Acta Univ. Sapientiae, Mathematica, **2**, 2 (2010) 206–214

On the positive correlations in Wiener space via fractional calculus

Toufik Guendouzi
Laboratory of Mathematics,
Djillali Liabes University,
PO. Box 89, 22000 Sidi Bel Abbès, Algeria
email: tf.guendouzi@gmail.com

Abstract. In this paper we study the correlation inequality in the Wiener space using the Malliavin and the fractional calculus. Under positivity and monotonicity conditions, we give a proof of the positive correlation between two random functionals F and G which are assumed smooth enough. The main argument is the Itô-Clark representation formula for the functionals of a fractional Brownian motion.

1 Introduction

It is well-known that the correlation inequalities are one of the most powerful tools of the stochastic analysis due to its vast range of applications. So, The theoretical study of these inequalities has matured tremendously since the seminal work of Fortuin, Kasteleyn and Ginibre [5]. In general, several authors have been interested in finding applications of these inequalities in some areas including statistical mechanics (see, for instance, Bakry and Michel [1], Preston [15]).

Recently, Mayer-Wolf, Üstünel and Zakai obtained general covariance inequalities on an abstract Wiener space. They consider such inequalities for functionals satisfying either monotonicity or convexity properties [13]. Hence Houdré and Pérez-Abreu in [9] used Malliavin calculus techniques to obtain

2010 Mathematics Subject Classification: 60E15, 60F15, 60G05, 60G10, 60H07, 60J60
Key words and phrases: Wiener space, positive correlations, Malliavin calculus, Clark formula, FKG inequality, fractional Brownian motion

covariance identities and inequalities for functionals of the Wiener and the Poisson processes.

The purpose of this paper is to use the Malliavin calculus techniques to study the positive correlations between two functionals on the Wiener space via fractional calculus. Our proofs rely in general on the Itô-Clark representation formula for the functionals of a fractional Brownian motion and the monotonicity condition for F and G on the Wiener space. Here, the fractional Brownian motion of index $H \in (0,1)$ is the centred Gaussian process whose covariance kernel is given by

$$R_H(s,t) = \mathbb{E}_H[W_s^H W_t^H],$$

and for f given in $[a,b]$, each of the expressions

$$(D_{a^+}^{\alpha} f)(x) = \left(\frac{d}{dx}\right)^{[\alpha]+1} I_{a^+}^{1-\{\alpha\}} f(x), \quad (D_{b^-}^{\alpha} f)(x) = \left(-\frac{d}{dx}\right)^{[\alpha]+1} I_{b^-}^{1-\{\alpha\}} f(x),$$

are respectively called right and left fractional derivative, where $[\alpha]$ denotes the integer part of α, $\{\alpha\} = \alpha - [\alpha]$ and $(I_{a^+}^{\alpha} f(x))(x)$, $(I_{b^-}^{\alpha} f(x))(x)$ are right and left fractional integral of the order $\alpha > 0$ (see [3]). Hence for $H \in (0,1)$ the integral transform $K_H f$ is defined as

$$\begin{aligned} K_H f &= I_{0^+}^{2H} x^{1/2-H} I_{0^+}^{1/2-H} x^{H-1/2} f, & H \leq 1/2 \\ K_H f &= I_{0^+}^{1} x^{H-1/2} I_{0^+}^{H-1/2} x^{1/2-H} f, & H \geq 1/2, \end{aligned}$$

K_H is an isomorphism from $L^2([0,1])$ onto $I_{0^+}^{H+1/2}(L^2([0,1]))$. If $H \geq 1/2$, $r \to K_H(t,r)$ is continuous on $(0,1]$.

The organization of this paper is as follows: in Section 2, we shall give some preparation and state main results. We begin by recalling the basic notions of Malliavin calculus, the gradient operator and Sobolev-type space $\mathcal{D}_{2,1}$, the Ornstein-Uhlenbeck semigroup, the Itô-Clark representation formula for functional of Brownian motion. In Section 3, we shall study the positive correlation between two functionals of the Wiener space satisfying monotonicity property.

