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Study of Γ-hyperrings by fuzzy hyperideals with respect to a t-norm

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Abstract. In this paper, we study the Γ-hyperrings via T-fuzzy hyperideals. By means of the use of a triangular norm T, we define, characterize and study the T-fuzzy left and right hyperideals, T-fuzzy quasi-hyperideal and bi-hyperideal in Γ-hyperrings and some related properties are investigated. Regular Γ-hyperrings are characterized in terms of T-fuzzy quasi-hyperideal and T-fuzzy bi-hyperideal. We also introduce the T-(λ, μ)-fuzzy bi-hyperideals in Γ-hyperrings and investigate some of their properties.

1 Introduction and preliminaries

The applications of mathematics in other disciplines, for example in informatics, play a key role and they represent, in the last decades, one of the purposes

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of the study of the experts of Hyperstructures Theory all over the world. Hyperstructures, as a natural extension of classical algebraic structures, in particular hypergroups, were introduced in 1934 by a French mathematician, Marty, at the 8th Congress of Scandinavian Mathematicians [29]. Since then, a lot of papers and several books have been written on this topic. Nowadays, hyperstructures have a lot of applications to several domains of mathematics and computer science(see [10, 24, 25, 33, 41, 44]) and they are studied in many countries of the world. This theory has been subsequently developed by Corsini [11, 9, 10], Mittas [27, 28], Stratigopoulos [40] and by various authors. Basic definitions and propositions about the hyperstructures are found in [9, 10, 15, 29, 41]. Krasner [22] has studied the notion of hyperfields, hyperrings, and then some researchers, namely, Ameri [2], Dasic [12], Davvaz [14, 30, 13], Gontineac [19], Massouros [26], Pianskool et al. [34], Sen and Dasgupta [38], Vougiouklis [41, 42] and others followed him.

Hyperrings are essentially rings with approximately modified axioms. There are different notions of hyperrings $(R, +, \cdot)$. If the addition $+$ is a hyperoperation and the multiplication $\cdot$ is a binary operation, then the hyperring is called Krasner (additive) hyperring [22]. Rota [35] introduced a multiplicative hyperring, where $+$ is a binary operation and the multiplication $\cdot$ is a hyperoperation. De Salvo [37] studied hyperrings in which the additions and the multiplications were hyperoperations. These hyperrings were also studied by Barghi [7] and by Asokkumar and Velrajan [5, 6]. In 2007, Davvaz and Leoreanu-Fotea [15] published a book titled Hyperring Theory and Applications.

Γ-rings were introduced by Nobusawa [31] as an algebraic tool for observing the relationship between the groups of homomorphisms $\mathrm{hom}(B, C)$ and $\mathrm{hom}(C, B)$ of commutative groups B and C. The class of Γ-rings contains not only all rings but also Hestenes ternary rings. Barnes [8] weakened slightly the conditions in the definition of Γ-ring in the sense of Nobusawa. The study of Γ-hyperrings as a generalization of rings, ternary rings, Γ-rings was initiated by Ameri et. al. [4].

After the introduction of the concept of fuzzy sets by Zadeh in 1965 [45], it has found manifold applications in the field of mathematics and related areas. This provides sufficient motivations for researchers to review various concepts and results from the realm of abstract algebra to a broader framework of fuzzy setting. The study of fuzzy algebraic structures was started with the introduction of the concepts of fuzzy subgroups by Rosenfeld [36]. The study of fuzzy hyperstructures is an interesting research topic of fuzzy sets.There is a considerable amount of work on the connections between fuzzy sets and hyperstructures. Davvaz [17] introduced the concept of fuzzy hyperideals in a semihypergroup. In 2009, Davvaz [18] gave the concept of fuzzy hyperideals in ternary semihyperrings. The relationships between the fuzzy sets and algebraic hyperstructures have been considered by Corsini, Davvaz, Leoreanu, Zhan, Zahedi, Ameri, Cristea and many other researchers. For more on fuzzy hyperstructures one can see [16]. The fuzzy hyperring notion is defined and studied in [23]. The fuzzy Γ-hyperring notion is defined and studied in [4, 43].

On the other hand, in 1960, Schweizer and Sklar [39] introduced the notion of triangular norm (t-norm) and triangular conorm (t-conorm) in order to generalize the ordinary triangle inequality in a metric space to the more general probabilistic metric space. Using t-norm, Anthony and Sherwood [1] first redefined Rosenfeld's [36] notion of fuzzy groups. Since then t-norm has played an important role in fuzzy algebra. In application, t-norm T and t-conorm S are the functions that map the unit square into the unit interval. In fuzzy sets theory, triangular norm (t-norm) is extensively used to model the logical connective: conjunction (AND). There are many applications of triangular norms in several fields of mathematics and artificial intelligence [21].

In this paper, we inquire further into the properties on some kind fuzzy hyperideals and we study the Γ-hyperrings via T-fuzzy hyperideals. By means of the use of a triangular norm T, we define, characterize and study the T-fuzzy left and right hyperideals, T-fuzzy quasi-hyperideal and bi-hyperideal in Γ-hyperrings and some related properties are investigated. We compare fuzzy hyperideal to T-fuzzy hyperideals. We have shown that Γ-hyperring is regular if and only if intersection of any T-fuzzy right hyperideal with T-fuzzy left hyperideal is equal to its product. We introduce the notion of T-fuzzy quasi-hyperideal and T-fuzzy bi-hyperideal. We discuss some of its properties. We have shown that the meet of T-fuzzy right and T-fuzzy left ideal is a T-fuzzy quasi hyperideal of a Γ-hyperring. We characterize regular Γ-hyperring with T-fuzzy quasi-hyperideal and T-fuzzy bi-hyperideal. We also introduce the T-(λ, μ)-fuzzy bi-hyperideals in Γ-hyperrings and investigate some of their properties.

Recall first the basic terms and definitions from the hyperstructure theory. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set.

A map $\circ : H \times H \to \mathcal{P}^*(H)$ is called *hyperoperation* or *join operation* on the set H, where H is a non-empty set and $\mathcal{P}^*(H) = \mathcal{P}(H)\backslash\{\emptyset\}$ denotes the set of all nonempty subsets of H. A *hyperstructure* is called the pair $(H, \circ)$ where $\circ$ is a hyperoperation on the set H. A hyperstructure $(H, \circ)$ is called a *semihypergroup* if for all $x, y, z \in H$, $(x \circ y) \circ z = x \circ (y \circ z)$, which means that

$$\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v.$$

If $x \in H$ and A, B are nonempty subsets of H, then

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, A \circ x = A \circ \{x\}, \text{ and } x \circ B = \{x\} \circ B.$$

A non-empty subset B of a semihypergroup H is called a *sub-semihypergroup* of H if $B \circ B \subseteq B$ and H is called in this case *super-semihypergroup* of B. Let $(H, \circ)$ be a semihypergroup. Then H is called a *hypergroup* if it satisfies the reproduction axiom, for all $a \in H$, $a \circ H = H \circ a = H$. An element e in a semihypergroup H is called *identity* if

$$x \circ e = e \circ x = \{x\}, \forall x \in H.$$

An element 0 in a semihypergroup H is called *zero element* if

$$x \circ 0 = 0 \circ x = \{0\}, \forall x \in H.$$

A non-empty set H with a hyperoperation $+$ is said to be a *canonical hypergroup* if the following conditions hold:

1. for every $x, y \in H, x + y = y + x$,
2. for every $x, y, z \in H, x + (y + z) = (x + y) + z$,
3. there exists $0 \in H$, (called neutral element of H) such that $0 + x = \{x\} = x + 0$ for all $x \in H$,
4. for every $x \in H$, there exists a unique element denoted by $-x \in H$ such that $0 \in x + (-x) \cap (-x) + x$,
5. for every $x, y, z \in H$, $z \in x + y$ implies $y \in -x + z$ and $x \in z - y$.

A comprehensive review of the theory of hypergroups appears in [9]. For any subset A of a canonical hypergroup H, $-A$ denotes the set $\{-a : a \in A\}$. A non-empty subset N of a canonical hypergroup of H is called a subcanonical hypergroup of H if N is a canonical hypergroup under the same hyperoperation as that of H. Equivalently, for every $x, y \in N, x - y \subseteq N$. In particular, for any $x \in N, x - x \subseteq N$. Since $0 \in x - x$, it follows that $0 \in N$.

There are several kinds of hyperrings that can be defined on a non-empty set R. In what follows, we shall consider one of the most general types of hyperrings. The definition of a hyperring given below is equivalent to one formulated by De Salvo [37] (see Corsini [9]).

