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Defining and investigating new soft ordered maps by using soft semi open sets

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Abstract. Here, we employ soft semi open sets to define new soft ordered maps, namely soft x-semi continuous, soft x-semi open, soft x-semi closed and soft x-semi homeomorphism maps, where x denotes the type of monotonicity. To show the relationships among them, we provide some illustrative examples. Then we give complete descriptions for each one of them. Also, we investigate "transmission" of these maps between soft and classical topological ordered spaces.

1 Introduction

In 1965, Nachbin [41] introduced new mathematical structure, namely topological ordered space. This structure consists of two independent concepts defined on a non-empty set X, one of them is a topological space (X, τ) and the other is a partially ordered set $(X, \preceq)$. McCartan [39] in 1968, studied separation axioms via topological ordered spaces. Kumar [35] defined the concepts of continuous and homeomorphism maps via topological ordered spaces. Recently, the authors of [1, 4, 6, 9, 10, 12, 23, 26, 28] have introduced and investigated many concepts via supra topological ordered spaces.

In 1999, Molotdsov [40] introduced the concept of soft sets for dealing with uncertainties and vagueness. Then, Maji et al. [38] put up the basis of soft

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set theory by defining some operations between two soft sets like soft subset and equality relations, and soft union and intersection. Shabir and Naz [45] initiated the idea of soft topological spaces and studied soft separation axioms. Later on, many researchers carried out several studies to discuss the topological notions on soft topologies (see, for example [2, 3, 7, 11, 15, 20, 22, 29, 30, 31, 36, 44]). Chen [24] and Mahanta and Das[37] displayed and probed the notions of soft semi open sets and soft semi separation axioms. Depend on soft semi open sets, some works were done (see, for example [5, 25, 33, 34]). At present, the notions of soft topological ordered spaces [16], supra soft topological ordered spaces [13] and soft ordered maps [13] were introduced and investigated.

This paper is organized as follows: In Section (2), we recall the previous definitions and results that we will need to prove our results. Section (3) gives other applications of soft semi open sets by defining some soft ordered maps, namely soft x-semi continuous, soft x-semi open, soft x-semi closed and soft x-semi homeomorphism maps for $x \in \{I, D, B\}$. These concepts are described and some examples are constructed to show the relationships among them. Also, we demonstrate the interrelationships between these soft maps and their counterparts of crisp ordered maps when the soft topology is extended. Section (4) concludes the paper.

2 Preliminaries

Let X and Ω be a universal set and a set of parameters, respectively. The power set of X is denoted by 2^X.

2.1 Soft sets

Definition 1 [40] *A notation G_Ω is said to be a soft set, terminologically, over X, if G is a map from Ω to 2^X. Usually, we write it as follows:*

$$G_\Omega = \{(\omega, G(\omega)) : \omega \in \Omega \text{ and } G(\omega) \in 2^X\}.$$

Through this article, $S(X_\Omega)$ denotes the family of soft sets on X with Ω.

Definition 2 [29, 45] *For $y \in X$ and G_Ω over X, we write that:*

1. $y \in G_\Omega$ *(resp. $y \notin G_\Omega$) if $y \in G(\omega)$ (resp. $y \notin G(\omega)$) for each $\omega \in \Omega$.*

2. $y \Subset G_\Omega$ *(resp. $y \not\Subset G_\Omega$) if $y \in G(\omega)$ (resp. $y \notin G(\omega)$) for some $\omega \in \Omega$.*

Definition 3 [38] *If* $G(\omega) = \emptyset$ *and* $F(\omega) = X$ *for each* $\omega \in \Omega$, *then* G_Ω *and* F_Ω *are respectively called null soft set and absolute soft set. They are respectively denoted by* $\widetilde{\emptyset}$ *and* $\widetilde{X}$.

Definition 4 [21] *The relative complement* G^c_Ω *of* G_Ω *is defined by* $G^c(\omega) = X \setminus G(\omega)$ *for each* $\omega \in \Omega$.

Definition 5 [46] *A soft mapping of* $S(X_\Omega,)$ *into* $S(Y_\Gamma)$, *denoted by* f_ϕ, *is a pair of mappings* $f : X \to Y$ *and* $\phi : \Omega \to \Gamma$ *such that the image of* $G_K \in S(X_\Omega,)$ *and pre-image of* $H_L \in S(Y_\Gamma)$ *are given by the following formulations:*

(i) $f_\phi(G_K) = (f_\phi(G))_\Gamma$ *is a soft subset of* $S(Y_\Gamma)$ *given by*

$$f_\phi(G)(\gamma) = \begin{cases} \bigcup_{a \in \phi^{-1}(\gamma) \bigcap K} f(G(a)) & : \quad \phi^{-1}(\gamma) \bigcap K \neq \emptyset \\ \emptyset & : \quad \phi^{-1}(\gamma) \bigcap K = \emptyset \end{cases}$$

for each $\gamma \in \Gamma$.

(ii) $f^{-1}_\phi(H_L) = (f^{-1}_\phi(H))_\Omega$ *is a soft subset of* $S(X_\Omega)$ *given by*

$$f^{-1}_\phi(H)(\omega) = \begin{cases} f^{-1}(H(\phi(\omega))) & : \quad \phi(\omega) \in L \\ \emptyset & : \quad \phi(\omega) \notin L \end{cases}$$

for each $\omega \in \Omega$.

Definition 6 [46] *If* f *and* ϕ *are injective (resp. surjective, bijective) maps, then* $f_\phi : S(X_\Omega) \to S(Y_\Gamma)$ *is said to be injective (resp. surjective, bijective).*

Proposition 1 [42] *Let* G_Ω *and* H_Γ *be soft subsets of* $S(X_\Omega)$ *and* $S(Y_\Gamma)$, *respectively. Then:*

(i) $G_\Omega \widetilde{\subseteq} f^{-1}_\phi f_\phi(G_\Omega)$. *If* f_ϕ *is injective, then* $G_\Omega = f^{-1}_\phi f_\phi(G_\Omega)$.

(ii) $f_\phi f^{-1}_\phi(H_\Gamma) \widetilde{\subseteq} H_\Gamma$. *If* f_ϕ *is surjective, then* $f_\phi f^{-1}_\phi(H_\Gamma) = H_\Gamma$.

Definition 7 [27], [42] *If there exist* $\omega \in \Omega$ *and* $x \in X$ *such that* $G(\omega) = \{x\}$ *and* $G(a) = \emptyset$ *for each* $a \in \Omega \setminus \{\omega\}$, *then* G_Ω *is called a soft point. Briefly, it is denoted by* P^x_ω.

