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More on decomposition of generalized continuity

Bishwambhar Roy
Department of Mathematics,
Women's Christian College,
6, Greek Church Row
Kolkata-700 026, India
email: bishwambhar_roy@yahoo.co.in

Abstract. In this paper a new class of sets termed as ω_μ^*-open sets has been introduced and studied. Using these concept, a unified theory for decomposition of (μ, λ)-continuity has been given.

1 Introduction

For the last one decade or so, the notion of generalized topological spaces and several classes of generalized types of open sets are being studied by different mathematicians. Our aim here is to study the notion of decomposition of continuity by using the concept of generalized topology introduced by Á. Császár [2]. On the otherhand the notion of decompositions of continuity was first introduced by Tong [18, 19] by defining $\mathcal{A}$-sets and $\mathcal{B}$-sets. After then decompositions of continuity and some of its weak forms have been studied by Ganster and Reilly [7, 8], Yalvac [20], Hatir and Noiri [10, 11], Przemski [14], Noiri and Sayed [13], Dontchev and Przemski [5], Erguang and Pengfei [6] and many others. Decompositions of regular open sets and regular closed sets are given by using PS-regular sets in [9]. Since then the notion of decompositions of continuity is one of the most important area of research.

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We first recall some definitions given in [2]. Let X be a non-empty set and $\exp X$ denote the power set of X. We call a class $\mu \subsetneq \exp X$ a generalized topology (briefly, GT) [1, 2], if $\varnothing \in \mu$ and μ is closed under arbitrary unions. A set X, with a GT μ on it is said to be a generalized topological space (briefly, GTS) and is denoted by (X, μ). A GT μ is said to be a quasi topology (briefly QT) [3] if $M, M^{'} \in \mu$ implies $M \cap M^{'} \in \mu$. The pair (X, μ) is said to be a QTS if μ is a QT on X. For a GTS (X, μ), the elements of μ are called μ-open sets and the complements of μ-open sets are called μ-closed sets. A GTS (X, μ) is called a μ-space [13] or a strong GTS [4] if $X \in \mu$. A subset A of a topological space (X, τ) is called ω-closed [12] if it contains all its condensation points. The complement of an ω-closed set is called an ω-open set. It is well known that a subset A of a space (X, τ) is ω-open if and only if for each $x \in A$, there exists $U \in \tau$ containing x such that $U \setminus A$ is countable.

The purpose of this paper is to introduce the decomposition theorem for the (μ, λ) continuous functions introduced in [1] which is a generalization of continuity and different weak forms of continuity. Throughout the paper, by (X, μ) and (Y, λ) we shall mean GTS unless otherwise stated.

2 ω_μ^*-open sets and its properties

Definition 1 *Let (X, μ) be a GTS. A subset A of X is called an ω_μ^*-open (ω_μ-open [15]) set if for each $x \in A$, there exists a μ-open set U containing x such that $U \setminus i_\mu(A)$ (resp. $U \setminus A$) is countable. The complement of an ω_μ^*-open (resp. ω_μ-open) set is known as an ω_μ^*-closed (resp. ω_μ-closed [15]) set.*

It follows from Definition 1 that every ω_μ^*-open set is an ω_μ-open set and every μ-open set is ω_μ^*-open set but the converses are false as shown in Example 3.

Remark 1 *Let μ be a GT on a topological space (X, τ). If $\tau \subsetneq \mu$, then the following relations hold:*

$$\begin{array}{ccccc} \omega\text{-open set} & \Leftarrow & \text{open set} & \Rightarrow & \mu\text{-open set} \\ \Downarrow & & & & \Downarrow \\ \omega_\mu\text{-open set} & & \Leftarrow & & \omega_\mu^*\text{-open set} \end{array}$$

Example 1 (a) *Let $X = \mathbb{R}$, τ be the usual topology on $\mathbb{R}$. Let $\mu = \{\varnothing, X, \mathbb{Q}\}$. Then μ is a GT on the topological space (X, τ). It is easy to see that $\mathbb{Q}$ is an ω_μ^*-open set but not an ω-open set.*