Definition 1 *A hyperring is a triple* $(R, +, \cdot)$, *where* R *is a non-empty set with a hyperaddition* $+$ *and a hypermultiplication* $\cdot$ *satisfying the following axioms:*

1. $(R, +)$ *is a canonical hypergroup,*
2. $(R, \cdot)$ *is a semihypergroup such that* $x \cdot 0 = 0 \cdot x = 0$ *for all* $x \in R$, *(i.e, 0 is a bilaterally absorbing element),*
3. *The hypermultiplication* $\cdot$ *is distributive with respect to the hyperoperation* $+$. *That is, for every* $x, y, z \in R, x \cdot (y + z) = x \cdot y + x \cdot z$, *and* $(x + y) \cdot z = x \cdot z + y \cdot z$.

Definition 2 [37, 9] *A non-empty subset* R' *of* R *is called a subhyperring of* $(R, +, \cdot)$ *if* $(R', +)$ *is a subhypergroup of* $(R, +)$ *and* $\forall x, y \in R', x \cdot y \in \mathcal{P}^*(R')$.

Example 1 *Let* $R = \{0, 1\}$ *be a set with two hyperoperations defined as follows:*

$+$	0	1
0	$\{0\}$	$\{0,1\}$
1	$\{0,1\}$	$\{0,1\}$

$\cdot$	0	1
0	$\{0\}$	$\{0\}$
1	$\{0\}$	$\{0,1\}$

Clearly, $(R, +, \cdot)$ *is a hyperring.*

Example 2 *Let* $R = \{0, a, b\}$ *be a set with two hyperoperations defined as follows.*

$+$	0	a	b
0	$\{0\}$	$\{a\}$	$\{b\}$
a	$\{a\}$	$\{a,b\}$	R
b	$\{b\}$	R	$\{a,b\}$

$\cdot$	0	a	b
0	$\{0\}$	$\{0\}$	$\{0\}$
a	$\{0\}$	R	R
b	$\{0\}$	R	R

Clearly, $(R, +, \cdot)$ *is a hyperring.*

Example 3 *Let* $(R, +, \cdot)$ *be a hyperring. Then* $(M_n(R), \oplus, \odot)$ *is a hyperring, where* $M_n(R)$ *is the set of all* $n \times n$ *matrices over* R *for some natural number* n *and the hyperoperations* $\oplus$ *and* $\odot$ *are defined as follows:*

For $x = (x_{ij}), y = (y_{ij}) \in M_n(R)$, $x \oplus y = \{z \in M_n(R) : z = (z_{ij}), z_{ij} \in x_{ij} + y_{ij}, 1 \leq i, j \leq n\}$ *and* $x \odot y = \{z \in M_n(R) : z = (z_{ij}), z_{ij} \in \sum\limits_{k=1}^{n} x_{ik} \cdot y_{kj}, 1 \leq i, j \leq n\}$.

Definition 3 [37, 9] *Let* $(R, +, \cdot)$ *be a hyperring. A non-empty subset* A *of* R *is called a hyperideal of* R *if* $(A, +)$ *is a subhypergroup of* $(R, +)$ *and* $\forall x \in R, \forall y \in A$, *both* $x \cdot y$ *and* $y \cdot x$ *are elements of* $\mathcal{P}^*(A)$.

Let R and Γ be two non-empty sets. A map from $R \times \Gamma \times R \to P^*(R)$ will be called a Γ-hypermultiplication in R and is denoted by $(\cdot)_\Gamma$. The result of this Γ-hypermultiplication for every two elements $a, b \in R$ and every element $\gamma \in \Gamma$ is denoted by $a\gamma b$. In the following we give the definition of Γ-hyperrings in a different way.

Definition 4 (cf. [4]) *A Γ-hyperring is called a five tuple $(R, \Gamma, +, \oplus, (\cdot)_\Gamma)$ where R, Γ are nonempty sets, $+$ is the hyperaddition in R, $\oplus$ is the hyperaddition in Γ, $(\cdot)_\Gamma$ is a Γ-hypermultiplication in R, such that:*

1. *$(R, +)$ is a canonical hypergroup;*
2. *$(\Gamma, \oplus)$ is a canonical hypergroup;*
3. $\forall (x, y, z, \alpha) \in R^3 \times \Gamma, (x + y)\alpha z = x\alpha z + y\alpha z, x\alpha(y + z) = x\alpha y + x\alpha z$*;*
4. $\forall (x, y, \alpha, \beta) \in R^2 \times \Gamma^2, x(\alpha \oplus \beta)y = x\alpha y + x\beta y$*;*
5. $\forall (x, y, z, \alpha, \beta) \in R^3 \times \Gamma^2, (x\alpha y)\beta z = x\alpha(y\beta z)$.

Definition 5 [20] *A Γ-semihypergroup R is called an ordered pair $(R, (\cdot)_\Gamma)$ where R and Γ are nonempty sets and $(\cdot)_\Gamma$ is a Γ-hypermultiplication on R which satisfies the following property: $\forall (a, b, c, \alpha, \beta) \in R^3 \times \Gamma^2, (a\alpha b)\beta c = a\alpha(b\beta c)$.*

Definition 6 *An weakly Γ-hyperring is called any triple $(R, +, (\cdot)_\Gamma)$ where R, Γ are nonempty sets, $+$ is the hyperaddition in R, $(\cdot)_\Gamma$ is a Γ-hypermultiplication in R, such that:*

1. *$(R, +)$ is a canonical hypergroup;*
2. *$(R, (\cdot)_\Gamma)$ is a Γ-semihypergroup;*
3. $\forall\, (x, y, z, \alpha) \in R^3 \times \Gamma, (x + y)\alpha z = x\alpha z + y\alpha z, x\alpha(y + z) = x\alpha y + x\alpha z$.

Examples of Γ-hyperrings can be found in [4, 43]. It is clear tht every hyperring is a Γ-hyperring.

In what follows, unless otherwise stated, an weakly Γ-hyperring $(R, +, \Gamma)$ always denotes a Γ-hyperring.

Definition 7 [4] *A non-empty subset R' of $(R, +, \Gamma)$ is called a sub-Γ-hyperring of R if $(R', +)$ is a subhypergroup of $(R, +)$ and $\forall x, y \in R', \gamma \in \Gamma, x\gamma y \subseteq \mathcal{P}^*(R')$.*

Definition 8 [4] *Let $(R, +, \Gamma)$ be a Γ-hyperring. A non-empty subset A of R is called a right (left) hyperideal of R if $(A, +)$ is a subhypergroup of $(R, +)$ and $A\Gamma R \subseteq A(R\Gamma A \subseteq A)$. A is called a hyperideal if it is both a left and a right hyperideal of R.*

Definition 9 *A non-empty subset Q of $(R, +, \Gamma)$ is said to be a quasi-hyperideal of R if $(Q, +)$ is a subhypergroup of $(R, +)$ and $Q\Gamma R \cap R\Gamma Q \subseteq Q$. A non-empty subset B of R is said to be a bi-hyperideal of $(R, +, \Gamma)$ if $(B, +)$ is a subhypergroup of $(R, +)$ and $B\Gamma R\Gamma B \subseteq B$.*

Definition 10 [32] *An element $a \in R$ is said to be regular if $a \in a\Gamma R\Gamma a$. That is, there exist an element $b \in R$ and $\alpha, \beta \in \Gamma$ such that $a \in a\alpha b\beta a$. A Γ-hyperring R is said to be regular if every element of R is regular.*

A mapping $\mu : X \to [0, 1]$, where X is an arbitrary non-empty set and is called a *fuzzy set* in X. For $\alpha \in [0, 1]$, the set $U(\mu; \alpha) = \{x \in X | \mu(x) \geq \alpha\}$ is called *level set* of μ. A fuzzy set μ in a Γ-hyperring R is called a *fuzzy left* (resp. *right*) *hyperideal* of R if it satisfies:

- $\inf\limits_{a \in x - y} \mu(a) \geq \min\{\mu(x), \mu(y)\}$,
- $\inf\limits_{a \in x\gamma y} \mu(a) \geq \mu(y)$ (resp. $\inf\limits_{a \in x\gamma y} \mu(a) \geq \mu(x)$) for all $x, y \in R$ and $\gamma \in \Gamma$.

A fuzzy set μ in a Γ-hyperring R is called a *fuzzy hyperideal* of R if μ is both a fuzzy left and a fuzzy right hyperideal of R [4].

A triangular norm (briefly, t-norm) (cf. Schweizer and Sklar [39]) is a function $T : [0, 1] \times [0, 1] \to [0, 1]$ satisfying for every $x, y, z \in [0, 1]$ the following conditions:

(T1) $T(x, y) = T(y, x)$ (commutative),

(T2) $T(x, y) \leq T(x, z)$ if $y \leq z$ (monotone in the right factor),

(T3) $T(x, T(y, z)) = T(T(x, y), z)$ (associative),

(T4) $T(x, 1) = x$ (having 1 as a right identity).