2 Preliminaries

This section gives some basic notions of analysis on the Wiener space $(W, \mathcal{F}^H, \mathbb{P}_H)$. The reader can consult [14] for a complete survey on this topic. Let W represented as $C_0([0,1], \mathbb{R})$ of continuous function $\omega : [0,1] \longrightarrow \mathbb{R}$ with

$w(0) = 0$, equipped with the $\|.\|_\infty$-norm i.e W is also a (separable) Banach-space, W^* is its topological dual and $(W_t)_{t\in[0,1]}$ be a canonical Brownian motion generating the filtration $(\mathcal{F}_t^H)_{t\in[0,1]}$. Random-variables on W are called Wiener functionals and the coordinate process $\omega(t)$ is a Brownian motion under P_H. So we write $\omega(t) = W(t,\omega) = W(t)$. Recall that P_H is the unique probability measure on W such that the canonical process $(W(t))_{t\in\mathbb{R}}$ is a centered Gaussian process with the covariance Kernel R_H:

$$E_H[W(t)W(s)] = R_H(t,s).$$

The Cameron-Martin space H_H is an subspace of W defined as

$$H_H = \{K_H\dot{h};\ \dot{h} \in L^2([0,1],dt)\},$$

i.e, any $h \in H_H$ can be represented as $h(t) = K_H\dot{h}(t) = \int_0^1 K_H(s,t)\dot{h}(s)ds$, $\dot{h}$ belongs to $L^2([0,1])$. The scalar product on the space H_H is given by $(h,g)_{H_H} = (K_H\dot{h}, K_H\dot{g})_{H_H} = (\dot{h},\dot{g})_{L^2([0,1])}$.

We note that for any $H \in (0,1)$, $R_H(t,s)$ can be written as

$$R_H(t,s) = \int_0^1 K_H(t,r)K_H(s,r)dr,$$

and $R_H = K_H K_H^*$, where K_H is the Hilbert-Schmidt operator introduced in the first section. R_H is also the injection from W^* into the space H_H and it can be decomposed as $R_H\eta = K_H(K_H^*\eta)$, for any η in W^* (see, [18]). The restriction of K_H^* to W^* is the injection from W^* into $L^2([0,1])$.

If y is an H_H-valued random variable, we denote by $\dot{y}$ the $L^2([0,1],\mathbb{R})$-valued random variable such that $y(\omega,t) = \int_0^t K_H(t,s)\dot{y}(\omega,s)ds$. Here, for $F \in \mathcal{S}(\chi)$ the H-Gross Sobolev derivative of F, denoted by ∇F and is the $H_H \otimes \chi$-valued mapping defined by

$$\nabla F(\omega) = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(\langle l_1,\omega\rangle,\ldots,\langle l_n,\omega\rangle)R_H(l_i)\otimes x, \qquad (1)$$

where χ is a separable Hilbert space, $\mathcal{S}(\chi)$ is the set of χ-valued smooth cylindric functionals, and for each $1 \le i \le n$, l_i is in W^* and x_i belongs to χ. Hence, for any $R_H\eta \in H_H$ we have by the Cameron-Martin theorem

$$E_H[F(\omega + R_H\eta)] = \int F(\omega)\exp\Big(\langle\eta,\omega\rangle - \|R_H\eta\|^2_{H_H}/2\Big)dP_H(\omega). \qquad (2)$$

The Ornstein-Uhlenbeck semigroup $\{T_t^H, t \geq 0\}$ of bounded operators which acts on $L^p(\mathbb{P}_H, \chi)$ for any $p \geq 1$ can be described by the Mehler formula:

$$(T_t^H F)(\omega) = \int_{\mathcal{W}} F\Big(e^{-t}\omega + \sqrt{1-e^{-2t}}\omega'\Big)\mathbb{P}_H(d\omega'). \tag{3}$$

The directional derivative of $F \in \mathcal{S}(\chi)$ in the the direction $R_H\eta \in \mathcal{H}_H$ is given by

$$(\nabla F, R_H\eta)_{\mathcal{H}_H} = \frac{d}{dt}F(\omega + t.R_H\eta)\Big|_{t=0} \tag{4}$$

and from (2) we have ∇F depends only on the equivalence classes with respect to $\mathbb{P}_H$ and $\mathbb{E}_H((\nabla F, R_H\eta)_{\mathcal{H}_H}) = \mathbb{E}_H(F\langle\omega,\eta\rangle)$.