If $x \in G(\omega)$, then $P^x_\omega \in G_\Omega$.

Definition 8 [16] *A triple* $(X, \Omega, \preceq)$ *is said to be a partially ordered soft set if* $(X, \preceq)$ *is a partially ordered set.*

$\preceq$ *is called linearly ordered if any pair of elements in the set of the relation are comparable, i.e., for each* $x, y \in X$ *either* $x \preceq y$ *or* $y \preceq x$.

Remark 1 *Through this paper, the notation* $\triangle$ *denotes a diagonal relation, i.e.* $\triangle = \{(x, x) : x \in X\}$.

Definition 9 [16] *An increasing soft operator* $i : (S(X_\Omega), \preceq) \to (S(X_\Omega), \preceq)$ *and a decreasing soft operator* $d : (S(X_\Omega), \preceq) \to (S(X_\Omega), \preceq)$ *are defined as follows: For each soft subset* G_Ω *of* $S(X_\Omega)$

1. $i(G_\Omega) = (iG)_\Omega$, *where a mapping* iG *of* Ω *into* 2^X *given by* $iG(\omega) = i(G(\omega)) = \{v \in X : y \preceq v$ *for some* $y \in G(\omega)\}$.

2. $d(G_\Omega) = (dG)_\Omega$, *where a mapping* dG *of* Ω *into* 2^X *given by* $dG(\omega) = d(G(\omega)) = \{v \in X : v \preceq y$ *for some* $y \in G(\omega)\}$.

Definition 10 [16] *A soft subset* G_Ω *of* $(X, \Omega, \preceq)$ *is said to be increasing (resp. decreasing) if* $G_\Omega = i(G_\Omega)$(*resp.* $G_\Omega = d(G_\Omega)$).

Theorem 1 [16] *If* $f_\phi : (S(X_\Omega), \preceq_1) \to (S(Y_\Gamma), \preceq_2)$ *is surjective and increasing (resp. decreasing), then the inverse image of each increasing (resp. decreasing) soft set is increasing (resp. decreasing).*

2.2 Soft topologies

Definition 11 [45] *A sub-collection* τ *of* $S(X_\Omega)$ *is called a soft topology on* X *provided that it is closed under finite soft intersection and arbitrary soft union.*

By a soft topological space we mean a triple (X, τ, Ω). *Every member of* τ *is called soft open and its relative complement is called soft closed.*

Proposition 2 [45] *In* (X, τ, Ω), *a class* $\tau_\gamma = \{G(\omega) : G_\Omega \in \tau\}$ *defines a classical topology on* X *for each* $\omega \in \Omega$.

Proposition 3 [42] *A class* $\tau^\star = \{G_\Omega : G(\omega) \in \tau_\gamma$ *for each* $\omega \in \Omega\}$ *defines a soft topology on* X *finer than* τ.

Henceforward, $\tau^\star$ is called an extended soft topology.

Definition 12 [24, 37] *A soft subset* H_Ω *of* (X, τ, Ω) *which satisfies* $H_\Omega \widetilde{\subseteq} cl(int(H_\Omega))$ *is said to be soft semi open. The relative complement of a soft semi open set is said to be soft semi closed.*

Definition 13 [24, 37, 45] *We associate a subset* H_Ω *of* (X, τ, Ω) *with the following four operators:*

(i) $\mathrm{int}(H_\Omega)$ (*resp.* $\mathrm{int}_s(H_\Omega)$) *is the largest soft open* (*resp. soft semi open*) *set contained in* H_Ω.

(ii) $\mathrm{cl}(H_\Omega)$ (*resp.* $\mathrm{cl}_s(H_\Omega)$) *is the smallest soft closed* (*resp. soft semi closed*) *set containing* H_Ω.

Definition 14 [24] $f_\phi : (X, \tau, A) \to (Y, \theta, B)$ *is said to be:*

(i) *soft semi continuous if* $f_\phi^{-1}(G_B)$ *is soft semi open for each* $G_B \in \theta$.

(ii) *soft semi open* (*resp. soft semi closed*) *if* $f_\phi(U_A)$ *is soft semi open* (*resp. soft semi closed*) *for each* $U_A (\mathrm{resp.}\ U_A^c) \in \tau$.

(iii) *a soft semi homeomorphism if it is bijective, soft semi continuous and soft semi open.*

Definition 15 [16] *We call a quadrable system* $(X, \tau, \Omega, \preceq)$ *a soft topological ordered space provided that* τ *is a soft topology and* $\preceq$ *is a partially ordered set on* X.

Henceforward, we use the two notations $(X, \tau, \Omega, \preceq_1)$ *and* $(Y, \theta, \Gamma, \preceq_2)$ *to denote soft topological ordered spaces.*

Definition 16 [17] *The composition of* $f_\phi : (X, \tau, \Omega, \preceq_1) \to (Y, \theta, \Gamma, \preceq_2)$ *and* $g_\lambda : (Y, \theta, \Gamma, \preceq_2) \to (Z, \upsilon, K, \preceq_3)$ *is a soft map* $f_\phi \circ g_\lambda : (X, \tau, \Omega, \preceq_1) \to (Z, \upsilon, K, \preceq_3)$ *and is given by* $(f_\phi \circ g_\lambda)(P_\omega^x) = f_\phi(g_\lambda(P_\omega^x))$.

Definition 17 [43] *A map* g *from* $(X, \tau, \preceq_1)$ *to* $(Y, \theta, \preceq_2)$ *is said to be:*

(i) *D* (*resp. I, B*) *-semi continuous if* $g^{-1}(G)$ *is D* (*resp. I, B*) *-semi open for each* $G \in \theta$.

(ii) *D* (*resp. I, B*) *-semi open if* $g(F)$ *is D* (*resp. I, B*) *-semi open for each* $F \in \tau$.

(iii) *D* (*resp. I, B*) *-semi closed if* $g(H)$ *is D* (*resp. I, B*) *-semi closed for each* $F^c \in \tau$.

(iv) *D* (*resp. I, B*) *-semi homeomorphism if it is bijective, D* (*resp. I, B*) *-semi continuous and D* (*resp. I, B*) *-semi open.*

3 New types of soft semi ordered maps

3.1 Soft $D(I, B)$-semi continuity

This subsection introduces the concepts of $D(I, B)$-semi continuity at soft point and ordinary point, where D, I and B denote "Decreasing", "Increasing" and "Balancing", respectively. We also give the equivalent terms for each one of these concepts at the ordinary points and provide some illustrative examples.