These four axioms are independent in the sense that none of them can be deduced from the other three. Obviously, the function min defined on $[0, 1] \times [0, 1]$ is a t-norm. Other t-norms which are frequently encountered in the study of probabilistic spaces are T_m and T_p defined by $T_m(a, b) = \max(a + b - 1, 0), T_p(a, b) = ab$ for every $a, b \in [0, 1]$. Replacing 1 by 0 in condition (T4) we obtain the concept of triangular conorm (t-conorm). In general every t-norm T satisfies the following conditions:

$$\text{(i) } T(x,0) = 0; T(0,0) = 0 \text{ and } T(1,1) = 1;$$
$$\text{(ii) } T(x,y) \leq \min(x,y), \forall x,y \in [0,1].$$

For a t-norm T on [0,1], we denote it $E_T = \{\alpha \in [0,1] | T(\alpha,\alpha) = \alpha\}$.

Let T_1 and T_2 be two t-norms. T_2 is said to be *dominate* T_1 and write $T_1 \ll T_2$ if for all $a,b,c,d \in [0,1]$, $T_1(T_2(a,c),T_2(b,d)) \leq T_2(T_1(a,b),T_1(c,d))$ and T_1 is said *weaker* then T_2 or T_2 is *stronger* then T_1 and write $T_1 \leq T_2$ if for all $x,y \in [0,1], T_1(x,y) \leq T_2(x,y)$. Since a triangular norm T is a generalization of the minimum function, Anthony and Sherwood in [1] replaced the axiom $\min\{\mu(x),\mu(y)\} \leq \mu(xy)$ occurring in the definition of a fuzzy subgroup by the inequality $T(\mu(x),\mu(y)) \leq \mu(xy)$.

Definition 11 *Let μ,λ be the fuzzy subsets of a set X. A fuzzy subset $\mu \cap \lambda$ is defined as $(\mu \cap \lambda)(x) = \min(\mu(x),\lambda(x))$.*

Definition 12 *Let μ,λ be the fuzzy subsets of a set X. A fuzzy subset $\mu \wedge \lambda$ is defined as $(\mu \wedge \lambda)(x) = T(\mu(x),\lambda(x))$.*

Definition 13 *Let μ,λ be the fuzzy subsets of a set X. The product of the fuzzy subset μ and λ is defined as $(\mu \circ \lambda)(x) = \sup\limits_{x \in y\gamma z} T(\mu(y),\lambda(z))$, $\gamma \in \Gamma$.*

2 T-fuzzy right and left hyperideals in Γ-hyperrings

In this section, we introduce the notions of T-fuzzy left hyperideal and T-fuzzy right hyperideal in Γ-hyperrings and some properties of them are studied. Also, the regular Γ-hyperrings are studied in terms of T-fuzzy left hyperideals and T-fuzzy right hyperideals.

Definition 14 *A fuzzy set μ in a Γ-hyperring R is called a fuzzy left (resp. right) hyperideal of R with respect to a* t*-norm T (briefly, a T-fuzzy left (resp. right) hyperideal of R) if it satisfies:*

1. $\inf\limits_{a \in x-y} \mu(a) \geq T(\mu(x),\mu(y))$,

2. $\inf\limits_{a \in x\gamma y} \mu(a) \geq \mu(y)$ (*resp.* $\inf\limits_{a \in x\gamma y} \mu(a) \geq \mu(x)$)

for all $x,y \in R$ and $\gamma \in \Gamma$.

Remark 1 1. *If we take* t*-norm as min-norm, T-fuzzy right hyperideal coincides with fuzzy right hyperideal* [3].

2. *T-fuzzy hyperideal is both T-fuzzy right and left hyperideal.*

Lemma 1 *Let* $(R, +, \Gamma)$ *be a* Γ*-hyperring. Every fuzzy right hyperideal of* R *is a* T*-fuzzy right hyperideal.*

Proof. Let μ be a fuzzy right hyperideal of R. Then $\inf\limits_{a \in x-y} \mu(a) \geq \min\{\mu(x), \mu(y)\} \geq T(\mu(x), \mu(y)\}$ and $\inf\limits_{a \in x\gamma y} \mu(a) \geq \mu(x)$ for all $x, y \in R$ and $\gamma \in \Gamma$. Hence μ is a T-fuzzy hyperideal. □

Corollary 1 *Let* $(R, +, \Gamma)$ *be a hyperring. If* A *is a right hyperideal of* R*, then* χ_A *is a* T*-fuzzy right hyperideal.*

Proof. Let A be a right hyperideal of a Γ-hyperring R. Then χ_A is a fuzzy right hyperideal. Therefore by Lemma 1, χ_A is a T-fuzzy right hyperideal. □

Note: Every T-fuzzy right (resp. left) hyperideal need not be a fuzzy hyperideal by the following examples.

Example 4 *Let* $R = \{0, a, b, c, d\}$ *be a set with two hyperoperations defined as follows.*

$+$	0	a	b	c	d
0	0	a	b	c	d
a	a	$\{0, a\}$	b	c	d
b	b	b	$\{0, a\}$	d	c
c	c	c	d	$\{0, a\}$	b
d	d	d	c	b	$\{0, a\}$

$\cdot$	0	a	b	c	d
0	0	0	0	0	0
a	0	$\{0, a\}$	$\{0, a\}$	$\{0, a\}$	$\{0, a\}$
b	0	$\{0, a\}$	$\{0, a\}$	$\{0, a\}$	$\{0, a\}$
c	0	$\{0, a\}$	$\{0, a\}$	$\{0, a\}$	$\{0, a\}$
d	0	$\{0, a\}$	$\{0, a\}$	$\{0, a\}$	$\{0, a\}$

Clearly, $(R, +, \cdot)$ *is a hyperring. Let* $\Gamma = \{0, a, b\}$ *be an hyperideal of* R*. Then* R *is a* Γ*-hyperring. Define* $\mu : R \to [0, 1]$ *on* R *as follows:*

$$\mu(0) = 0.8, \mu(a) = 0.6, \mu(b) = 0.5, \mu(c) = 0.4, \mu(d) = 0.3$$

It can be easily verified that μ *is a* T*-fuzzy right hyperideal under* T_p*. But* μ *is not a fuzzy hyperideal of* R*.*

Example 5 [4] *Let* $I_1 \subset I_2 \subset .... \subset I_n \subset ...$ *be a strictly increasing sequence of left hyperideals of an arbitrary* Γ*-hyperring* R *and* $\{t_j\}_{j=1}^{\infty}$ *be a strictly increasing sequence in* $[0, 1]$*. Define* μ *in* R *as follows:*

$$\mu(x) = t_j \text{ if } x \in I_j \backslash I_{j-1}, \text{ where } t_{j-1} < t_j, j = 1, 2, ... \text{ and } \mu(x) = 0, \text{ if } x \in R \backslash \bigcup_{j=1}^{\infty} I_j$$

It can be easily verified that μ *is a* T*-fuzzy right hyperideal under* T_p*. But* μ *is not a fuzzy hyperideal of* R*, it is only a left fuzzy hyperideal of* R*.*

Theorem 1 *Let* $(R, +, \Gamma)$ *be a* Γ*-hyperring. If* μ, λ *are* T*-fuzzy right hyperideals of* R*, then* $\mu \wedge \lambda$ *is a* T*-fuzzy right hyperideal of* R*.*

Proof. Let $x, y \in R, \gamma \in \Gamma$,

$$\begin{aligned}
\inf_{a \in x-y} (\mu \wedge \lambda)(a) &\geq T(\inf_{a \in x-y} \mu(a), \inf_{a \in x-y} \lambda(a)) \\
&\geq T(T(\mu(x), \mu(y)), T(\lambda(x), \lambda(y))) \\
&= T(T(T(\mu(x), \mu(y)), \lambda(x)), \lambda(y)) \\
&\geq T(T(T(\mu(x), \lambda(x)), \mu(y), \lambda(y))) \\
&= T(T(\mu(x), \lambda(x)), T(\mu(y), \lambda(y))) \\
&= T((\mu \wedge \lambda)(x), (\mu \wedge \lambda)(y)).
\end{aligned}$$

Since $\inf_{a \in x\gamma y} \mu(a) \geq \mu(x)$ and $\inf_{a \in x\gamma y} \lambda(a) \geq \lambda(x)$, we have $T(\inf_{a \in x\gamma y} \mu(a), \inf_{a \in x\gamma y} \lambda(a)) \geq T(\mu(x), \lambda(x))$. Then $\inf_{a \in x\gamma y} (\mu \wedge \lambda)(a) \geq (\mu \wedge \lambda)(x)$. Thus $\mu \wedge \lambda$ is a T-fuzzy right hyperideal of R. □

Corollary 2 *Let* $(R, +, \Gamma)$ *be a* Γ*-hyperring. If* μ, λ *are fuzzy right hyperideals of* R*, then* $\mu \cap \lambda$ *is a fuzzy right hyperideal of* R*.*

Proof. By taking min as t-norm T in Theorem 1, we get the required result. □

Lemma 2 *Let* $(R, +, \Gamma)$ *be a hyperring.* R *is regular if and only if* $A\Gamma B = A \cap B$ *for any right hyperideal* A *and left hyperideal* B *of* R*.*

Proof. Let $(R, +, \Gamma)$ be a regular Γ-hyperring and A, B be the right and left hyperideals of R respectively. Clearly, $A\Gamma B \subseteq A \cap B$. Since R is regular, for $x \in R$ we have $x \in x\alpha a\beta x$ for some $a \in R$ and $\alpha, \beta \in \Gamma$. Now let $x \in A \cap B$. Then $x\alpha a \subseteq A$ and $x \in B$, thus $x \in x\alpha a\beta x \subseteq A \cdot B$. Hence $A\Gamma B = A \cap B$.