(b) *Let* $X = \mathbb{R}$ *and* μ *be the usual topology on* $\mathbb{R}$*. Then* μ *is a GT on* X*. It is easy to see that* $\mathbb{I}$*, the set of irrationals is an* ω_μ*-open set but not an* ω^*_μ*-open set.*

(c) *Let* $X = \mathbb{R}$ *and* $\mu = \{A \subseteqq X : 0 \in A\} \cup \{\varnothing\}$*. Then* μ *is a GT on the set* X*. It is easy to see that* $\mathbb{I}$*, the set of irrationals is* ω_μ*-open but not* μ*-open.*

(d) *Let* $X = \mathbb{R}$*,* $\mu = \{A : A$ *is uncountable*$\} \cup \{\varnothing\}$ *and* $\tau = \{\varnothing, X, \mathbb{Q}\}$*. Then* μ *is a GT on the topological space* (X, τ)*. It can be easily verified that* $\mathbb{Q}$ *is an* ω*-open set but not an* ω^*_μ*-open set.*

The family of all ω^*_μ-open sets of a GTS (X, μ) is denoted by $\omega^*_\mu(X)$ or simply by ω^*_μ.

Proposition 1 (a) *In a GTS* (X, μ)*, the collection of all* ω^*_μ*-open sets forms a GT on* X*.*
(b) *If* (X, μ) *is a QTS, then the collection of all* ω^*_μ*-open sets forms a QT on* X*.*

Proof. (a) It is obvious that $\varnothing$ is an ω^*_μ-open set. Let $\{A_\alpha : \alpha \in \Lambda\}$ be a collection of ω^*_μ-open subsets of X. Then for each $x \in \cup\{A_\alpha : \alpha \in \Lambda\}$, $x \in A_\alpha$ for some $\alpha \in \Lambda$. Thus there exists $U \in \mu$ containing x such that $U \setminus i_\mu(A_\alpha)$ is countable. Now as $U \setminus i_\mu(\cup\{A_\alpha : \alpha \in \Lambda\}) \subseteqq U \setminus i_\mu(A_\alpha)$, thus $U \setminus i_\mu(\cup\{A_\alpha : \alpha \in \Lambda\})$ is countable. Hence $\cup\{A_\alpha : \alpha \in \Lambda\}$ is an ω^*_μ-open set.

(b) It follows from (a) that (X, ω^*_μ) is a GTS. Let A and B be two ω^*_μ-open sets and $x \in A \cap B$. Then there exist μ-open sets U and V containing x such that $U \setminus i_\mu(A)$ and $V \setminus i_\mu(B)$ are countable. Then $U \cap V$ is a μ-open set containing x and $(U \cap V) \setminus i_\mu(A \cap B) = (U \cap V) \setminus i_\mu(A) \cap i_\mu(B) \subseteqq [U \setminus i_\mu(A)] \cup [V \setminus i_\mu(B)]$. Thus $(U \cap V) \setminus i_\mu(A \cap B)$ is countable so that ω^*_μ is a QT on X. □

Theorem 1 *A subset* A *of a GTS* (X, μ) *is an* ω^*_μ*-open set if and only if for each* $x \in A$*, there exist* $U_x \in \mu$ *containing* x *and a countable subset* C *such that* $U_x \setminus C \subseteqq i_\mu(A)$*.*

Proof. Let A be an ω^*_μ-open set in X and $x \in A$. Then there exists $U_x \in \mu$ containing x such that $U_x \setminus i_\mu(A)$ is countable. Let $C = U_x \setminus i_\mu(A) = U_x \cap (X \setminus i_\mu(A))$. Then $U_x \setminus C \subseteqq i_\mu(A)$.

Conversely, let $x \in A$ and there exist $U_x \in \mu$ containing x and a countable subset C such that $U_x \setminus C \subseteqq i_\mu(A)$. Then $U_x \setminus i_\mu(A) \subseteqq C$ and hence $U_x \setminus i_\mu(A)$ is a countable set. Thus A is an ω^*_μ-open set in X. □

Theorem 2 *Let* (X, μ) *be a GTS and* $C \subseteq X$. *If* C *is* ω^*_μ*-closed, then* $C \subseteq K \cup B$ *for some* μ*-closed set* K *and a countable subset* B.