For any $p \geq 1$ we define Sobolev space $\mathcal{D}_{p,k}^H(\chi)$, $k \in \mathbb{Z}$, as the completion of $\mathcal{S}(\chi)$ with respect to the norm

$$\|F\|_{p,k,H} = \|F\|_{L_H^p} + \|\nabla^k F\|_{L^p(\mathbb{P}_H, \chi)} [illegible]$$

hence the operator ∇ can be extended as continuous linear operator from $\mathcal{D}_{p,k}^H(\chi)$ to $\mathcal{D}_{p,k-1}^H(\mathcal{H}_H \otimes \chi)$ for any $p > 1$ and $k \in \mathbb{Z}$ (see [18]). Thus $\nabla : \mathcal{D}_{p,k}^H(\chi) \to \mathcal{D}_{p,k-1}^H(\mathcal{H}_H \otimes \chi)$ its formal adjoint with respect to $\mathbb{P}_H$ is the operator δ_H in the sense that $\forall F \in \mathcal{S}$, $\forall y \in$ [illegible], $\mathbb{E}_H[F\delta_H y] = \mathbb{E}_H\Big[(\nabla F, y)_{\mathcal{H}_H}\Big]$, and since ∇ has continuous extensions, δ_H has also a continuous linear extension from $\mathcal{D}_{p,k}^H(\mathcal{H}_H)$ to $\mathcal{D}_{p,k-1}^H$ for any $p > 1$ and $k \in \mathbb{N}$.

Recall the following, unique, Wiener-Itô chaos expansion for all $\mathbb{P}_H$-square integrable functional F from $\mathcal{W}$ to $\mathbb{R}$

$$F = \mathbb{E}F + \sum_1^\infty J_n^H F, \tag{5}$$

where $J_n^H F$ is the n-fold iterated Itô integral of F. If $y \in \mathcal{H}_H$ and $\vartheta_1^y = \exp(\delta_H y - 1/2\|y\|^2_{\mathcal{H}_H})$, then we have

$$J_n^H \vartheta_1^y = \frac{1}{n!}\delta_H^{(n)} y^{\otimes n}. \tag{6}$$

More precisely, if $F \in \cup_{k\in\mathbb{Z}}\mathcal{D}_{2,k}^H$,

$$J_n^H F = \frac{1}{n!}\delta_H^{(n)}\Big(\mathbb{E}_H \nabla^{(n)} F\Big).$$

For $\mathsf{H} \in (0,1)$, let $\{\pi_t^{\mathsf{H}}; t \in [0,1]\}$ be the family of orthogonal projection in $\mathcal{H}_{\mathsf{H}}$ defined by

$$\pi_t^{\mathsf{H}}(\mathsf{K}_{\mathsf{H}} y) = \mathsf{K}_{\mathsf{H}}(y \mathbf{1}_{[0,1]}), \quad y \in \mathsf{L}^2([0,1]). \tag{7}$$

The operator $\Upsilon(\pi_t^{\mathsf{H}})$ is the second quantization of π_t^{H} from $\mathsf{L}^2(\mathbb{P}_{\mathsf{H}})$ into itself defined by

$$\mathsf{F} = \sum_{n \geq 0} \delta_{\mathsf{H}}^{(n)} f_n \mapsto \Upsilon \pi_t^{\mathsf{H}}(\mathsf{F}) = \sum_{n \geq 0} \delta_{\mathsf{H}}^{(n)} \Big((\pi_t^{\mathsf{H}})^{\otimes n} f_n \Big).$$