Conversely, let $x \in R$. Now $\langle x \rangle_r = \{w | w \in x\gamma r + nx | r \in R, n \in Z, \gamma \in \Gamma\}$ is a right hyperideal generated by x and $\langle x \rangle_l = \{w | w \in r\gamma x + nx | r \in R, n \in Z, \gamma \in \Gamma\}$ is a left hyperideal generated by x. Then $x\gamma 0 + 1 \cdot x = 0\gamma x + 1 \cdot x = x \in \langle x \rangle_r \cap \langle x \rangle_l = \langle x \rangle_r \Gamma \langle x \rangle_l$. Therefore $x \in x\Gamma R\Gamma x$ or $x \in n_1 \cdot x^3, n_1 \in Z$ where $x^3 = x\Gamma x\Gamma x$. Hence R is regular. □

Theorem 2 *Let* $(R, +, \Gamma)$ *be a* Γ*-hyperring.* R *is regular if and only if* $\lambda \circ \mu = \lambda \wedge \mu$ *for any* T*-fuzzy right hyperideal* λ *and* T*-fuzzy left hyperideal* μ *of* R*.*

Proof. Let R be a regular Γ-hyperring. Let λ, μ be the T-fuzzy right and left hyperideals of Γ-hyperring R respectively. Let $x \in R, \gamma \in \Gamma$.

$$\begin{aligned}(\lambda \circ \mu)(x) &= \sup_{x \in y\gamma z} T(\lambda(y), \mu(z)) \leq \sup_{x \in y\gamma z} T(\lambda(x), \mu(x)) \\ &= (\lambda \wedge \mu)(x).\end{aligned}$$

Thus $\lambda \circ \mu \subseteq \lambda \wedge \mu$. Since R is regular, for $x \in R$ we have $x \in x\alpha a\beta x$ for some $a \in R, \alpha, \beta \in \Gamma$.

$$\begin{aligned}(\lambda \circ \mu)(x) &= \sup_{x \in y\gamma z} T(\lambda(y), \mu(z)) \\ &\geq T(\sup_{s \in x\alpha a} \lambda(s), \mu(x)) \\ &\geq T(\lambda(x), \mu(x)) \\ &= (\lambda \wedge \mu)(x).\end{aligned}$$

Hence $\lambda \circ \mu = \lambda \wedge \mu$.

Conversely, let us assume that $\lambda \circ \mu = \lambda \wedge \mu$ for any T-fuzzy right hyperideal λ and T-fuzzy left hyperideal μ. Let A, B be the right and left hyperideals of Γ-hyperring R respectively. Then χ_A, χ_B are the T-fuzzy right and left hyperideals of Γ-hyperring R respectively. Clearly, $A\Gamma B \subseteq A \cap B$. Now $x \in A \cap B$. Then $\chi_A(x) = \chi_B(x) = 1$. Thus $(\lambda \wedge \mu)(x) = T(\lambda(x), \mu(x)) = 1$. Therefore $(\lambda \circ \mu)(x) = 1$. Then there is $a \in A, b \in B, \gamma \in \Gamma$ such that $x \in a\gamma b$. Thus $x \in A\Gamma B$. Hence $A\Gamma B = A \cap B$. Then by Lemma 2, R is regular. □

Corollary 3 *Let $(R, +, \Gamma)$ be a Γ-hyperring. R is regular if and only if $\lambda \circ \mu = \lambda \cap \mu$ for any fuzzy right hyperideal λ and fuzzy left hyperideal μ of R.*

Proof. By taking min as t-norm T in Theorem 2, we get the required result. □

Let $\{\mu_i | i \in \Lambda\}$ be a family of fuzzy sets in the Γ-hyperring $(R, +, \Gamma)$. We define the join $\bigvee_{i \in \Lambda} \mu_i$ and meet $\bigwedge_{i \in \Lambda} \mu_i$ as follows:

$$\left(\bigvee_{i \in \Lambda} \mu_i\right)(x) = \sup\{\mu_i(x) | i \in \Lambda\}, \left(\bigwedge_{i \in \Lambda} \mu_i\right)(x) = \inf\{\mu_i(x) | i \in \Lambda\},$$

for all $x \in R$, where Λ is any index set.

Theorem 3 *Let $(R, +, \Gamma)$ be a Γ-hyperring. Then the family of T-fuzzy hyperideals in R is a completely distributive lattice with respect to meet "$\wedge$" and join "$\vee$".*

Proof. Since [0,1] is a completely distributive lattice with respect to the usual ordering in [0,1], it is sufficient to show that $\bigvee_{i\in\Lambda}\mu_i$ and $\bigwedge_{i\in\Lambda}\mu_i$ are T-fuzzy hyperideals of R for a family of T-fuzzy hyperideals $\{\mu_i | i \in \Lambda\}$. For any $x, y \in R$, we have

$$\begin{aligned}
\inf_{a\in x-y}\left(\bigvee_{i\in\Lambda}\mu_i\right)(a) &= \sup\left\{\inf_{a\in x-y}\mu_i(a) | i\in\Lambda\right\}\\
&\geq \sup\{T(\mu_i(x),\mu_i(y)) | i\in\Lambda\}\\
&\geq T(\sup\{\mu_i(x) | i\in\Lambda\},\sup\{\mu_i(y) | i\in\Lambda\})\\
&= T\left(\left(\bigvee_{i\in\Lambda}\mu_i\right)(x),\left(\bigvee_{i\in\Lambda}\mu_i\right)(y)\right),\\
\inf_{a\in x-y}\left(\bigwedge_{i\in\Lambda}\mu_i\right)(a) &= \inf\left\{\inf_{a\in x-y}\mu_{ia}(a) | i\in\Lambda\right\}\\
&\geq \inf\{T(\mu_i(x),\mu_i(y)) | i\in\Lambda\}\\
&\geq T(\inf\{\mu_i(x) | i\in\Lambda\},\inf\{\mu_i(y) | i\in\Lambda\})\\
&= T\left(\left(\bigwedge_{i\in\Lambda}\mu_i\right)(x),\left(\bigwedge_{i\in\Lambda}\mu_i\right)(y)\right).
\end{aligned}$$

Now let $x, y \in R, \gamma \in \Gamma$. Then

$$\begin{aligned}
\inf_{a\in x\gamma y}\left(\bigvee_{i\in\Lambda}\mu_i\right)(a) &= \sup\{\inf_{a\in x\gamma y}\mu_i(a) | i\in\Lambda\}\\
&\geq \sup\{\mu_i(y) | i\in\Lambda\}\\
&= \left(\bigvee_{i\in\Lambda}\mu_i\right)(y),\\
\inf_{a\in x\gamma y}\left(\bigwedge_{i\in\Lambda}\mu_i\right)(a) &= \inf\{\inf_{a\in x\gamma y}\mu_i(a) | i\in\Lambda\}\\
&\geq \inf\{\mu_i(y) | i\in\Lambda\}\\
&= \left(\bigwedge_{i\in\Lambda}\mu_i\right)(y).
\end{aligned}$$

Hence $\bigvee_{i\in\Lambda}\mu_i$ and $\bigwedge_{i\in\Lambda}\mu_i$ are T-fuzzy hyperideals of R. □

Definition 15 *Let* $(R, +, \Gamma)$ *be a* Γ*-hyperring and* T *be a* t*-norm. A fuzzy set* μ *in* R *is said to satisfy imaginable property if* $Im(\mu) \subseteq E_T = \{\alpha \in [0, 1] \mid T(\alpha, \alpha) = \alpha\}$.