Definition 18 *A soft subset* H_Ω *of* $(X, \tau, \Omega, \preceq_1)$ *which is:*

(i) *soft semi open and increasing (resp. decreasing, balancing) is said to be SI (resp. SD, SB) -semi open.*

(ii) *soft semi closed and increasing (resp. decreasing, balancing) is said to be SI (resp. SD, SB) -semi closed.*

Definition 19 $f_\phi : (X, \tau, \Omega, \preceq_1) \to (Y, \theta, \Gamma, \preceq_2)$ *is called:*

1. *SI (resp. SD, SB) -semi continuous at* $P^x_\omega \in \widetilde{X}$ *if for each soft open set* H_Γ *containing* $f_\phi(P^x_\omega)$*, there exists an SI (resp. SD, SB) -semi open set* G_Ω *containing* P^x_ω *such that* $f_\phi(G_\Omega) \widetilde{\subseteq} H_\Gamma$.

2. *SI (resp. SD, SB) -semi continuous at* $x \in X$ *if it is SI (resp. SD, SB) -semi continuous at each* P^x_ω.

3. *SI (resp. SD, SB) -semi continuous if it is SI (resp. SD, SB) -semi continuous at each* $x \in X$.

Theorem 2 $f_\phi : (X, \tau, \Omega, \preceq_1) \to (Y, \theta, \Gamma, \preceq_2)$ *is SI (resp. SD, SB) -semi continuous iff the inverse image of each soft open set is SI (resp. SD, SB) -semi open.*

Proof. When f_ϕ is SD-semi continuous.
Necessity: Let $G_\Gamma \in \theta$. Without loss of generality, consider $f^{-1}(G_\Gamma) \neq \widetilde{\emptyset}$. By choosing $P^x_\omega \in X$ s.t. $P^x_\omega \in f^{-1}_\phi(G_\Gamma)$, we obtain $f_\phi(P^x_\omega) \in G_\Gamma$. Then there is an SD-semi open set H_Ω containing P^x_ω s.t. $f_\phi(H_\Omega) \widetilde{\subseteq} G_\Gamma$. Since P^x_ω is chosen arbitrary, then $f^{-1}_\phi(G_\Gamma) = \widetilde{\bigcup}_{P^x_\omega \in f^{-1}_\phi(G_\Gamma)} H_\Omega$; therefore, $f^{-1}_\phi(G_\Gamma)$ is an SD-semi open set.

Sufficiency: Let $G_\Gamma \in \theta$ such that $f_\phi(P^x_\omega) \in \theta$. Then $P^x_\omega \in f^{-1}_\phi(G_\Gamma)$. By hypothesis, $f^{-1}_\phi(G_\Gamma)$ is an SD-semi open set. Since $f_\phi(f^{-1}_\phi(G_\Gamma)) \widetilde{\subseteq} G_\Gamma$, then f_ϕ is an SD-semi continuous map at P^x_ω and since P^x_ω is selected randomly, then f_ϕ is an SD-semi continuous map. □

Remark 2 *It can be seen from Definition (19) the following.*

1. *Every SI (SD, SB) -semi continuous map is soft semi continuous.*
2. *Every SB-semi continuous map is SI (SD) -semi continuous.*

Examples given below manifest that the two results of the remark above are not reversible.

Example 1 *Let* $\Omega = \{\omega_1, \omega_2\}$ *be a parameters set and* $X = \{1, 2, 3, 4\}$ *be a universe set and consider* $\phi : \Omega \to \Omega$ *and* $f : X \to X$ *are two identity maps. Let* $\preceq = \triangle \bigcup \{(1, 3)\}$ *be a partial order relation on* X *and consider* $\tau = \{\widetilde{\emptyset}, \widetilde{X}, F_\Omega G_\Omega\}$ *and* $\theta = \{\widetilde{\emptyset}, \widetilde{Y}, H_\Omega L_\Omega\}$ *are two soft topologies on* X*, where* $F_\Omega = \{(\omega_1, \{1\}), (\omega_2, \{3, 4\})\}$*,* $G_\Omega = \{(\omega_1, \emptyset), (\omega_2, \{3\})\}$*,* $H_\Omega = \{(\omega_1, \{1\}), (\omega_2, \{2, 3\})\}$ *and* $L_\Omega = \{(\omega_1, \{1\}), (\omega_2, \{3\})\}$*. For a soft map* $f_\phi : (X, \tau, \Omega, \preceq) \to (X, \theta, \Omega, \preceq)$*, we note that* $f_\phi^{-1}(H_\Omega) = H_\Omega$ *and* $f_\phi^{-1}(L_\Omega) = L_\Omega$ *are soft semi open sets. So* f_ϕ *is a soft semi continuous map. But,* $f_\phi^{-1}(H_\Omega)$ *is neither an SD-semi open set nor an SI-semi open set. Hence* f_ϕ *is not SI (SD, SB)-semi continuous.*

Example 2 *By replacing a partial order relation (in the above example) by* $\preceq = \triangle \bigcup \{(2, 4)\}$*(resp.* $\preceq = \triangle \bigcup \{(4, 1)\}$*), we obtain a soft map* f_ϕ *is SD-semi continuous (resp. SI-continuous), but is not SB-semi continuous.*

Definition 20 *For any set* H_Ω *in* $(X, \tau, \Omega, \preceq)$*, we introduce the next operators:*

(i) H_Ω^{iso} *(resp.* $H_\Omega^{dso}, H_\Omega^{bso}$*) is the largest SI (resp. SD, SB) -semi open set contained in* H_Ω*.*

(ii) H_Ω^{iscl} *(resp.* $H_\Omega^{dscl}, H_\Omega^{bscl}$*) is the smallest SI (resp. SD, SB) -semi closed set containing* H_Ω*.*

Lemma 1 *The next properties are satisfied for a set* H_Ω *in* $(X, \tau, \Omega, \preceq)$*.*

(i) $(H_\Omega^{dscl})^c = (H_\Omega^c)^{iso}$.

(ii) $(H_\Omega^{iscl})^c = (H_\Omega^c)^{dso}$.

(iii) $(H_\Omega^{bscl})^c = (H_\Omega^c)^{bso}$.

Proof.

