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ps-ro fuzzy strongly α-irresolute function

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Abstract. The prime objective of this paper is to introduce and characterize a new type of function in a fuzzy topological spaces called ps-ro fuzzy strongly α-irresolute function. The interrelations of this function with the parallel existing allied concepts are established. The independence of ps-ro fuzzy strongly α-irresolute and well known concept of fuzzy strongly α-irresolute function motivate authors to explore it. Also, this function is found to be stronger than ps-ro fuzzy continuity, ps-ro fuzzy semicontinuity, ps-ro fuzzy precontinuity and ps-ro fuzzy α-continuity. Further, several characterizations of these functions along with different conditions for their existence are obtained.

1 Introduction

Ever since the introduction of the concept of fuzzy logic and fuzzy sets by L. A. Zadeh [13] and fuzzy topological space by C. L. Chang [3], several concepts of general topology has been generalized successfully in fuzzy settings by different mathematicians in different directions. Fuzzy α-open sets and fuzzy α-continuity were introduced and studied in [2]. After the initiation of the idea of ps-ro fuzzy topology [8], several forms of fuzzy continuous type of functions viz ps-ro fuzzy continuous, ps-ro fuzzy semi continuous, ps-ro fuzzy α-continuous and ps-ro fuzzy precontinuous functions were introduced

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and explored in [9, 10], [4], [5] and [7] respectively. Here, the idea of ps-ro fuzzy strongly α-irresolute function is initiated. The interrelations of this function with the existing similar types of functions are explored. It is seen that this function neither implies nor implied by the existing concept of fuzzy strongly α-irresolute function. Also, ps-ro fuzzy continuity, ps-ro fuzzy semicontinuity, ps-ro fuzzy precontinuity and ps-ro fuzzy α-continuity are all found to be weaker than ps-ro fuzzy strongly α-irresoluteness.

2 Preliminaries

A fuzzy set A on a nonempty set X is a function from X to $I = [0, 1]$. For two fuzzy sets A and B, A is subset of B written as $A \leq B$ if $A(t) \leq B(t) \forall t \in X$. A fuzzy point x_t is a fuzzy set with value $t (0 < t \leq 1)$ at x, elsewhere the value is 0. x_t is said to be quasi-coincident (q-coincident, in short) with a fuzzy set A if $A(x) + t > 1$. A and B are said to be q-coincident, written as AqB if for some $x \in X$, $A(x) + B(x) > 1$ [11]. Throughout this paper, a fuzzy topological space(fts, for short) in the sense of Chang [3] is denoted by (X, τ) or simply by X. A fuzzy set A on X is called fuzzy α-open if $A \leq int(cl(intA))$, where $intA$ and clA are fuzzy interior and closure of a fuzzy set A on X [2].

Corresponding to a fts (X, τ) one can establish a family of general topological spaces $(X, i_\alpha(\tau))$, where $i_\alpha(\tau) = \{A^\alpha : A \in \tau\}$ and $A^\alpha = \{x \in X : A(x) > \alpha\} \forall \alpha \in I_1 = [0, 1)$. Fuzzy regular openness of A in (X, τ) does not imply regular openness of A^α in $(X, i_\alpha(\tau))$ and also regular openness of A^α in $(X, i_\alpha(\tau))$ does not guarantee fuzzy regular openness of A in (X, τ). This gave birth of ps-ro fuzzy topology, which is a fuzzy topology on X and is generated by pseudo regular open fuzzy sets on (X, τ) which are defined as those members of τ whose corresponding crisp set on $(X, i_\alpha(\tau)) \forall \alpha \in I_1$ are regular open. Members of ps-ro fuzzy topology are called ps-ro open and their complements as ps-ro closed fuzzy sets on X [8, 9].

In a fts (X, τ), fuzzy ps-closure and ps-interior of A, denoted by ps-$cl(A)$ and ps-$int(A)$ are given by ps-$cl(A) = \wedge\{B : A \leq B,\ B$ is ps-ro closed fuzzy set on $X\}$ and ps-$int(A) = \vee\{B : B \leq A,\ B$ is ps-ro open fuzzy set on $X\}$ [9, 10]. A fuzzy set A on a fts (X, τ) is said to be ps-ro semiopen [4] (ps-ro α-open [5], ps-ro preopen [7]) fuzzy set if $A \leq$ ps-cl(ps-$int(A)$)(resp. $A \leq$ ps-int(ps-cl(ps-$int(A)$)), $A \leq$ ps-int(ps-$cl(A)$)).
A function f between two fts (X, τ_1) and (Y, τ_2) is

(i) fuzzy strongly α-irresolute function if $f^{-1}(A) \in \tau_1$, for each fuzzy α-open A on Y [12].