Proof. If C be ω^*_μ-closed, then $X \setminus C$ is ω^*_μ-open and hence for each $x \in X \setminus C$, there exist $U \in \mu$ containing x and a countable subset B such that $U \setminus B \subseteq i_\mu(X \setminus C) = X \setminus c_\mu(C)$. Thus $c_\mu(C) \subseteq X \setminus (U \setminus B) = X \setminus (U \cap (X \setminus B)) = (X \setminus U) \cup B$. Let $K = X \setminus U$. Then K is μ-closed such that $C \subseteq K \cup B$. □

Proposition 2 *In a GTS* (X, μ), $\omega^*_\mu = \omega^*_{\omega^*_\mu}$, *where* ω^*_μ *denotes the family of* ω^*_μ*-open sets of the GTS* (X, μ).

Proof. By Remark 1, we have $\omega^*_\mu \subseteq \omega^*_{\omega^*_\mu}$. Let $A \in \omega^*_{\omega^*_\mu}$. Then for each $x \in A$, there exist $U_x \in \omega^*_\mu$ with $x \in U_x$ and a countable set C_x such that $U_x \setminus C_x \subseteq i_\mu(A)$. Furthermore there exist a $V_x \in \mu$ with $x \in V_x$ and a countable set D_x such that $V_x \setminus D_x \subseteq i_\mu(U_x)$. Thus $V_x \setminus (C_x \cup D_x) = (V_x \setminus D_x) \setminus C_x \subseteq U_x \setminus C_x \subseteq i_\mu(A)$. Since $C_x \cup D_x$ is a countable set, we obtain $A \in \omega^*_\mu$. □

Remark 2 *If* (X, μ) *be a* μ*-space, then* (X, ω^*_μ) *is an* ω^*_μ*-space.*

Definition 2 *A subset* A *of a GTS* (X, μ) *is called an* (i) (ω_μ, ω)*-set if* $i_{\omega^*_\mu}(A) = i_{\omega_\mu}(A)$.
(ii) (ω_μ, μ)*-set if* $i_{\omega^*_\mu}(A) = i_\mu(A)$.

Remark 3 *Every* ω^*_μ*-open set is an* (ω_μ, ω)*-set and every* μ*-open set is an* (ω_μ, μ)*-set but the converses are usually not true.*

Example 2 (a) *Let* $X = \mathbb{R}$, $\mu = \{\varnothing, \mathbb{I}, X\}$. *Then* μ *is a GT on* X. *It can be checked that* $\mathbb{N}$ *(= the set of natural numbers) is not an* ω^*_μ*-open but an* (ω_μ, ω)*-set.*

(b) *Let* $X = \mathbb{R}$, $\mu = \{\varnothing, (2, 3), X\}$. *Then* μ *is a GT on* X. *The set* $(1, \frac{3}{2})$ *is an* (ω_μ, μ)*-set but not a* μ*-open set.*

Theorem 3 *A subset* A *of a GTS* (X, μ) *is* ω^*_μ*-open if and only if* A *is* ω_μ*-open and an* (ω_μ, ω)*-set.*

Proof. Since every ω^*_μ-open set is ω_μ-open (from definition) and an (ω_μ, ω)-set (by Remark 3), we have nothing to show.

Conversely, let A be an ω_μ-open and an (ω_μ, ω)-set. Then $A = i_{\omega_\mu}(A) = i_{\omega^*_\mu}(A)$. Thus A is an ω^*_μ-open set. □

Theorem 4 *A subset A of a GTS (X,μ) is μ-open if and only if A is ω_μ^*-open and an (ω_μ,μ)-set.*

Proof. One part follows from the fact that every μ-open set is ω_μ^*-open and (ω_μ,μ)-set.