(i) $(H^{dscl}_{\Omega})^c = \{\widetilde{\bigcup} F_{\Omega} : F_{\Omega}$ is an SD-semi closed set containing $H_{\Omega}\}^c$ $= \widetilde{\bigcap}\{F^c_{\Omega} : F^c_{\Omega}$ is an SI-semi open set contained in $H^c_{\Omega}\} = (H^c_{\Omega})^{iso}$.

By analogy with **(i)**, one can prove **(ii)** and **(iii)**. $\square$

Theorem 3 *The next properties of* $f_{\phi} : (X, \tau, \Omega, \preceq_1) \rightarrow (Y, \theta, \Gamma, \preceq_2)$ *are equivalent:*

1. f_{ϕ} *is SI-semi continuous;*
2. $f^{-1}_{\phi}(L_{\Gamma})$ *is an SD-semi closed subset of* $\widetilde{X}$ *for any soft closed set* L_{Γ} *in* $\widetilde{Y}$*;*
3. $(f^{-1}_{\phi}(M_{\Gamma}))^{dscl} \widetilde{\subseteq} f^{-1}_{\phi}(cl(M_{\Gamma}))$ *for every* $M_{\Gamma} \widetilde{\subseteq} \widetilde{Y}$*;*
4. $f_{\phi}(N^{dscl}_{\Omega}) \widetilde{\subseteq} cl(f_{\phi}(N_{\Omega}))$ *for every* $N_{\Omega} \widetilde{\subseteq} \widetilde{X}$*;*
5. $f^{-1}_{\phi}(int(M_{\Gamma})) \widetilde{\subseteq} (f^{-1}_{\phi}(M_{\Gamma}))^{iso}$ *for every* $M_{\Gamma} \widetilde{\subseteq} \widetilde{Y}$*.*

Proof. $1 \Rightarrow 2$: Suppose L_{Γ} is a soft closed set in $\widetilde{Y}$. Then, $f^{-1}_{\phi}(L^c_{\Gamma})$ is an SI-semi open set in $\widetilde{X}$. Now, $f^{-1}_{\phi}(L^c_{\Gamma}) = (f^{-1}_{\phi}(L_{\Gamma}))^c$; hence, $f^{-1}_{\phi}(L_{\Gamma})$ is an SD-semi closed set.
$2 \Rightarrow 3$: It comes from 2 that $f^{-1}_{\phi}(cl(M_{\Omega}))$ is an SD-semi closed set in $\widetilde{X}$ for any $M_{\Gamma} \widetilde{\subseteq} \widetilde{Y}$. Therefore, $(f^{-1}_{\phi}(M_{\Gamma}))^{dscl} \widetilde{\subseteq} (f^{-1}_{\phi}(cl(M_{\Gamma}))^{dscl} = f^{-1}_{\phi}(cl(M_{\Gamma}))$.
$3 \Rightarrow 4$: We know that that $N^{dscl}_{\Omega} \widetilde{\subseteq} (f^{-1}_{\phi}(f_{\phi}(N_{\Omega}))^{dscl}$; according to 3 we have $(f^{-1}_{\phi}(f_{\phi}(N_{\Omega}))^{dscl} \widetilde{\subseteq} f^{-1}_{\phi}(cl(f_{\phi}(N_{\Omega}))$. Hence, $f_{\phi}(N^{dscl}_{\Omega}) \widetilde{\subseteq} cl(f_{\phi}(N_{\Omega}))$.
$4 \Rightarrow 5$: For any soft set M_{Γ} in $\widetilde{Y}$, we obtain from Lemma (1) that $f_{\phi}(\widetilde{X} - (f^{-1}_{\phi}(N_{\Omega}))^{iso}) = f_{\phi}(((f^{-1}_{\phi}(N_{\Omega}))^c)^{dscl})$. It follows from statement 4, that $f_{\phi}(((f^{-1}_{\phi}(N_{\Omega}))^c)^{dscl}) \widetilde{\subseteq} cl(f_{\phi}(f^{-1}_{\phi}(N_{\Omega}))^c) = cl(f_{\phi}(f^{-1}_{\phi}(N^c_{\Omega}))) \widetilde{\subseteq} cl(\widetilde{Y} - N_{\Omega}) = \widetilde{Y} - int(N_{\Omega})$. Therefore $(\widetilde{X} - (f^{-1}_{\phi}(N_{\Omega}))^{iso}) \widetilde{\subseteq} f^{-1}_{\phi}(\widetilde{Y} - int(N_{\Omega})) = \widetilde{X} - f^{-1}_{\phi}(int(N_{\Omega}))$. Thus $f^{-1}_{\phi}(int(N_{\Omega})) \widetilde{\subseteq} (f^{-1}_{\phi}(N_{\Omega}))^{iso}$.
$5 \Rightarrow 1$: Consider M_{Γ} is a soft open set in $\widetilde{Y}$. Then $f^{-1}_{\phi}(M_{\Gamma}) = f^{-1}_{\phi}(int(M_{\Gamma})) \widetilde{\subseteq} (f^{-1}_{\phi}(M_{\Gamma}))^{iso}$. So $(f^{-1}_{\phi}(M_{\Gamma}))^{iso} = f^{-1}_{\phi}(M_{\Gamma})$ and this means that $f^{-1}_{\phi}(M_{\Gamma})$ is an SI-semi open set in $\widetilde{X}$. $\square$

Theorem 4 *The next properties of* $f_{\phi} : (X, \tau, \Omega, \preceq_1) \rightarrow (Y, \theta, \Gamma, \preceq_2)$ *are equivalent:*

1. f_ϕ *is SD-semi continuous (resp. SB-semi continuous);*
2. $f_\phi^{-1}(L_\Gamma)$ *is an SI-semi closed (resp. SB-semi closed) set in* $\widetilde{X}$ *for each soft closed set* L_Γ *in* $\widetilde{Y}$*;*
3. $(f_\phi^{-1}(M_\Gamma))^{iscl}\widetilde{\subseteq}f_\phi^{-1}(cl(M_\Gamma))$ *(resp.* $(f_\phi^{-1}(M_\Gamma))^{bscl}\widetilde{\subseteq}f_\phi^{-1}(cl(M_\Gamma)))$ *for every* $M_\Gamma\widetilde{\subseteq}\widetilde{Y}$*;*
4. $f_\phi(N_\Omega^{iscl})\widetilde{\subseteq}cl(f_\phi(N_\Omega))$ *(resp.* $f_\phi(N_\Omega^{bscl})\widetilde{\subseteq}cl(f_\phi(N_\Omega)))$ *for every* $N_\Omega\widetilde{\subseteq}\widetilde{X}$*;*
5. $f_\phi^{-1}(int(M_\Gamma))\widetilde{\subseteq}(f_\phi^{-1}(M_\Gamma))^{dso}$ *(resp.* $f_\phi^{-1}(int(M_\Gamma))\widetilde{\subseteq}(f_\phi^{-1}(M_\Gamma))^{bso}$*) for every* $M_\Gamma\widetilde{\subseteq}\widetilde{Y}$*.*