Theorem 4 *Let* $(R, +, \Gamma)$ *be a* Γ*-hyperring,* T *be a* t*-norm and* μ *be an imaginable fuzzy set in* R*. If each non-empty upper level set* $U(\mu; \alpha)$ *of* μ *is a hyperideal of* R*, then* μ *is imaginable* T*-fuzzy hyperideal of* R.

Proof. Let us suppose that each non-empty upper level set $U(\mu; \alpha)$ of μ is a hyperideal of R. Then we first show that $\inf\limits_{a \in x-y} \mu(a) \geq \min(\mu(x), \mu(y))$ for all $x, y \in R$. In fact, if not, then there exist $x_0, y_0 \in R$ such that $\inf\limits_{a \in x_0 - y_0} \mu(a) < \min(\mu(x_0), \mu(y_0))$. Taking

$$\alpha_0 = \frac{1}{2}\left(\inf_{a \in x_0 - y_0} \mu(a) + \min(\mu(x_0), \mu(y_0))\right),$$

we get $\inf\limits_{a \in x_0 - y_0} \mu(a) < \alpha_0 < \min(\mu(x_0), \mu(y_0))$ and thus $x_0, y_0 \in U(\mu; \alpha_0)$ and $x_0 - y_0 \not\subseteq U(\mu; \alpha_0)$. This is a contradiction. Hence

$$\inf_{a \in x-y} \mu(a) \geq \min(\mu(x), \mu(y)) \geq T(\mu(x), \mu(y))$$

for all $x, y \in R$. Now if the condition (2) of Definition 14 is not true, then $\inf\limits_{b \in x_0 \gamma y_0} \mu(b) < \mu(y_0)$ for some $x_0, y_0 \in R, \gamma \in \Gamma$. Taking $s_1 = \frac{1}{2}\left(\inf\limits_{b \in x_0 \gamma y_0} \mu(b) + \mu(y_0)\right)$, then $0 \leq s_1 < \mu(y_0)$ and $\inf\limits_{b \in x_0 \gamma y_0} \mu(b) < s_1$. Hence $y_0 \in U(\mu; s_1)$ and $x_0 \gamma y_0 \not\subseteq U(\mu; s_1)$, a contradiction. This completes the proof. $\square$

3 T-fuzzy quasi-hyperideals and T-fuzzy bi-hyperideals in Γ-hyperrings

In this section, we introduce the notions of fuzzy quasi(bi)-hyperideal and T-fuzzy quasi(bi)-hyperideal in Γ-hyperrings and some properties of them are studied. Also, the regular Γ-hyperrings are studied in terms of T-fuzzy quasi-hyperideals and T-fuzzy bi-hyperideals.

Definition 16 *Let* $(R, +, \Gamma)$ *be a* Γ*-hyperring. A fuzzy subset* μ *of* R *is called* T*-fuzzy quasi-hyperideal if*

1. $\inf\limits_{a \in x-y} \mu(a) \geq T(\mu(x), \mu(y))$,

2. $(\mu \circ \chi_R) \wedge (\chi_R \circ \mu) \subseteq \mu$ *for all* $x, y \in R$.

If $T = \min$, then μ is called *fuzzy quasi-hyperideal* of R.

Lemma 3 *Let* $(R, +, \cdot)$ *be a hyperring. A fuzzy subset* μ *is a* T*-fuzzy quasi-hyperideal of* R *if and only if*

(a) $\mu(x) \geq T[\sup_{x \in y\gamma z} \mu(y), \sup_{x \in y\gamma z} \mu(z)]$, $\forall\, x \in R, \gamma \in \Gamma$ *and*

(b) $\inf_{a \in x-y} \mu(a) \geq T(\mu(x), \mu(y))$, $\forall\, x, y \in R$.

Proof. Let μ be a T-fuzzy quasi-hyperideal of R. Let $x \in R$. Then

$$\begin{aligned}
\mu(x) &\geq T((\mu \circ \chi_R)(x), (\chi_R \circ \mu(x))) \\
&= T\left(\sup_{x \in y\gamma z} T(\mu(y), \chi_R(z)) \right), \sup_{x \in y\gamma z} T((\chi_R)(y), \mu(z)) \\
&= T\left(\sup_{x \in y\gamma z} \mu(y), \sup_{x \in y\gamma z} \mu(z) \right).
\end{aligned}$$

Conversely,

$$\begin{aligned}
\mu(x) &\geq T(\sup_{x \in y\gamma z} \mu(y), \sup_{x \in y\gamma z} \mu(z)) \\
&= T\left(\sup_{x \in y\gamma z} T(\mu(y), \chi_R(z)) \right), \sup_{x \in y\gamma z} T((\chi_R)(y), \mu(z))) \\
&= T((\mu \circ \chi_R)(x), (\chi_R \circ \mu)(x)) \\
&= ((\mu \circ \chi_R) \wedge (\chi_R \circ \mu))(x).
\end{aligned}$$

Then μ is a T-fuzzy quasi-hyperideal of R. □

Corollary 4 *Let* $(R, +, \cdot)$ *be a* Γ*-hyperring. A fuzzy subset* μ *is a fuzzy quasi-hyperideal of* R *if and only if*

1. $\mu(x) \geq \min\{\sup_{x \in y\gamma z} \mu(y), \sup_{x \in y\gamma z} \mu(z)\}$, $\forall x \in R, \gamma \in \Gamma$ *and*

2. $\inf_{a \in x-y} \mu(a) \geq \min\{\mu(x), \mu(y)\}$, $\forall x, y \in R$.

Proof. By taking min as t-norm T in Lemma 3, we get the required result. □

Lemma 4 *Let* $(R, +, \Gamma)$ *be a* Γ*-hyperring. Every fuzzy quasi-hyperideal of* R *is a* T*-fuzzy quasi-hyperideal of* R.

Proof. Let μ be a fuzzy quasi-hyperideal of R. Let $x, y \in R$. Then

$$\inf_{a \in x-y} \mu(a) \geq \min\{\mu(x), \mu(y)\} \geq T(\mu(x), \mu(y))$$

and

$$\mu(x) \geq \min\{\sup_{x \in y\gamma z} \mu(y), \sup_{x \in y\gamma z} \mu(z)\} \geq T[\sup_{x \in y\gamma z} \mu(y), \sup_{x \in y\gamma z} \mu(z)].$$

Thus μ is a T-fuzzy quasi-hyperideal of R. □

Corollary 5 *Let* $(R, +, \Gamma)$ *be a* Γ*-hyperring. If* Q *is a quasi-hyperideal in* R*, then* χ_Q *is a* T*-fuzzy quasi-hyperideal of* R.

Theorem 5 *Let* $(R, +, \Gamma)$ *be a* Γ*-hyperring. Every* T*-fuzzy right hyperideal of* R *is* T*-fuzzy quasi-hyperideal. Moreover, every* T*-fuzzy left hyperideal of* R *is* T*-fuzzy quasi-hyperideal.*

Proof. Let μ be a T-fuzzy right hyperideal of R. Let $x \in R$. Then $\inf_{a \in y\gamma z} \mu(a) \geq \mu(y)$. If $x \in y\gamma z$, then

$$\begin{aligned}\mu(x) \geq \mu(y) &\geq \min\left\{\sup_{x \in y\gamma z} \mu(y), \sup_{x \in y\gamma z} \mu(z)\right\} \\ &\geq T\left[\sup_{x \in y\gamma z} \mu(y), \sup_{x \in y\gamma z} \mu(z)\right].\end{aligned}$$

Therefore μ is a T-fuzzy quasi-hyperideal. Similarly, if λ is a T-fuzzy left hyperideal of R, then λ is a T-fuzzy quasi-hyperideal. □

Theorem 6 *Let* $(R, +, \Gamma)$ *be a* Γ*-hyperring. If* μ, λ *are* T*-fuzzy quasi-hyperideals of* R*, then* $\mu \wedge \lambda$ *is a* T*-fuzzy quasi-hyperideal of* R.