Thus we have, for $y \in \mathcal{H}_{\mathsf{H}}$,

$$\Upsilon(\pi_t^{\mathsf{H}}) \Big(\vartheta_1^y \Big) = \exp(\delta_{\mathsf{H}}(\pi_t^{\mathsf{H}} y) - 1/2 \|\pi_t^{\mathsf{H}} y\|^2_{\mathcal{H}_{\mathsf{H}}}) = \vartheta_1^y, \tag{8}$$

hence the bijectivity of the operator K_{H} has the following consequence

$$\mathcal{F}_t^{\mathsf{H}} = \sigma\{\delta_{\mathsf{H}}(\pi_t^{\mathsf{H}} y), y \in \mathcal{H}_{\mathsf{H}}\} \vee \mathcal{N}_{\mathsf{H}}$$

where $\mathcal{N}_{\mathsf{H}}$ is the set of the $\mathbb{P}_{\mathsf{H}}$-negligible events.

We also note that for any $\mathsf{F} \in \mathsf{L}^2(\mathbb{P}_{\mathsf{H}})$,

$$\Upsilon(\pi_t^{\mathsf{H}})\mathsf{F} = \mathbb{E}_{\mathsf{H}}[\mathsf{F}|\mathcal{F}_t^{\mathsf{H}}],$$

and in particular

$$\mathbb{E}_{\mathsf{H}}[W_t|\mathcal{F}_t^{\mathsf{H}}] = \int_0^t \mathsf{K}_{\mathsf{H}}(t,s) \mathbf{1}_{[0,1]}(s) \delta_{\mathsf{H}} W_s,$$

$$\mathbb{E}_{\mathsf{H}}[\exp(\delta_{\mathsf{H}} y - 1/2 \|y\|^2_{\mathcal{H}_{\mathsf{H}}}) | \mathcal{F}_t^{\mathsf{H}}] = \exp(\delta_{\mathsf{H}}(\pi_t^{\mathsf{H}} y) - 1/2 \|\pi_t^{\mathsf{H}} y\|^2_{\mathcal{H}_{\mathsf{H}}}),$$

for any $y \in \mathcal{H}_{\mathsf{H}}$.

We shall recall the following results

Theorem 1 ([3]) *Let* F *be* $\mathcal{D}_{2,1}^{\mathsf{H}}$. *Then* F *belongs to* $\mathcal{F}_t^{\mathsf{H}}$ *iff* $\nabla \mathsf{F} = \pi_t^{\mathsf{H}} \nabla \mathsf{F}$.

Theorem 2 (Itô-Clark representation formula) *For any* $\mathsf{F} \in \mathcal{D}_{2,1}^{\mathsf{H}}$,

$$\begin{aligned} \mathsf{F} - \mathbb{E}_{\mathsf{H}}[\mathsf{F}] &= \int_0^1 \mathbb{E}_{\mathsf{H}}[\mathsf{K}_{\mathsf{H}}^{-1}(\nabla \mathsf{F})(s)|\mathcal{F}_s^{\mathsf{H}}] \delta_{\mathsf{H}} W_s \\ &= \delta_{\mathsf{H}} \Big(\mathsf{K}_{\mathsf{H}}(\mathbb{E}_{\mathsf{H}}[\mathsf{K}_{\mathsf{H}}^{-1}(\nabla \mathsf{F})(.)|\mathcal{F}_.]) \Big). \end{aligned}$$

3 Monotonicity and positive correlations

Our method relies on the Itô-Clark formula which plays a crucial role to establish positive correlation between two random functionals under some hypotheses. Thus we recall here the following correlation identity in the first lemma, which is based on the Clark formula and the Itô isometry. We refer to [3] and [9] for tutorial references on this identity.