Proof. Similar to the proof of Theorem (3). □

Theorem 5 *Let a soft topology* $\tau^\star$ *be extended. Then* $g_\phi : (X, \tau^\star, \Omega, \preceq_1) \to (Y, \theta, \Gamma, \preceq_2)$ *is SI (resp. SD, SB) -semi continuous iff a crisp map* $g : (X, \tau^\star_\gamma, \preceq_1) \to (Y, \theta_{\phi(\omega)}, \preceq_2)$ *is I (resp. D, B) -semi continuous.*

Proof. $\Rightarrow$: Consider U is an open set in $(Y, \theta_{\phi(\omega)}, \preceq_2)$. Then there is a soft open set G_Γ in $(Y, \theta, \Gamma, \preceq_2)$ s.t. $G(\phi(\omega)) = U$. Since g_ϕ is an SI (resp. SD, SB) -semi continuous map, then $g_\phi^{-1}(G_\Gamma)$ is an SI (resp. SD, SB) -semi open set. From Definition (5), a soft set $g_\phi^{-1}(G_\Gamma) = (g_\phi^{-1}(G))_\Omega$ in $(X, \tau, \Omega, \preceq_1)$ is given by $g_\phi^{-1}(G)(\omega) = g^{-1}(G(\phi(\omega)))$ for any $\omega \in \Omega$. Now, $\tau^\star$ is extended; thus, a set $g^{-1}(G(\phi(\omega))) = g^{-1}(U)$ in $(X, \tau_\gamma, \preceq_1)$ is I (resp. D, B) -semi open. This proves that g is I (resp. D, B) -semi continuous.
$\Leftarrow$: Consider G_Γ is a soft open set in $(Y, \theta, \Gamma, \preceq_2)$. Then a soft set $g_\phi^{-1}(G_\Gamma) = (g_\phi^{-1}(G))_\Omega$ in $(X, \tau^\star, \Omega, \preceq_1)$ isgiven by $g_\phi^{-1}(G)(\omega) = g^{-1}(G(\phi(\omega)))$ for any $\omega \in \Omega$. Since amap g is I (resp. D, B) -semi continuous, a set $g^{-1}(G(\phi(\omega)))$ in $(X, \tau^\star_\gamma, \preceq_1)$ is I (resp. D, B) -semi open. Now, $\tau^\star$ is extended; thus, $g_\phi^{-1}(G_\Gamma)$ is an SI (resp. SD, SB) -semi open set in $(X, \tau^\star, \Omega, \preceq_1)$. This proves that a soft map g_ϕ is SI (resp. SD, SB) -semi continuous. □

Proposition 4 *Let* $f_\phi : (X, \tau, \Omega, \preceq_1) \to (Y, \theta, \Gamma, \preceq_2)$ *be SB-semi continuous and surjective. If* $\preceq_1$ *is linearly ordered, then* θ *is the indiscrete soft topology.*

3.2 Soft $I(D, B)$-semi open and soft $I(D, B)$-semi closed maps

In the following part, we present the notions of soft $I(D, B)$-semi open and soft $I(D, B)$-semi closed maps. Then, we elucidate the relationships among them with the help of examples. Finally, we characterize each one of these concepts and study some properties.

Definition 21 $f_\phi : (X, \tau, \Omega, \preceq_1) \to (Y, \tau, \Gamma, \preceq_2)$ *is called:*

(i) *SI (resp. SD, SB) -semi open if the image of any soft open set in* $\widetilde{X}$ *is an SI (resp. SD, SB) -semi open set in* $\widetilde{Y}$.

(ii) *SI (resp. SD, SB) -semi closed if the image of any soft closed set in* $\widetilde{X}$ *is an SI (resp. SD, SB) -semi closed set in* $\widetilde{Y}$.

Remark 3 *Note that:*

1. *an SI (SD, SB) -semi open map is soft semi open.*
2. *an SI (SD, SB) -semi closed map is soft semi closed.*
3. *an SB-semi open (resp. SB-semi closed) map is SI (SD) -semi open (resp. SI (SD) -semi closed).*

Examples given below manifest that the three results of the remark above are not reversible.

Example 3 *Let* Ω, X, $\phi : \Omega \to \Omega$, $f : X \to X$ *and* $\preceq$ *be the same as in Example (1). Consider* $\tau = \{\widetilde{\emptyset}, \widetilde{X}, F_\Omega\}$ *and* $\theta = \{\widetilde{\emptyset}, \widetilde{Y}, L_\Omega\}$ *are two soft topologies on* X, *where* $F_\Omega = \{(\omega_1, \{1\}), (\omega_2, \{3, 4\})\}$ *and* $L_\Omega = \{(\omega_1, \{1\}), (\omega_2, \{3\})\}$. *For a soft map* $f_\phi : (X, \tau, \Omega, \preceq) \to (X, \theta, \Omega, \preceq)$, *we note that* $f_\phi(F_\Omega) = F_\Omega$ *is a soft semi open set. So* f_ϕ *is a soft semi open map. On the other hand,* $f_\phi(F_\Omega)$ *is neither an SD-semi open set nor an SI-semi open set. Hence* f_ϕ *is not SI (SD, SB)-semi open. Also,* f_ϕ *is a soft semi closed map, but it is not SI (SD, SB)-semi closed.*

Example 4 *By replacing a partial order relation (given in the example above) by* $\preceq = \triangle \bigcup \{(2, 4)\}$, *we obtain* f_ϕ *is SI-semi open and SD-semi closed, but it is neither an SB-semi open map nor an SB-semi closed map. Also, if we replace only the partial order relation by* $\preceq = \triangle \bigcup \{(1, 2)\}$, *then the soft map* f_ϕ *is SD-open and SI-semi closed, but it is neither an SB-semi open map nor an SB-semi closed map.*

Theorem 6 *The next properties of* $f_\phi : (X, \tau, \Omega, \preceq_1) \to (Y, \theta, \Gamma, \preceq_2)$ *are equivalent:*

1. f_ϕ *is SI-semi open;*

2. $\mathrm{int}(f_\phi^{-1}(M_\Gamma))\widetilde{\subseteq}f_\phi^{-1}(M_\Gamma^{iso})$ *for any* $M_\Gamma\widetilde{\subseteq}\widetilde{Y}$;

3. $f_\phi(\mathrm{int}(N_\Omega))\widetilde{\subseteq}(f_\phi(N_\Omega))^{iso}$ *for any* $N_\Omega\widetilde{\subseteq}\widetilde{X}$.

