. *If* $f \in \mathcal{A}$ *such that*

$$\left(\frac{I_\lambda^1(f(z))}{z}\right)^\delta \in \mathcal{H}[q(0), 1] \cap Q,$$

$\left(1-\frac{\alpha}{\lambda}\right)\left(\frac{I_\lambda^1(f(z))}{z}\right)^\sigma + \frac{\alpha}{\lambda}\left(\frac{I_\lambda^1(f(z))}{z}\right)^\sigma \cdot \frac{f(z)}{I_\lambda^1(f(z))}$ *is univalent in* U *and satisfies*

$$q_1(z)+\frac{\alpha}{\delta}zq_1'(z) \prec \left(1-\frac{\alpha}{\lambda}\right)\left(\frac{I_\lambda^1(f(z))}{z}\right)^\sigma + \frac{\alpha}{\lambda}\left(\frac{I_\lambda^1(f(z))}{z}\right)^\sigma \cdot \frac{f(z)}{I_\lambda^1(f(z))} \prec q_2(z)+\frac{\alpha}{\delta}zq_2'(z)$$

then $q_1(z) \prec \left(\frac{I_\lambda^1(f(z))}{z}\right)^\sigma \prec q_2(z)$ *and* q_1, q_2 *are the best subordinant and the best dominant respectively .*

If $\lambda = 1$ in Theorem 13 we get the following corollary.

Corollary 15. *Let* q_1, q_2 *be convex in* U, *with* $q_1(0) = q_2(0) = 1$, $\alpha \in \mathbb{C}$ *with* $\Re\alpha > 0$, $\delta > 0$. *If* $f \in \mathcal{A}$ *such that*

$$\left(\frac{I_1^{m+1}(f(z))}{z}\right)^\delta \in \mathcal{H}[q(0), 1] \cap Q,$$

$(1-\alpha)\left(\frac{I_1^{m+1}(f(z))}{z}\right)^\sigma + \alpha\left(\frac{I_1^{m+1}(f(z))}{z}\right)^\sigma \cdot \frac{I_1^m(f(z))}{I_1^{m+1}(f(z))}$ *is univalent in* U *and satisfies*

$$q_1(z)+\frac{\alpha}{\delta}zq_1'(z) \prec (1-\alpha)\left(\frac{I_1^{m+1}(f(z))}{z}\right)^\sigma + \alpha\left(\frac{I_1^{m+1}(f(z))}{z}\right)^\sigma \cdot \frac{I_1^m(f(z))}{I_1^{m+1}(f(z))} \prec q_2(z)+\frac{\alpha}{\delta}zq_2'(z)$$

then $q_1(z) \prec \left(\frac{I_1^{m+1}(f(z))}{z}\right)^\sigma \prec q_2(z)$ *and* q_1, q_2 *are the best subordinant and the best dominant respectively .*

References

[1] T. Bulboacă, *Classes of first-order differential superordinations*, Demonstr. Math., 35, (2)(2002), 287-292.

[2] T. Bulboacă, *Differential Subordinations and Superordinations*, Recent Results, House of Scientific Book Publ., Cluj-Napoca, 2005.

[3] S.S. Miller and P.T. Mocanu, *Differential Subordinations: Theory and Applications*, in Series on Monographs and Textbooks in Pure and Applied Mathematics, No. 225, Marcel Dekker, New York, (2000).

[4] S.S. Miller and P.T. Mocanu, *Subordinants of differential superordinations*, Compelx Variables, 48(10)(2003), 815-826.

[5] S.S. Miller and P.T. Mocanu, *Briot-Bouquet differential superordinations and sandwich theorems*, J. Math., Anal. Appl., 329(1), (2007), 327-335.

[6] J. Patel, *Inclusion relations and convolution properties of certain subclasses of analytic functions defined by generalized Sălăgean operator*, Bull., Belg. Math. Soc. Simon Stevin, 15(2008), 33-47.

[7] G. Şt. Sălăgean, *Subclasses of univalent functions*, Complex Analysis, Fifth Romanian-Finnish Seminar, Part 1 (Bucharest, 1981), Lecture Notes in Math., vol. 1013, Springer, Berlin, 1983, 362-372.

Luminiţa-Ioana Cotîrlă
Department of Mathematics,
Technical University of Cluj-Napoca,
Cluj-Napoca, Romania,
email: *luminita.cotirla@yahoo.com*, *Luminita.Cotirla@math.utcluj.ro*

Adriana Cătaş
Department of Mathematics and Computer Science,
University of Oradea, Romania
email: *acatas@gmail.com*