Lemma 1 *For any* $F, G \in L^2(\mathbb{P}_H)$ *we have*

$$\mathrm{Cov}(F,G) = \mathbb{E}_H\Big[\int_0^1 \mathbb{E}_H[K_H^{-1}(\nabla F)(s)|\mathcal{F}_s^H]\mathbb{E}_H[K_H^{-1}(\nabla G)(s)|\mathcal{F}_s^H]ds\Big]. \qquad (9)$$

Proof. We have

$$\begin{aligned}\mathrm{Cov}(F,G) &= \mathbb{E}_H\Big[(F-\mathbb{E}_H[F])(G-\mathbb{E}_H[G])\Big]\\ &= \mathbb{E}_H\Big[\int_0^1 \mathbb{E}_H[K_H^{-1}(\nabla F)(s)|\mathcal{F}_s^H]\delta_H W_s \int_0^1 \mathbb{E}_H[K_H^{-1}(\nabla G)(s)|\mathcal{F}_s^H]\delta_H W_s\Big]\\ &= \mathbb{E}_H\Big[\int_0^1 \mathbb{E}_H[K_H^{-1}(\nabla F)(s)|\mathcal{F}_s^H]\mathbb{E}_H[K_H^{-1}(\nabla G)(s)|\mathcal{F}_s^H]ds\Big].\end{aligned}$$

□

Proposition 1 *Let* G *be a* $\mathcal{F}_t^H$*-measurable element of* $\mathcal{D}_{2,1}^H$*. Then the identity* (9) *can be written as*

$$\begin{aligned}\mathrm{Cov}(F,G) &= \mathbb{E}_H\Big[\int_0^1 \mathbb{E}_H[K_H^{-1}(\nabla F)(s)|\mathcal{F}_s^H]K_H^{-1}(\nabla G)(s)ds\Big]\\ &= \mathbb{E}_H\Big[\int_0^1 \mathbb{E}_H[K_H^{-1}(\nabla F)(s)|\mathcal{F}_s^H]K_H^{-1}(\pi_t^H\nabla G)(s)ds\Big].\end{aligned}$$

The next result is an immediate consequence of (9).

Lemma 2 *Let* $F, G \in L^2(\mathbb{P}_H)$ *such that*

$$\mathbb{E}_H[K_H^{-1}(\nabla F)(s)|\mathcal{F}_s^H]\mathbb{E}_H[K_H^{-1}(\nabla G)(s)|\mathcal{F}_s^H] \geq 0, \quad ds\times d\mathbb{P}-a.s.$$

Then F *and* G *are positively correlated and we have* $\mathrm{Cov}(F,G)\geq 0$.

The main results of this section are the following:

Corollary 1 *If* $F, G \in \mathcal{D}_{2,1}^H$ *satisfy* $K_H^{-1}[\nabla F](t) \geq 0, K_H^{-1}[\nabla G](t) \geq 0 a.s.$*, then* F *and* G *are positively correlated.*

Corollary 2 *If* $G \in \mathcal{D}^H_{2,1}$, *and if* $\mathbb{E}_H[K_H^{-1}(\nabla F)(s)|\mathcal{F}^H_s] \geq 0$, $K_H^{-1}[\nabla G](t) \geq 0 a.s.$, *then* F *and* G *are positively correlated.*

The next theorem studies the positivity of $K_H^{-1}[\nabla F](t)$, for any functional $F \in \mathcal{D}^H_{2,1}$ under the monotonicity assumption.

Theorem 3 *For any increasing functional* $F \in \mathcal{D}^H_{2,1}$ *we have*

$$K_H^{-1}[\nabla F](t) \geq 0, \quad dt \times d\mathbb{P}_H - a.s.$$

Proof. Let F be increasing functional i.e. $F(.+y) \geq F(.)$ a.s., for all $y \in \mathcal{H}_H$ and $\{u^t_n, n \geq 0\}$ be an orthonormal basis of $L^2([0,1])$, for $H \in (0,1)$, $\mathcal{V}^t_n$ be the σ field generated by $\{\delta_H K_H u^t_i, i \leq n\}$. Since $\vee_n \mathcal{V}^t_n = \mathcal{F}^H_t$, the sequence $F_n = \mathbb{E}_H\Big[F/\mathcal{V}^t_n\Big]$ converge to F in $\mathcal{D}^H_{2,1}$, and for $\pi^H_t K_H u^t_n = K_H u^t_n$, for F_n we have $\nabla F_n = \pi^H_t \nabla F_n$ and $\nabla F = \pi^H_t \nabla F$ a.s. Hence, by the Cameron-Martin formula (2) we have for any $\mathcal{V}^t_n$-measurable and square-integrable random variable ϑ^t_n,