Proof. Let μ, λ be the T-fuzzy quasi-hyperideal of R. Let $x, y \in R$. From the proof of the Theorem 1, we have $\inf_{a \in x-y} (\mu \wedge \lambda)(a) \geq T((\mu \wedge \lambda)(x), (\mu \wedge \lambda)(y))$. We have

$$(\mu \wedge \lambda)(x) = T(\mu(x), \lambda(x))$$

$$\begin{aligned}
&\geq T\Big(T\Big(\sup_{x\in y\gamma z}\mu(y), \sup_{x\in y\gamma z}\mu(z)\Big), T\Big(\sup_{x\in y\gamma z}\lambda(y), \sup_{x\in y\gamma z}\lambda(z)\Big)\Big)\\
&= T\Big(T\Big(T\Big(\sup_{x\in y\gamma z}\mu(y), \sup_{x\in y\gamma z}\mu(z)\Big), \sup_{x\in y\gamma z}\lambda(y)\Big), \sup_{x\in y\gamma z}\lambda(z)\Big)\\
&= T\Big(T\Big(T\Big(\sup_{x\in y\gamma z}\mu(y), \sup_{x\in y\gamma z}\lambda(y)\Big), \sup_{x\in y\gamma z}\mu(z)\Big), \sup_{x\in y\gamma z}\lambda(z)\Big)\\
&= T\Big(T\Big(\sup_{x\in y\gamma z}\mu(y), \sup_{x\in y\gamma z}\lambda(y)\Big), T\Big(\sup_{x\in y\gamma z}\mu(z), \sup_{x\in y\gamma z}\lambda(z)\Big)\Big)\\
&\geq T(T(\mu(y),\lambda(y)), T(\mu(z),\lambda(z)))\\
&\geq T\Big(\sup_{x\in y\gamma z}T(\mu(y),\lambda(y)\Big), \sup_{x\in y\gamma z}T(\mu(z),\lambda(z)\Big)\Big)\\
&= T\Big(\sup_{x\in y\gamma z}(\mu\wedge\lambda)(y), \sup_{x\in y\gamma z}(\mu\wedge\lambda)(z)\Big).
\end{aligned}$$

Therefore $\mu\wedge\lambda$ is a T-fuzzy quasi-hyperideal of R. □

Theorem 7 *Let* $(R,+,\Gamma)$ *be a* Γ*-hyperring. If* μ,λ *are* T*-fuzzy right and* T*-fuzzy left hyperideals of* R *respectively, then* $\mu\wedge\lambda$ *is a* T*-fuzzy quasi-hyperideal of* R*.*

Proof. Let μ and λ be T-fuzzy right and T-fuzzy left hyperideal of R respectively. Then by Theorem 5, μ and λ are T-fuzzy quasi-hyperideals of R. By Theorem 6, $\mu\wedge\lambda$ is a T-fuzzy quasi-hyperideal of R. □

Definition 17 *Let* $(R,+,\Gamma)$ *be a* Γ*-hyperring. A fuzzy subset* μ *of* R *is called* T*-fuzzy bi-hyperideal if*

1. $\inf_{a\in x-y}\mu(a)\geq T(\mu(x),\mu(y))$,
2. $\inf_{a\in x\gamma_1 y\gamma_2 z}\mu(a)\geq T(\mu(x),\mu(z))$ *for all* $x,y,z\in R, \gamma_1,\gamma_2\in\Gamma$.

If $T=\min$, then μ is called *fuzzy bi-hyperideal* of R.

Lemma 5 *Let* B *be a bi-hyperideal of a* Γ*-hyperring* R*. For any* $0<\alpha<1$*, there exists a fuzzy bi-hyperideal* μ *such that* $U(\mu;\alpha)=B$*.*

Proof. Let B be a bi-hyperideal of R. Define $\mu: R\to[0,1]$ by

$$\mu(x)=\begin{cases}\alpha, & \text{if } x\in B\\ 0, & \text{if } x\notin B\end{cases}$$

where α is a fixed number in $(0,1)$. It is clear that $U(\mu;\alpha) = B$. Let $x, y \in R$. If $x, y \in B$, then $\inf_{a \in x-y} \mu(a) = \alpha = \min\{\mu(x), \mu(y)\}$. If at least one of x and y is not in B, then $\inf_{a \in x-y} \mu(a) = 0 = \min\{\mu(x), \mu(y)\}$. Let $x, y, z \in R$. If $x, z \in B$, then $\mu(x) = \alpha, \mu(z) = \alpha$. Also $\inf_{a \in x\gamma y \cdot z} \mu(a) = \alpha \geq \min\{\mu(x), \mu(z)\}$. If at least one of x and z is not in B, then $\inf_{a \in x\gamma y \cdot z} \mu(a) = 0 = \min\{\mu(x), \mu(z)\}$. Thus μ is a fuzzy bi-hyperideal of R. □

Lemma 6 *Let* B *be a non-empty subset of a* Γ*-hyperring* R. B *is a bi-hyperideal of* R *if and only if* χ_B *is a fuzzy bi-hyperideal of* R.

Proof. (i) Let $x, y \in R$.

Case 1: $x, y \in B$. Then $\chi_B(x) = \chi_B(y) = 1$. Therefore $\inf_{a \in x-y} \chi_B(a) = 1 \geq \min\{\chi_B(x), \chi_B(y)\}$.

Case 2: $x \in B$ and $y \notin B$. Then $\chi_B(x) = 1$ and $\chi_B(y) = 0$. Therefore $\inf_{a \in x-y} \chi_B(a) = 0 \geq \min\{\chi_B(x), \chi_B(y)\}$.

Case 3: $x \notin B$ and $y \in B$. Then $\chi_B(x) = 0$ and $\chi_B(y) = 1$. Therefore $\inf_{a \in x-y} \chi_B(a) = 0 \geq \min\{\chi_B(x), \chi_B(y)\}$.

Case 4: $x \notin B$ and $y \notin B$. Then $\chi_B(x) = 0$ and $\chi_B(y) = 0$. Therefore $\inf_{a \in x-y} \chi_B(a) = 0 \geq \min\{\chi_B(x), \chi_B(y)\}$.

(ii) Let $x, y, z \in R, \gamma_1, \gamma_2 \in \Gamma$.

Case 1: $x, z \in B$. Then $\chi_B(x) = \chi_B(z) = 1$. Therefore $\inf_{a \in x\gamma_1 y\gamma_2 z} \chi_B(a) = 1 \geq \min\{\chi_B(x), \chi_B(z)\}$.

Case 2: $x \in B$ and $z \notin B$. Then $\chi_B(x) = 1$ and $\chi_B(z) = 0$. Therefore $\inf_{a \in x\gamma_1 y\gamma_2 z} \chi_B(a) = 0 \geq \min\{\chi_B(x), \chi_B(z)\}$.

Case 3: $x \notin B$ and $z \in B$. Then $\chi_B(x) = 0$ and $\chi_B(z) = 1$. Therefore $\inf_{a \in x\gamma_1 y\gamma_2 z} \chi_B(a) = 0 \geq \min\{\chi_B(x), \chi_B(z)\}$.

Case 4: $x \notin B$ and $z \notin B$. Then $\chi_B(x) = 0$ and $\chi_B(z) = 0$. Therefore $\inf_{a \in x\gamma_1 y\gamma_2 z} \chi_B(a) = 0 \geq \min\{\chi_B(x), \chi_B(z)\}$.

Thus χ_B is a fuzzy bi-hyperideal of R. Conversely, let us suppose that χ_B is a fuzzy bi-hyperideal of R. Then, by Lemma 5, χ_B is two-valued. Hence B is a bi-hyperideal of R. □

Lemma 7 *Let* $(R, +, \Gamma)$ *be a* Γ*-hyperring. A fuzzy subset* μ *is a* T*-fuzzy bi-hyperideal of* R *if and only if* $\mu \circ \chi_R \circ \mu \subseteq \mu$.

Proof. Let μ be a T-fuzzy bi-hyperideal of R. Let $x \in R, \gamma_1, \gamma_2$. If $x \notin a\gamma_1 b\gamma_2 c$ for $a, b, c \in R$, then $\mu(x) \geq (\mu \circ \chi_R \circ \mu)(x) = 0$. If $x \in a\gamma_1 b\gamma_2 c$, then

$$\begin{aligned}\mu(x) &\geq T(\mu(a), \mu(c)) = T(\mu(a), T(1, \mu(c))) \\ &= T(\mu(a), T(\chi_R(b), \mu(c))).\end{aligned}$$

Therefore,

$$\mu(x) \geq \sup_{x \in a\gamma_1 y} T(\mu(a), \sup_{y \in b\gamma_2 c} T(\chi_R(b), \mu(c))) \geq (\mu \circ \chi_R \circ \mu)(x).$$

Conversely, it is clear that $\inf\limits_{a \in x\alpha y\beta z} \mu(a) \geq T(\mu(x), \mu(z))$. □

Theorem 8 *Let* $(R, +, \Gamma)$ *be a* Γ*-hyperring. Every* T*-fuzzy quasi-hyperideal of* R *is a* T*-fuzzy bi-hyperideal of* R.

Proof. Let μ be a T-fuzzy quasi-hyperideal of R. Let $x, y, z \in R, \alpha, \beta$. Then $\inf\limits_{a \in x\alpha y\beta z} \mu(a) \geq T(\mu(x), \inf\limits_{s \in y\beta z} \mu(s)) \geq T(\mu(x), \mu(z))$. Therefore μ is a T-fuzzy bi-hyperideal of R. □

Theorem 9 *Let* $(R, +, \Gamma)$ *be a* Γ*-hyperring. Every fuzzy bi-hyperideal of* R *is a* T*-fuzzy bi-hyperideal of* R.