Conversely, let A be an ω_μ^*-open set and an (ω_μ,μ)-set. Then $A = i_{\omega_\mu^*}(A) = i_\mu(A)$. Thus A is μ-open. $\square$

Definition 3 *A GTS (X,μ) is called*
(i) *μ-locally countable if each $x \in X$ is contained in a countable μ-open set.*
(ii) *anti μ-locally countable if each non-empty μ-open subsets are uncountable.*

Theorem 5 *Let (X,μ) be a μ-locally countable space. Then*
(i) *for any subset A of X, A is ω_μ^*-open.*
(ii) *A is ω_μ^*-open if and only if A is ω_μ-open.*

Proof. (i) Let $A \subsetneqq X$ and $x \in A$. Then there is a countable μ-open set U containing x. Then $U \setminus i_\mu(A)$ is a countable set. Thus A is ω_μ^*-open.

(ii) One part follows from the fact that every ω_μ^*-open is ω_μ-open.

Conversely, let A be ω_μ-open. Then by (i), A is ω_μ^*-open. $\square$

Remark 4 *Let (X,μ) be a countable GTS. Then for any subset A of X, A is ω_μ^*-open.*

Theorem 6 (i) *Let (X,μ) be a GTS and $A \subsetneqq X$. If (X,μ) is anti μ-locally countable, then so is (X,ω_μ^*).*
(ii) *Let (X,μ) be a QTS which is anti μ-locally countable. Then for any μ-open subset A of X, $c_\mu(A) = c_{\omega_\mu^*}(A)$.*

Proof. (i) Let A be an ω_μ^*-open set and $x \in A$. Then there exist $U_x \in \mu$ containing x and a countable subset C such that $U_x \setminus C \subsetneqq i_\mu(A)$. Thus $i_\mu(A)$ is uncountable and hence A is uncountable.

(ii) Let $x \in c_\mu(A)$ and G be an ω_μ^*-open set containing x. Then there exist $U_x \in \mu$ containing x and a countable subset C such that $U_x \setminus C \subsetneqq i_\mu(G)$. Then $(U_x \setminus C) \cap A \subsetneqq i_\mu(G) \cap A$ i.e., $(U_x \cap A) \setminus C \subsetneqq i_\mu(G) \cap A$. Since U_x is a μ-open set, $U_x \cap A \neq \emptyset$ and thus $U_x \cap A$ is a non-empty μ-open set. Hence by anti μ-locally countableness of (X,μ), it follows that $U_x \cap A$ is uncountable and hence $(U_x \cap A) \setminus C$ is also uncountable. Thus $i_\mu(G) \cap A$ is uncountable. Hence $G \cap A \neq \emptyset$. So $x \in c_{\omega_\mu^*}(A)$ i.e, $c_\mu(A) \subsetneqq c_{\omega_\mu^*}(A)$. The other part is obvious. $\square$

Definition 4 *A μ-space (X, μ) is said to be μ-Lindelöf* [17] *if every cover of X by μ-open sets has a countable subcover.*

A subset A of a μ-space (X, μ) is said to be μ-Lindelöf relative to X [17] *if every cover of A by μ-open sets of X has a countable subcover.*

Theorem 7 *Let (X, μ) be a μ-Lindelöf GTS and A be a μ-closed, ω_μ^*-open subset of X. Then $A \setminus i_\mu(A)$ is countable.*

Proof. Clearly A is a μ-Lindelöf space (see Corollary 3.6 of [15]). For each $x \in A$, there exist $U_x \in \mu$ containing x and a countable subset C such that $U_x \setminus C \subseteqq i_\mu(A)$. Thus $\{U_x : x \in A\}$ is cover of A by μ-open subsets of X. Hence by μ-Lindelöfness of A, it has a countable subcover $\{U_n : n \in \mathbb{N}\}$. Since $A \setminus i_\mu(A) \subseteqq \cup\{U_n \setminus i_\mu(A) : n \in \mathbb{N}\}$, $A \setminus i_\mu(A)$ becomes countable. □