Proof. $1 \Rightarrow 2$: It is clear that $\mathrm{int}(f_\phi^{-1}(M_\Gamma))$ is a soft open set in $\widetilde{X}$ for any soft set M_Γ in $\widetilde{Y}$. Now, $f_\phi(\mathrm{int}(f_\phi^{-1}(M_\Gamma)))$ is an SI-semi open set in $\widetilde{Y}$. Since $f_\phi(\mathrm{int}(f_\phi^{-1}(M_\Gamma)))\widetilde{\subseteq}f_\phi(f_\phi^{-1}(M_\Gamma))\widetilde{\subseteq}M_\Gamma$, then $\mathrm{int}(f_\phi^{-1}(M_\Gamma))\widetilde{\subseteq}f_\phi^{-1}(M_\Gamma^{iso})$.
$2 \Rightarrow 3$: Given a soft set N_Ω in $\widetilde{X}$, according to 2 $\mathrm{int}(f_\phi^{-1}(f_\phi(N_\Omega)))\widetilde{\subseteq} f_\phi^{-1}((f_\phi(N_\Omega))^{iso})$. Since $\mathrm{int}(N_\Omega)\widetilde{\subseteq}f_\phi^{-1}(f_\phi(\mathrm{int}(f_\phi^{-1}(f_\phi(N_\Omega)))))\widetilde{\subseteq}f_\phi^{-1}((f_\phi(N_\Omega))^{iso})$, then $f_\phi(\mathrm{int}(N_\Omega))\widetilde{\subseteq}(f_\phi(N_\Omega))^{iso}$.
$3 \Rightarrow 1$: Let G_Ω be a soft open set in $\widetilde{X}$. Then $f_\phi(\mathrm{int}(G_\Omega)) = f_\phi(G_\Omega)\widetilde{\subseteq}(f_\phi(G_\Omega))^{iso}$. Hence, f_ϕ is an SI-semi open map. □

Following similar technique, the following two theorems are proved.

Theorem 7 *The following three properties of* $f_\phi : (X, \tau, \Omega, \preceq_1) \to (Y, \theta, \Gamma, \preceq_2)$ *are equivalent:*

1. f_ϕ *is SD-semi open (resp. SB-semi open);*

2. $\mathrm{int}(f_\phi^{-1}(M_\Gamma))\widetilde{\subseteq}f_\phi^{-1}(M_\Gamma^{dso})$ (*resp.* $\mathrm{int}(f_\phi^{-1}(M_\Gamma))\widetilde{\subseteq}f_\phi^{-1}(M_\Gamma^{bso})$) *for every* $M_\Gamma\widetilde{\subseteq}\widetilde{Y}$;

3. $f_\phi(\mathrm{int}(N_\Omega))\widetilde{\subseteq}(f_\phi(N_\Omega))^{dso}$ (*resp.* $f_\phi(\mathrm{int}(N_\Omega))\widetilde{\subseteq}(f_\phi(N_\Omega))^{bso}$) *for every* $N_\Omega\widetilde{\subseteq}\widetilde{X}$.

Theorem 8 *The next statements hold for* $f_\phi : (X, \tau, \Omega, \preceq_1) \to (Y, \theta, \Gamma, \preceq_2)$:

1. f_ϕ *is SI-semi closed iff* $(f_\phi(G_\Omega))^{iscl}\widetilde{\subseteq}f_\phi(\mathrm{cl}(G_\Omega))$ *for every* $G_\Omega\widetilde{\subseteq}\widetilde{X}$.

2. f_ϕ *is SD-semi closed iff* $(f_\phi(G_\Omega))^{dscl}\widetilde{\subseteq}f_\phi(\mathrm{cl}(G_\Omega))$ *for every* $G_\Omega\widetilde{\subseteq}\widetilde{X}$.

3. f_ϕ *is SB-semi closed iff* $(f_\phi(G_\Omega))^{bscl}\widetilde{\subseteq}f_\phi(\mathrm{cl}(G_\Omega))$ *for every* $G_\Omega\widetilde{\subseteq}\widetilde{X}$.

Proof. We only prove 1.
Necessity: Since f_ϕ is SI-semi closed, $f_\phi(\mathrm{cl}(G_\Omega))$ is an SI-semi closed set in $\widetilde{Y}$ and since $f_\phi(G_\Omega)\widetilde{\subseteq}f_\phi(\mathrm{cl}(G_\Omega))$, $(f_\phi(G_\Omega))^{iscl}\widetilde{\subseteq}f_\phi(\mathrm{cl}(G_\Omega))$.
Sufficiency: Consider H_Ω is a soft closed set in $\widetilde{X}$. Then $f_\phi(H_\Omega)\widetilde{\subseteq}(f_\phi(H_\Omega))^{iscl}\widetilde{\subseteq} f_\phi(\mathrm{cl}(H_\Omega)) = f_\phi(H_\Omega)$. Therefore, $f_\phi(H_\Omega) = (f_\phi(H_\Omega))^{iscl}$. This means that $f_\phi(H_\Omega)$ is an SI-semi closed set. □

Theorem 9 *The next hold for a bijective soft map* $f_\phi : (X, \tau, \Omega, \preceq_1) \to (Y, \theta, \Gamma, \preceq_2)$*:*

(i) f_ϕ *is SI (resp. SD, SB) -semi open if and only if* f_ϕ *is SD (resp. SD, SB) -semi closed.*

(ii) f_ϕ *is SI (resp. SD, SB) -semi open if and only if* f_ϕ^{-1} *is SI (resp. SD, SB) -semi continuous.*

(iii) f_ϕ *is SD (resp. SI, SB) -semi closed if and only if* f_ϕ^{-1} *is SI (resp. SD, SB) -semi continuous.*

Proof. We prove the cases outside the parenthesis and the cases between parenthesis can be made similarly.