(ii) ps-ro fuzzy continuous [9], [10] (ps-ro semicontinuous [4], ps-ro α-continuous [5], ps-ro precontinuous [7]) if $f^{-1}(A)$ is ps-ro open (resp. ps-ro semiopen, ps-ro α-open, ps-ro preopen)fuzzy set on X for each ps-ro open fuzzy set A on Y.

(iii) ps-ro fuzzy α-irresolute [6] if $f^{-1}(A)$ is ps-ro α-open on X, for each ps-ro α-open A on Y.

3 ps-ro fuzzy strongly α-irresolute function

Definition 1 *A function* f *between two* fts (X, τ_1) *and* (Y, τ_2) *is said to be* ps-ro *fuzzy strongly* α*-irresolute function if for each* ps-ro α*-open fuzzy set* U *on Y,* $f^{-1}(U)$ *is* ps-ro *open fuzzy set on* X.

Remark 1 *It follows directly from the definition that* ps-ro *fuzzy strongly* α*-irresoluteness implies* ps-ro *fuzzy* α*-irresoluteness,* ps-ro *fuzzy* α*-continuity,* ps-ro *fuzzy semicontinuity and* ps-ro *fuzzy precontinuity. But the converses are not true in general is given by the example below:*

Example 1 *Let* $X = \{a, b, c, d\}$ *and* $Y = \{w, x, y, z\}$. *Let* A, B *and* C *be fuzzy sets on* X *defined by* $A(a) = 0.2, A(b) = 0.2, A(c) = 0.2$ *and* $A(d) = 0.3$*;* $B(t) = 0.2,\ \forall\ t \in X$ *and* $C(t) = 0.5,\ \forall\ t \in X$. *Let* D, E, F *and* G *be fuzzy sets on* Y *defined by* $D(t) = 0.4\ \forall\ t \in Y$*;* $E(w) = 0.5, E(x) = 0.5, E(y) = 0.5,$ *and* $E(z) = 0.6$*;* $F(t) = 0.3,\ \forall\ t \in Y$*;* $G(w) = 0.3, G(x) = 0.3, G(y) = 0.3, G(z) = 0.4$. *Clearly,* $\tau_1 = \{0, 1, A, B, C\}$ *and* $\tau_2 = \{0, 1, D, E, F, G\}$ *are fuzzy topologies on* X *and* Y *respectively. In the corresponding general topological space* $(X, i_\alpha(\tau_1))$, $\forall \alpha \in I_1 = [0, 1)$*, the open sets are* ϕ, X, A^α, B^α *and* C^α*, where* $A^\alpha = \begin{cases} X, & \text{for } \alpha < 0.2 \\ \{d\}, & \text{for } 0.2 \le \alpha < 0.3 \\ \phi, & \text{for } \alpha \ge 0.3 \end{cases}$, $B^\alpha = \begin{cases} X, & \text{for } \alpha < 0.2 \\ \phi, & \text{for } \alpha \ge 0.2 \end{cases}$ *and*

$$C^\alpha = \begin{cases} X, & \text{for } \alpha < 0.5 \\ \phi, & \text{for } \alpha \ge 0.5 \end{cases}$$

For $0.2 \le \alpha < 0.3$*, the closed sets on* $(X, i_\alpha(\tau_1))$ *are* ϕ, X *and* $X - \{d\}$. *Therefore,* $int(cl(A^\alpha)) = X$. *So,* A^α *is not regular open on* $(X, i_\alpha(\tau_1))$ *for* $0.2 \le \alpha < 0.3$. *Thus,* A *is not pseudo regular open fuzzy set on* (X, τ_1). *So, the* ps-ro *fuzzy topology on* X *is* $\{0, 1, B, C\}$. *Similarly,* E *and* G *are not pseudo regular open fuzzy set on* Y *for* $0.5 \le \alpha < 0.6$ *and* $0.3 \le \alpha < 0.4$ *respectively and hence the* ps-ro *fuzzy topology on* Y *is* $\{0, 1, D, F\}$. *Define a function* f *from* (X, τ_1) *to* (Y, τ_2) *by* $f(a) = w$, $f(b) = x$, $f(c) = x$ *and* $f(d) = z$. G *is* ps-ro α*-open* *fuzzy set on* Y *but* $f^{-1}(G)$ *is not* ps-ro *open fuzzy set on* X. *Hence,* f *is not* ps-ro *fuzzy strongly* α*-irresolute.* $0, 1, D, F$ *and* U *satisfying* $F \leq U \leq D$ *are* ps-ro α*-open fuzzy sets on* Y. $f^{-1}(V)$ *is* ps-ro α*-open fuzzy set on* X *for all* ps-ro α*-open fuzzy set* V *on* Y *proving* f *to be* ps-ro *fuzzy* α*-irresolute function. Similarly, it can be verified that* f *is* ps-ro *fuzzy* α*-irresolute,* ps-ro *fuzzy* α*-continuous,* ps-ro *fuzzy semicontinuous and* ps-ro *fuzzy precontinuous.*