3 Decomposition of continuity by ω_μ^*-open sets

Definition 5 *A function $f : (X, \mu) \to (Y, \lambda)$ is said to be ω_μ^*-continuous (resp. ω_μ-continuous* [15], *(μ, λ)-continuous* [1]) *if for every $x \in X$ and every λ-open set V of Y containing $f(x)$, there exists an ω_μ^*-open (resp. ω_μ-open, μ-open) set U containing x such that $f(U) \subseteqq V$.*

Definition 6 *A function $f : (X, \mu) \to (Y, \lambda)$ is said to be weakly ω_μ^*-continuous if for every $x \in X$ and every λ-open set V of Y containing $f(x)$, there exists an ω_μ^*-open set U containing x such that $f(U) \subseteqq c_\lambda(V)$.*

Theorem 8 *For a function $f : (X, \mu) \to (Y, \lambda)$ the following properties are equivalent:*
(i) *f is ω_μ^*-continuous;*
(ii) *$f : (X, \omega_\mu^*) \to (Y, \lambda)$ is (ω_μ^*, λ)-continuous;*
(iii) *$f^{-1}(V) \in \omega_\mu^*$ for every $V \in \lambda$.*

Proof. Obvious. □

Remark 5 *Let $f : (X, \mu) \to (Y, \lambda)$ be a function. Then the following relations hold: (μ, λ)-continuity $\Rightarrow$ ω_μ^*-continuity $\Rightarrow$ ω_μ-continuity $\Rightarrow$ weakly ω_μ^*-continuity.*

Example 3 (a) *Let $X = \{a, b, c\}$, $\mu = \{\varnothing, \{b\}, \{a, c\}, \{b, c\}, X\}$ and $\lambda = \{\varnothing, \{a, c\}, \{a, b\},$*

$X\}$. *Then* μ *and* λ *are two GT's on* X. *It can be verified that the identity mapping* $f : (X, \mu) \to (X, \lambda)$ *is* ω^*_μ*-continuous but not* (μ, λ)*-continuous.*

(b) *Let* $X = \mathbb{R}$, $\mu =$ *the usual topology on* $\mathbb{R}$, $Y = \{a, b, c, d\}$ *and* $\lambda = \{\varnothing, Y, \{a\}, \{a, b\}, \{c, d\}, \{a, b, c\}, Y\}$. *Consider the mapping* $f : (X, \mu) \to (Y, \lambda)$ *defined by*

$$f(x) = \begin{cases} a, & \text{if} \quad x \in \mathbb{I} \\ b, & \text{if} \quad x \notin \mathbb{I} \end{cases}$$

It can be checked that f *is* ω_μ*-continuous but not* ω^*_μ*-continuous.*

(c) *Let* $X = \mathbb{R}$ *be the set of real numbers,* $\mu = \{\varnothing, \mathbb{R}, \mathbb{I}\}$, $Y = \{a, b, c, d\}$ *and* $\lambda = \{\varnothing, Y, \{d\}, \{c, d\}, \{a, b, c\}\}$. *Consider the mapping* $f : (X, \mu) \to (Y, \lambda)$ *defined by*

$$f(x) = \begin{cases} a, & \text{if} \quad x \in \mathbb{I} \cup \{0\} \\ b, & \text{if} \quad x \notin \mathbb{I} \cup \{0\} \end{cases}$$

It can be verified that f *is* ω_μ*-continuous but not weakly* ω^*_μ*-continuous.*

Definition 7 *A function* $f : (X, \mu) \to (Y, \lambda)$ *is said to be* (ω^*_μ, ω)*-continuous (resp.* (ω^*_μ, μ)*-continuous) if for every* λ*-open set* A *of* Y, $f^{-1}(A)$ *is an* (ω_μ, ω)*-set (resp. an* (ω_μ, μ)*-set).*

Remark 6 *Every* (μ, λ)*-continuous function is* (ω^*_μ, μ)*-continuous and every* ω^*_μ*-continuous function is* (ω^*_μ, ω)*-continuous but the converses are not true.*