(i) Necessity: Let H_Ω be a soft closed set in $\widetilde{X}$ and consider f_ϕ is an SI-semi open map. Then H^c_Ω is soft open and $f_\phi(H^c_\Omega)$ is SI-semi open. Bijectiveness of f_ϕ leads to that $f_\phi(H^c_\Omega) = [f_\phi(H_\Omega)]^c$. This automatically implies that $f_\phi(H_\Omega)$ is SD-semi closed. Thus, f_ϕ is an SD-semi closed map. Following similar technique, the sufficient condition is proved.

(ii) Necessity: Let G_Ω be a soft open set in $\widetilde{X}$ and consider f_ϕ is an SI-semi open map. Then $f_\phi(G_\Omega)$ is SI-semi open. Bijectiveness of f_ϕ leads to that $f_\phi(G_\Omega) = (f_\phi^{-1})^{-1}(G_\Omega)$. This automatically implies that $(f_\phi^{-1})^{-1}(G_\Omega)$ is SI-semi open. Thus f_ϕ^{-1} is an SI-semi continuous map. Following similar technique, the sufficient condition is proved.

(iii) It follows from **(i)** and **(ii)**.

□

Theorem 10 *Let a soft topology* $\theta^\star$ *be extended and a map* ϕ *be injective. Then* $g_\phi : (X, \tau, \Omega, \preceq_1) \to (Y, \theta^\star, \Gamma, \preceq_2)$ *is SI (resp. SD, SB) -semi open iff a crisp map* $g : (X, \tau_\gamma, \preceq_1) \to (Y, \theta^\star_{\phi(\omega)}, \preceq_2)$ *is I (resp. D, B) -semi open.*

Proof. Let U be an open set in $(X, \tau_\gamma, \preceq_1)$ and $\phi(\omega) = f$. Then there is a soft open set G_Ω in $(X, \tau, \Omega, \preceq_1)$ s.t. $G(\omega) = U$. Since g_ϕ is an SI (resp. SD, SB) -semi open map, then $g_\phi(G_\Omega)$ is an SI (resp. SD, SB) -semi open set. Now, a soft set $g_\phi(G_\Omega) = (g_\phi(G))_\Gamma$ in $(Y, \theta, \Gamma, \preceq_2)$ is given by $g_\phi(G)(f) = \bigcup_{\omega \in \phi^{-1}(f)} g(G(\omega))$ for each $f \in \Gamma$. View of $\theta^\star$ is extended, a set $\bigcup_{\omega \in \phi^{-1}(f)} g(G(\omega)) = g(U)$ in $(Y, \theta_{\phi(\omega)}, \preceq_2)$ is I (resp. D, B) -semi open.

Hence a map g is I (resp. D, B) -semi open. Conversely, consider G_Ω is a soft open set in $(X, \tau, \Omega, \preceq_1)$. Then a soft set $g_\phi(G_\Omega) = (g_\phi(G))_\Gamma$ in $(Y, \theta^\star, \Gamma, \preceq_2)$ is given by $g_\phi(G)(f) = \bigcup_{\omega \in \phi^{-1}(f)} g(G(\omega))$ for each $f \in \Gamma$. Since a map g is I (resp. D, B) -semi open, a set $\bigcup_{\omega \in \phi^{-1}(f)} g(G(\omega))$ in $(Y, \theta^\star_{\phi(\omega)}, \preceq_2)$ is I (resp. D, B) -semi open. Now, $\theta^\star$ is an extended soft topology on Y, $g_\phi(G_\Omega)$ is an SI (resp. SD, SB) -semi open subset of $(Y, \theta^\star, \Gamma, \preceq_2)$. Hence a soft map g_ϕ is SI (resp. SD, SB) -semi open. □

Theorem 11 *Let a soft topology $\theta^\star$ be extended and a map ϕ is injective. Then $g_\phi : (X, \tau, \Omega, \preceq_1) \to (Y, \theta^\star, \Gamma, \preceq_2)$ is SI (resp. SD, SB) -semi closed iff a crisp map $g : (X, \tau_\gamma, \preceq_1) \to (Y, \theta^\star_{\phi(\omega)}, \preceq_2)$ is I (resp. D, B) -semi closed.*

Proposition 5 *Let $x \in \{I, D, B\}$ and consider $f_\phi : (X, \tau, \Omega, \preceq_1) \to (Y, \theta, \Gamma, \preceq_2)$ and $g_\lambda : (Y, \theta, \Gamma, \preceq_2) \to (Z, \upsilon, K, \preceq_3)$ are soft maps. Then:*

(i) *If f_ϕ is an Sx-semi continuous map and g_λ is a soft continuous map, then $g_\lambda \circ f_\phi$ is an Sx-continuous map.*

(ii) *If f_ϕ is a soft open (resp. soft closed) map and g_λ is an Sx-semi open (resp. Sx-semi closed) map, then $g_\lambda \circ f_\phi$ is an Sx-semi open (resp. Sx-semi closed) map.*

(iii) *If $g_\lambda \circ f_\phi$ is an Sx-open map and f_ϕ is surjective soft continuous, then g_λ is an Sx-open map.*

3.3 Soft $I(D, B)$-semi homeomorphism

We define and investigate in this subsection, the concepts of soft $I(D, B)$-semi homeomorphism maps. We discussed their main features and verify some findings related to them.

Definition 22 *A bijective soft map $g_\phi : (X, \tau, \Omega, \preceq_1) \to (Y, \theta, \Gamma, \preceq_2)$ is called SI (resp. SD, SB) -semi homeomorphism if it is SI-semi continuous and SI-semi open (resp. SD-semi continuous and SD-semi open, SB-semi continuous and SB-semi open).*

Remark 4 *Note that:*

1. *an SI (SD, SB) -semi homeomorphism map is soft semi homeomorphism.*
2. *an SB-semi homeomorphism map is SI-semi homeomorphism or SD-semi homeomorphism.*

Examples given below manifest that the results of the remark above are not reversible.