Proof. Let μ be a fuzzy bi-hyperideal. Let $x, y, z \in R, \alpha, \beta \in \Gamma$. Then

$$\inf_{a \in x-y} \mu(a) \geq \min\{\mu(x), \mu(y)\} \geq T(\mu(x), \mu(y))$$

and

$$\inf_{a \in x\alpha y\beta z} \mu(a) \geq \min\{\mu(x), \mu(z)\} \geq T(\mu(x), \mu(z)).$$

Thus μ is a T-fuzzy bi-hyperideal of R. □

Corollary 6 *Let* $(R, +, \Gamma)$ *be a* Γ*-hyperring. If* B *is a bi-hyperideal in* R*, then* χ_B *is a* T*-fuzzy bi-hyperideal of* R.

Theorem 10 *Let* $(R, +, \Gamma)$ *be a* Γ*-hyperring. If* μ *and* λ *are* T*-fuzzy bi-hyperideals of* R*, then* $\mu \wedge \lambda$ *is a* T*-fuzzy bi-hyperideal of* R.

Proof. Let μ and λ be T-fuzzy bi-hyperideals of R. Then

$$\begin{aligned}\inf_{a\in x\alpha y\beta z}(\mu\wedge\lambda)(a) &\geq T(T(\mu(x),\mu(z)),T(\lambda(x),\lambda(z)))\\ &= T(T(\mu(x),\lambda(x)),T(\mu(x),\lambda(z)))\\ &= T((\mu\wedge\lambda)(x),(\mu\wedge\lambda)(z)).\end{aligned}$$

Hence $\mu\wedge\lambda$ is a T-fuzzy bi-hyperideal of R. □

Corollary 7 *Let $(R,+,\Gamma)$ be a Γ-hyperring. If μ and λ are fuzzy bi-hyperideals of R, then $\mu\cap\lambda$ is a fuzzy bi-hyperideal of R.*

Lemma 8 *Let $(R,+,\cdot)$ be a Γ-hyperring. Then the following statements are equivalent:*

1. R *is regular.*
2. $B = B\Gamma R\Gamma B$*, for any bi-hyperideal* B *in* R*.*
3. $Q = Q\Gamma R\Gamma Q$ *for any quasi-hyperideal* Q *in* R*.*

Proof. (1) $\Rightarrow$ (2). Let us assume that R is a regular Γ-hyperring. Let B be a bi-hyperideal of R and $a\in B$. Then there exist $x\in R,\alpha,\beta\in\Gamma$ such that $a\in a\alpha x\beta a\subseteq B\Gamma R\Gamma B$. Hence, $B\subseteq B\Gamma R\Gamma B$. Since B is a bi-hyperideal, then $B\Gamma R\Gamma B\subseteq B$. Therefore $B = B\Gamma R\Gamma B$.

(2) $\Rightarrow$ (3). Let Q be a quasi-hyperideal of R, that is $Q\Gamma R\cap R\Gamma Q\subseteq Q$. Since every quasi-hyperideal is a bi-hyperideal of R, then we have $Q\Gamma R\Gamma Q = Q$.

(3) $\Rightarrow$ (1). Let us assume that (3) holds. Let I and J be any right hyperideal and left hyperideal of R, respectively. Then we have $(I\cap J)\Gamma R\cap R\Gamma(I\cap J)\subseteq I\cap J$. It is easy to see that $I\cap J$ is a quasi-hyperideal of R. By (3) and Lemma 2, we have $I\cap J\subseteq (I\cap J)\Gamma R\Gamma(I\cap J)\subseteq I\Gamma R\Gamma J\subseteq I\Gamma J\subseteq I\cap J$. Hence, $I\Gamma J = I\cap J$ and so R is regular. □

Theorem 11 *Let $(R,+,\Gamma)$ be a Γ-hyperring. Then the following statements are equivalent:*

1. R *is regular.*
2. $\lambda\wedge\lambda\subseteq\lambda\circ\chi_R\circ\lambda\subseteq\lambda$ *for any* T*-fuzzy bi-hyperideal* λ *of* R*.*
3. $\mu\wedge\mu\subseteq\mu\circ\chi_R\circ\mu\subseteq\mu$ *for any* T*-fuzzy quasi-hyperideal* μ *of* R*.*

Proof. (1) ⇒ (2). Let $(R, +, \Gamma)$ be a regular Γ-hyperring and let λ be a T-fuzzy bi-hyperideal of R. Let $x \in R$. By Lemma 7, $\lambda(x) \geq (\lambda \circ \chi_R \circ \lambda)(x)$. For $x \in R$ there is $a \in R, \alpha, \beta \in \Gamma$ such that $x \in x\alpha a\beta x$. Thus $\lambda(x) \geq \inf\limits_{s \in x\alpha a\beta x} \lambda(s) \geq T(\lambda(x), \lambda(x)) = (\lambda \wedge \lambda)(x)$. Hence $\lambda \wedge \lambda \subseteq \lambda \circ \chi_R \circ \lambda \subseteq \lambda$.

(2) ⇒ (3). It is trivial.

(3) ⇒ (1). Let Q be any quasi-hyperideal in R. Then by Corollary 5, χ_Q is a T-fuzzy quasi-hyperideal. It is clear that $Q\Gamma R\Gamma Q \subseteq Q$. Let $x \in Q$. Then $\chi_Q(x) = 1$. Thus $(\chi_Q \circ \chi_R \circ \chi_Q)(x) \geq T(\chi_Q(x), \chi_Q(x)) = 1$. Then $(\chi_Q \circ \chi_R \circ \chi_Q)(x) = 1$. Thus $x \in Q\Gamma R\Gamma Q$. Therefore $Q \subseteq Q\Gamma R\Gamma Q$. Hence $Q = Q\Gamma R\Gamma Q$. By Lemma 8, R is regular. □

Corollary 8 *Let $(R, +, \Gamma)$ be a Γ-hyperring. Then the following statements are equivalent:*

1. R *is regular.*
2. $\lambda = \lambda \circ \chi_R \circ \lambda$ *for any fuzzy bi-hyperideal* λ *of* R.
3. $\mu = \mu \circ \chi_R \circ \mu$ *for any fuzzy quasi-hyperideal* μ *of* R.

Proof. By taking min as t-norm T in Theorem 11, we get the required result. □

Theorem 12 *Let $(R, +, \Gamma)$ be a regular Γ-hyperring. If μ is an imaginable T-fuzzy bi-hyperideal of R, then μ is a T-fuzzy quasi-hyperideal of R.*

Proof. Let μ be an imaginable T-fuzzy bi-hyperideal of a regular Γ-hyperring R and $a \in R$. Suppose that $(\mu \circ \chi_R)(a) \leq \mu(a)$. Then we get:

$$\begin{aligned} \mu(a) &\geq (\mu \circ \chi_R)(a) \geq \min\{(\chi_R \circ \mu)(a), (\mu \circ \chi_R)(a)\} \\ &\geq T((\chi_R \circ \mu)(a), (\mu \circ \chi_R)(a)) \\ &= (\chi_R \circ \mu \wedge \mu \circ \chi_R)(a) \end{aligned}$$

Suppose that $(\mu \circ \chi_R)(a) > \mu(a)$. Then $\sup\limits_{a \in x\gamma y} T(\mu(x), \chi_R(y)) > \mu(a)$. Thus $T(\mu(x), \chi_R(y)) > \mu(a)$. So $\mu(x) > \mu(a)$.

Since R is a regular Γ-hyperring, then there exists $b \in R, \alpha, \beta \in \Gamma$, such that $a \in a\alpha b\beta a \subseteq x\gamma y\alpha b\beta a$. Since μ is an imaginable T-fuzzy bi-hyperideal, then $\mu(a) \geq T(\mu(x), \mu(a))$. Then $T(\mu(a), \mu(a)) \geq T(\mu(x), \mu(a))$. This is a contradiction to $\mu(x) > \mu(a)$. Thus $(\mu \circ \chi_R)(x) \leq \mu(a)$.