Example 4 (a) *Let* $X = \mathbb{R}$, $\mu = \{\varnothing, (2, 3), X\}$. *Then* μ *is a GT on* X. *Let* $B = (1, \frac{3}{2})$ *and* $\lambda = \{\varnothing, B, X \setminus B, X\}$. *Consider the mapping* $f : (X, \mu) \to (X, \lambda)$ *defined by*

$$f(x) = \begin{cases} \frac{5}{4}, & \text{if} \quad x \in (1, 2) \\ 4, & \text{if} \quad x \notin (1, 2) \end{cases}$$

It can be verified that f *is* (ω^*_μ, μ)*-continuous but not* (μ, λ)*-continuous.*

(b) *Let* $X = \mathbb{R}$, $\mu = \{\varnothing, \mathbb{I}, X\}$ *where* $\mathbb{I}$ *is the set of irrationals. Then* μ *is a GT on* X. *Consider the mapping* $f : (X, \mu) \to (X, \mu)$ *defined by*

$$f(x) = \begin{cases} \sqrt{2}, & \text{if} \quad x \in \mathbb{N} \\ 1, & \text{if} \quad x \notin \mathbb{N} \end{cases}$$

It can be verified that f *is* (ω^*_μ, ω)*-continuous but not* ω^*_μ*-continuous.*

Definition 8 *For any subset* A *of a GTS* (X, μ), *the* μ*-frontier* [16] *of* A *is denoted by* $\mathrm{Fr}_\mu(A)$ *and defined by* $\mathrm{Fr}_\mu(A) = c_\mu(A) \cap c_\mu(X \setminus A)$.

Definition 9 *A function* $f : (X, \mu) \to (Y, \lambda)$ *is said to be co-weakly* (ω^*_μ, λ)*-continuous if* $f^{-1}(Fr_\lambda(V))$ *is* ω^*_μ*-closed in* X *for every* λ*-open set* V *in* Y.

Theorem 9 *For a function* $f : (X, \mu) \to (Y, \lambda)$ *the following are equivalent:*
(i) f *is* ω^*_μ*-continuous.*
(ii) f *is* ω_μ*-continuous and* (ω_μ, ω)*-continuous.*

Proof. (i) $\Leftrightarrow$ (ii) Follows from Theorem 3. $\square$

Theorem 10 *Let* (X, μ) *be a QTS. Then for a function* $f : (X, \mu) \to (Y, \lambda)$ *the following are equivalent:*
(i) f *is* ω^*_μ*-continuous.*
(ii) f *is co-weakly* (ω^*_μ, λ)*-continuous and weakly* ω^*_μ*-continuous.*

Proof. (i) $\Rightarrow$ (ii): Obvious.
(ii) $\Rightarrow$ (i): Let f be a co-weakly (ω^*_μ, λ)-continuous and weakly ω^*_μ-continuous function. Let $x \in X$ and V be a λ-open set of Y containing $f(x)$. As f is a weakly ω^*_μ-continuous function, there exists a ω^*_μ-open set U containing x such that $f(U) \subsetneqq c_\lambda(V)$. Since $Fr_\lambda(V) = c_\lambda(V) \cap c_\lambda(X \setminus V) = c_\lambda(V) \setminus V$, we have $f(x) \notin Fr_\lambda(V)$. Since f is co-weakly (ω^*_μ, λ)-continuous, $x \in U \setminus f^{-1}(Fr_\lambda(V))$, which is ω^*_μ-open in X. Then for every $y \in f(U \setminus f^{-1}(Fr_\lambda(V)))$, $y = f(x_1)$ for a point $x_1 \in U \setminus f^{-1}(Fr_\lambda(V))$. Thus we have $f(x_1) = y \in f(U) \subsetneqq c_\lambda(V)$ and $y \notin Fr_\lambda(V)$ with $f(x_1) \in V$. Thus $f(U \setminus f^{-1}(Fr_\lambda(V))) \subsetneqq V$. Hence f is ω^*_μ-continuous. $\square$

Theorem 11 *For a function* $f : (X, \mu) \to (Y, \lambda)$ *the following are equivalent:*
(i) f (μ, λ)*-continuous.*
(ii) f *is* ω^*_μ*-continuous and* (ω^*_μ, μ)*-continuous.*

Proof. Follows from Theorem 4. $\square$

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