Example 5 *Let Ω, X, $\phi : \Omega \to \Omega$, $f : X \to X$ and $\preceq$ be the same as in Example (1). Consider $\tau = \{\widetilde{\emptyset}, \widetilde{X}, F_\Omega, L_\Omega\}$ and $\theta = \{\widetilde{\emptyset}, \widetilde{Y}, L_\Omega\}$ are two soft topologies on X, where $F_\Omega = \{(\omega_1, \{1\}), (\omega_2, \{3,4\})\}$ and $L_\Omega = \{(\omega_1, \{1\}), (\omega_2, \{3\})\}$. Then we find that $f_\phi : (X, \tau, \Omega, \preceq) \to (X, \theta, \Omega, \preceq)$ is a soft semi homeomorphism map, but it is neither an SD-semi homeomorphism map nor an SI-semi homeomorphism map. Hence f_ϕ is not SI (SD, SB)-semi homeomorphism.*

Example 6 *By replacing a partial order relation (given in example above) by $\preceq = \triangle \bigcup \{(2,4)\}$, we find that a soft map f_ϕ is SI-semi homeomorphism, but it is not an SB-semi homeomorphism map. Also, replacing a partial order relation (given in example above) by $\preceq = \triangle \bigcup \{(1,2)\}$ leads to that a soft map f_ϕ is SD-homeomorphism, but it is not an SB-semi homeomorphism map.*

Theorem 12 *Consider $f_\phi : (X, \tau, \Omega, \preceq_1) \to (Y, \theta, \Gamma, \preceq_2)$ is a bijective soft map and let $(\gamma, \lambda) \in \{(Is, dscl), (Ds, iscl), (Bs, bscl)\}$. Then f_ϕ is soft γ-homeomorphism if and only if $(f_\phi(G_\Omega))^\lambda = f_\phi(cl(G_\Omega)) = cl(f_\phi(G_\Omega)) = f_\phi(G_\Omega^\lambda)$ for every $G_\Omega \widetilde{\subseteq} \widetilde{X}$.*

Proof. We only prove the case of $(\gamma, \lambda) = (Is, dscl)$.
Necessity: The property f_ϕ is an SI-semi homeomorphism map implies that $f_\phi(G_\Omega^{dscl}) \widetilde{\subseteq} cl(f_\phi(G_\Omega))$ and $(f_\phi(G_\Omega))^{dscl} \widetilde{\subseteq} f_\phi(cl(G_\Omega))$ for every $G_\Omega \widetilde{\subseteq} \widetilde{X}$. So $f_\phi(cl(G_\Omega)) \widetilde{\subseteq} f_\phi(G_\Omega^{dscl}) \widetilde{\subseteq} cl(f_\phi(G_\Omega)) \widetilde{\subseteq} (f_\phi(G_\Omega))^{dscl}$ and $cl(f_\phi(G_\Omega)) \widetilde{\subseteq} (f_\phi(G_\Omega))^{dscl} \widetilde{\subseteq} f_\phi(cl(G_\Omega)) \widetilde{\subseteq} f_\phi(G_\Omega^{dscl})$. By the preceding two inclusion relations, we obtain the required equality relation.
Sufficiency: The equality relation $(f_\phi(G_\Omega))^{dscl} = f_\phi(cl(G_\Omega)) = cl(f_\phi(G_\Omega)) = f_\phi(G_\Omega^{dscl})$ implies that $f_\phi(G_\Omega^{dscl}) \widetilde{\subseteq} cl(f_\phi(G_\Omega))$ and $(f_\phi(G_\Omega))^{dscl} \widetilde{\subseteq} f_\phi(cl(G_\Omega))$. So f_ϕ is SI-semi continuous and SD-semi closed map. Hence the desired result is proved. □

Theorem 13 *If a bijective soft map $f_\phi : (X, \tau, \Omega, \preceq_1) \to (Y, \theta, \Gamma, \preceq_2)$ is SI-semi continuous (resp. SD-semi continuous, SB-semi continuous), Then the following three statements are equivalent:*

1. *f_ϕ is SI-semi homeomorphism (resp. SD-semi homeomorphism, SB-semi homeomorphism);*
2. *f_ϕ^{-1} is SI-semi continuous (resp. SD-semi continuous, SB-semi continuous);*

3. f_ϕ *is SD-semi closed (resp. SI-semi closed, SB-semi closed).*

Proof. $1 \Rightarrow 2$: Since f_ϕ is an SI-semi homeomorphism (resp. SD-semi homeomorphism, SB-semi homeomorphism) map, f_ϕ is SI-semi open (resp. SD-semi open, SB-semi open). It comes from item 2 of Theorem (9) that f_ϕ^{-1} is SI-semi continuous (resp. SD-semi continuous, SB-semi continuous).
$2 \Rightarrow 3$: It comes from item 3 of Theorem (9).
$3 \Rightarrow 1$: It suffices to prove that f_ϕ is an SI-semi open (resp. SD-semi open, SB-semi open) map. This comes from item 1 of Theorem (9). □

Theorem 14 *Let soft topologies* $\tau^\star$ *and* $\theta^\star$ *be extended on* X *and* Y*, respectively. Then a soft map* $g_\phi : (X, \tau^\star, \Omega, \preceq_1) \to (Y, \theta^\star, \Gamma, \preceq_2)$ *is SI (resp. SD, SB) -semi homeomorphism iff amap* $g : (X, \tau^\star_\gamma, \preceq_1) \to (Y, \theta^\star_{\phi(\omega)}, \preceq_2)$ *is I (resp. D, B) -semi homeomorphism.*

Proof. Directly from Theorem (5) and Theorem (10). □

Proposition 6 *Let the two soft topologies* τ *and* θ *on* X *and* Y*, respectively, do not belong to {soft discrete topology, soft indiscrete topology}. If a soft map* $f_\phi : (X, \tau, \Omega, \preceq_1) \to (Y, \theta, \Gamma, \preceq_2)$ *is SB-semi homeomorphism, then* $\preceq_1$ *and* $\preceq_2$ *is not linearly ordered.*

4 Conclusion

Study topological concepts in the ordered domain is an important issue because it helps to obtain some properties induced from the interaction between topology and algebra. Also, it helps to describe and solve some practical problems; see [8]

To this end, we [16] have formulated the concept of soft topological ordered spaces as an extension of the concept of soft topological spaces. Then we [17] have utilized monotone soft sets to define some soft ordered maps and investigated their main properties. In this work, we have used soft semi open sets to give the concepts of soft x-semi continuous, soft x-semi open, soft x-semi closed and soft x-semi homeomorphism maps for $x \in \{I, D, B\}$. We have given various characterizations for these concepts and have shown the relationships among them with the help of examples. It should be noted that results obtained herein and results obtained in [14] are independent of each other. Also, they are special case of results obtained in [32, 19] and are genuine generalizations of results obtained in [18].

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