In similar way, we can prove $(\chi_R \circ \mu)(a) \leq \mu(a)$. So $\mu \circ \chi_R \wedge \chi_R \circ \mu \subseteq \mu$. Hence μ is a T-fuzzy quasi-hyperideal of R. □

4 On T-(λ,μ)-fuzzy bi-hyperideals

Definition 18 *Let* $(R,+,\Gamma)$ *be a* Γ*-hyperring and* A *be a fuzzy subset of* R. *Then* A *is called a* T-(λ,μ)*-fuzzy bi-hyperideal of* R *if for all* $x,y,z\in R$, $\alpha,\beta\in\Gamma$, *we have*

1. $\inf_{a\in x-y} A(a)\vee\lambda\geq T(T(A(x),A(y)),\mu)$
2. $\inf_{a\in x\alpha y\beta z} A(a)\vee\lambda\geq T(T(A(x),A(z)),\mu)$

Theorem 13 *Let* $(R,+,\Gamma)$ *be a* Γ*-hyperring and* A *be a fuzzy subset of* R. *Then* A *is a* T-(λ,μ)*-fuzzy bi-hyperideal of* R *iff* A_α *is a bi-hyperideal of* R *for all* $\alpha\in(\lambda,\mu)$

Proof. Let A be a T-(λ,μ)-fuzzy bi-hyperideal of R. Let $x,y\in A_\alpha$. Then $A(x)\geq\alpha, A(y)\geq\alpha$. Consider $\inf_{a\in x-y} A(a)\vee\lambda\geq T(T(A(x),A(y)),\mu)\geq T(T(\alpha,\alpha),\mu)=\alpha$ (since $\alpha>\lambda$). That is, $\inf_{a\in x-y} A(a)\vee\lambda\geq\alpha$. This implies that $\inf_{a\in x-y} A(a)\geq\alpha$, that is, $x-y\subseteq A_\alpha$. Hence A_α is a subhypergroup of R. Let $x,z\in A_\alpha$, $y\in R$ and $\gamma,\beta\in\Gamma$. This implies that $A(x)\geq\alpha, A(z)\geq\alpha$. Then $\inf_{a\in x-y} A(a)\vee\lambda\geq T(T(A(x),A(z)),\mu)\geq T(T(\alpha,\alpha),\mu)=\alpha$ (since $\alpha>\lambda$). That is, $\inf_{a\in x\gamma y\beta z} A(a)\vee\lambda\geq\alpha$. This implies that $\inf_{a\in x\gamma y\beta z} A(a)\geq\alpha$ and so $x\gamma y\beta z\subseteq A_\alpha$. Hence A_α is a bi-hyperideal of R.

Conversely, let us suppose that A_α is a bi-hyperideal of R for all $\alpha\in(\lambda,\mu)$. Suppose that $\inf_{a\in x-y} A(a)\vee\lambda<T(T(A(x),A(y)),\mu)=\alpha$. Then $\inf_{a\in x-y} A(a)\vee\lambda<\alpha$ which implies that $\inf_{a\in x-y} A(a)<\alpha$ (since $\alpha>\lambda$). This implies that $x-y\nsubseteq A_\alpha$ which is impossible since A_α is a bi-hyperideal of R. Hence $\inf_{a\in x-y} A(a)\vee\lambda\geq T(T(A(x),A(y)),\mu)$. Similarly, we can prove that $\inf_{a\in x\alpha y\beta z} A(a)\vee\lambda\geq T(T(A(x),A(z)),\mu)$. Hence A is a T-(λ,μ)-fuzzy bi-hyperideal of R. □

Definition 19 *Let* R *and* R' *be* Γ *and* Γ'*-hyperrings, respectively,* $\varphi:R\to R'$ *and* $f:\Gamma\to\Gamma'$ *be two maps. Then,* (φ,f) *is called a* (Γ,Γ')*-homomorphism if*

1. $\forall(x,y)\in R^2$, $\varphi(x+y)=\{\varphi(z)|z\in x+y\}\subseteq\varphi(x)+\varphi(y)$,
2. $\forall(x,y,\alpha)\in R^2\times\Gamma$, $\varphi(x\alpha y)=\{\varphi(z)|z\in x\alpha y\}\subseteq\varphi(x)f(\alpha)\varphi(y)$.

Let φ be a mapping from a set X to a set Y. Let μ be a fuzzy subset of X and λ be a fuzzy subset of Y. Then the inverse image $\varphi^{-1}(\lambda)$ of λ is the fuzzy subset of X defined by $\varphi^{-1}(\lambda)(x) = \lambda(\varphi(x))$ for all $x \in X$. The image $\varphi(\mu)$ of μ is the fuzzy subset of Y defined by

$$\varphi(\mu)(y) = \begin{cases} \sup\{\mu(t) | t \in \varphi^{-1}(y)\}, & \text{if } \varphi^{-1}(y) \neq \emptyset; \\ 0, & \text{otherwise.} \end{cases}$$

for all $y \in Y$.

Theorem 14 *Let* (φ, f) *be a onto* (Γ, Γ')*-homomorphism of* $(R, +, \Gamma)$ *and* $(R', +, \Gamma')$ *respectively and let* A *be a* T-(λ, μ)*-fuzzy bi-hyperideal of* R. *Then* $\varphi(A)$ *is a* T-(λ, μ)*-fuzzy bi-hyperideal of* R'.

Proof. Let $y_1, y_2 \in R'$. Then we have

$$\begin{aligned}
\inf_{z \in y_1 - y_2} \{\varphi(A)(z)\} \vee \lambda &= \sup_{z \in y_1 - y_2} \left\{ \inf_{a \in \varphi^{-1}(y_1) - \varphi^{-1}(y_2)} A(a) \right\} \vee \lambda \\
&= \sup_{z \in y_1 - y_2} \left\{ \inf_{a \in \varphi^{-1}(y_1) - \varphi^{-1}(y_2)} A(a) \vee \lambda \right\} \\
&\geq \sup_{\varphi(z) \in \varphi(y_1) - \varphi(y_2)} \{A((\varphi)(z))\} \vee \lambda \\
&\geq \sup\{T(T(A((\varphi)(x_1)), A((\varphi)(x_2))), \mu) \mid \\
&\qquad x_1 \in (\varphi)^{-1}(y_1), x_2 \in (\varphi)^{-1}(y_2)\} \\
&\geq T(T(\sup\{A((\varphi)(x_1))\}, \sup\{A((\varphi)(x_2))\}), \mu) \mid \\
&\qquad x_1 \in (\varphi)^{-1}(y_1), x_2 \in (\varphi)^{-1}(y_2)\} \\
&= T(T((\varphi(A))(y_1), (\varphi(A))(y_2)), \mu)
\end{aligned}$$

Therefore, $\inf_{z \in y_1 - y_2} \{\varphi(A)(z)\} \vee \lambda \geq T(T((\varphi(A))(y_1), (\varphi(A))(y_2)), \mu)$.

Similarly, we can prove that

$$\inf_{a \in y_1 \alpha y_2 \beta y_3} (\varphi(A))(a) \vee \lambda \geq T(T((\varphi(A))(y_1), (\varphi(A))(y_3)), \mu).$$

Hence $\varphi(A)$ is a T-(λ, μ)-fuzzy bi-hyperideal of R'. □

Theorem 15 *Let* (φ, f) *be a onto* (Γ, Γ')*-homomorphism of* $(R, +, \Gamma)$ *and* $(R', +, \Gamma')$ *respectively and let* B *be a* T-(λ, μ)*-fuzzy bi-hyperideal of* R'. *Then* $\varphi^{-1}(B)$ *is a* T-(λ, μ)*-fuzzy bi-hyperideal of* R.

Proof. (1) Suppose that $x, y \in R$ and $\gamma \in \Gamma$. Then we have

$$\inf_{z \in x - y} \{\varphi^{-1}(B)(z)\} \vee \lambda = \inf_{z \in x - y} \{B((\varphi)(z))\} \vee \lambda$$

$$\begin{aligned}
&\geq \inf_{\varphi(z)\in\varphi(x)-\varphi(y)}\{B((\varphi)(z))\}\vee\lambda \\
&\geq T(T(B((\varphi)(x)),B((\varphi)(y))),\mu) \\
&= T(T((\varphi^{-1}(B))(x),(\varphi^{-1}(B))(y)),\mu)
\end{aligned}$$

Therefore, $\inf_{z\in x-y}\{\varphi^{-1}(B)(z)\}\vee\lambda \geq T(T((\varphi^{-1}(B))(x),(\varphi^{-1}(B))(y)),\mu)$.

(2) Similarly we can prove that

$$\inf_{a\in x\alpha y\beta z}(\varphi^{-1}(B))(a)\vee\lambda \geq T(T((\varphi^{-1}(B))(x),(\varphi^{-1}(B))(z)),\mu).$$

Hence $\varphi^{-1}(B)$ is a T-(λ,μ)-fuzzy bi-hyperideal of R. □

5 Conclusions

In this paper, we have studied the Γ-hyperrings via T-fuzzy left and right hyperideals, T-fuzzy quasi-hyperideal and bi-hyperideal and some related properties were investigated. As a future work, one can extend these results applying the intuitionistic fuzzy theory and also to extend these results in other algebraic hyperstructure such as (m,n)-hyperrings etc.

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