$$\begin{aligned}
\mathbb{E}_H\left[F_n(\omega + y^t_n)\right] &= \mathbb{E}_H\Big[\exp(\delta_H y^t_n - 1/2\|y^t_n\|^2_{\mathcal{H}_H})F_n(\omega)\Big] \\
&= \mathbb{E}_H\left[\vartheta^t_n F_n(\omega)\right] \\
&= \mathbb{E}_H\left[\vartheta^t_n \mathbb{E}_H[F/\mathcal{V}^t_n](\omega)\right] \\
&= \mathbb{E}_H\left[\mathbb{E}_H[\vartheta^t_n F/\mathcal{V}^t_n](\omega)\right] \\
&= \mathbb{E}_H[\vartheta^t_n F(\omega)] \\
&= \mathbb{E}_H[F(\omega + y^t_n)].
\end{aligned}$$

On the other hand, for any square-integrable function f on $[0,1]^n$ we have

$$\begin{aligned}
F_n(\omega + y) &= F_n(\omega + y^t) \\
&= f\Big(\delta_H K_H u^t_0 + (K_H u^t_0, K_H^{-1} y_t)_{L^2([0,1])}, \ldots, \\
&\qquad \ldots, \delta_H K_H u^t_n + (K_H u^t_n, K_H^{-1} y_t)_{L^2([0,1])}\Big) \\
&= f\Big(\delta_H K_H u^t_0 + (K_H u^t_0, \pi^H_t K_H^{-1} y^t_n)_{L^2([0,1])}, \ldots, \\
&\qquad \ldots, \delta_H K_H u^t_n + (K_H u^t_n, \pi^H_t K_H^{-1} y^t_n)_{L^2([0,1])}\Big) \\
&= F_n(\omega + y^t_n) \\
&= \mathbb{E}_H[F(\omega + y^t_n)/\mathcal{V}^t_n] \\
&\geq \mathbb{E}_H[F/\mathcal{V}^t_n](\omega) \\
&= F_n(\omega) - a.s.
\end{aligned}$$

Thus, we conclude that the smooth function $F_n(\omega + \tau y)$ is increasing in τ, for any $\tau \in \mathbb{R}$ where $\pi_t^H K_H^{-1} y_n^t = K_H^{-1} y_n^t$ is positive, hence we have from (4) that $(\nabla F_n, K_H^{-1} y)_{L^2([0,1])}$ is positive. Since ∇F_n is positive and $\nabla F_n \to \nabla F$ then ∇F is also positive.

To complete the proof, it suffices to use the fact that

$$\delta_H(\pi_t^H \nabla F) = \int_0^t K_H^{-1}[\nabla F](s) \delta_H W_s$$

and because ∇F positive we get $K_H^{-1}[\nabla F](s) \geq 0$, ds-a.s. □

Theorem 4 *For any increasing functional* $F \in L^2(\mathbb{P}_H)$ *we have*

$$\mathbb{E}_H[K_H^{-1}(\nabla F)(s) | \mathcal{F}_s^H] \geq 0, \quad dt \times d\mathbb{P}_H - a.s.$$

Proof. Let $\{T_t^H, t \geq 0\}$ be a semigroup defined as in (3), and assume that F is increasing functional in $L^2(\mathbb{P}_H)$. Taking $t = 1/n$, $\forall n \geq 1$, we have $T_{1/n}^H F$ is also increasing from (3) and element of $\mathcal{D}_{2,1}^H$. Hence from lemma 3, $\nabla T_{1/n}^H F$ is positive and also $K_H^{-1}[\nabla T_{1/n}^H F](s) \geq 0$, a.s. then $\mathbb{E}_H[K_H^{-1}(\nabla T_{1/n}^H F)(t) | \mathcal{F}_t^H]$ follows. Finally, using the fact that $T_{1/n}^H F \to F$ as n goes to infinity we get the result. □

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Received: February 25, 2010