Remark 2 *Clearly,* ps-ro *fuzzy strongly* α*-irresoluteness implies* ps-ro *fuzzy continuity but the converse is not true is shown below:*

Example 2 *Let* $X = \{a, b, c, d\}$ *and* $Y = \{w, x, y, z\}$. *Let* A, B, C *and* D *be fuzzy sets on* X *given by* $A(a) = 0.2, A(b) = 0.2, A(c) = 0.3$ *and* $A(d) = 0.3$; $B(t) = 0.3, \forall\ t \in X$; $C(t) = 0.2\ \forall\ t \in X$ *and* $D(t) = 0.7\ \forall\ t \in X$. *Let* E, F *and* G *be fuzzy sets on* Y *defined by* $E(t) = 0.3\ \forall\ t \in Y$; $F(w) = 0.2, F(x) = 0.2, F(y) = 0.3$ *and* $F(z) = 0.3$ *and* $G(t) = 0.2\ \forall\ t \in Y$. $\tau_1 = \{0, 1, A, B, C, D\}$ *and* $\tau_2 = \{0, 1, E, F, G\}$ *are fuzzy topologies on* X *and* Y *respectively.* A *is not pseudo regular open fuzzy set on* X *for* $0.2 \leq \alpha < 0.3$. *So, the* ps-ro *fuzzy topology on* X *is* $\{0, 1, B, C, D\}$. *Again* F *is not pseudo regular open fuzzy set for* $0.2 \leq \alpha < 0.3$ *on* Y. *So, the* ps-ro *fuzzy topology on* Y *is* $\{0, 1, E, G\}$. *Let* f *be a function from* (X, τ_1) *to* (Y, τ_2) *given by* $f(a) = w$, $f(b) = w$, $f(c) = y$ *and* $f(d) = z$. $f^{-1}(U)$ *is* ps-ro *open fuzzy set on* X *for every* ps-ro *open fuzzy set* U *on* Y, *proving that* f *is* ps-ro *fuzzy continuous function.* F *is* ps-ro α*-open fuzzy set on* Y *but* $f^{-1}(F)$ *is not* ps-ro *open fuzzy set on* X. *Hence,* f *is not* ps-ro *fuzzy strongly* α*-irresolute.*

Now, we find the relation of ps-ro fuzzy strongly α-irresoluteness with well known existing concept of fuzzy strongly α-irresoluteness.

Remark 3 *In Example 2, fuzzy* α*-open sets on* Y *are* 0, 1, E, F, G *and* T *where* $G \leq T \leq E$. $f^{-1}(U) \in \tau_1$ *forall fuzzy* α*-open set* U *on* Y. *Therefore* f *is fuzzy strongly* α*-irresolute but* f *is not* ps-ro *fuzzy strongly* α*-irresolute.*

Example 3 *Let* $X = \{a, b, c, d\}$ *and* $Y = \{w, x, y, z\}$. *Let* A, B *and* C *be fuzzy sets on* X *given by* $A(t) = 0.3, \forall\ t \in X$; $B(t) = 0.4, \forall\ t \in X$ *and* $C(a) = 0.5, C(b) = 0.5, C(c) = 0.5, C(d) = 0.6$. *Let* E, F, G *and* H *be fuzzy sets on* Y *given by* $E(w) = 0.3, E(x) = 0.4, E(y) = 0.4, E(z) = 0.4$; $F(t) = 0.3\ \forall\ t \in Y$; $G(t) = 0.4\ \forall\ t \in Y$ *and* $H(w) = 0.1, H(x) = 0.1, H(y) = 0.1$ *and* $H(z) = 0.3$. $\tau_1 = \{0, 1, A, B, C\}$ *and* $\tau_2 = \{0, 1, E, F, G, H\}$ *are fuzzy topologies on* X *and* Y *respectively.* ps-ro *fuzzy topology on* X *and* Y *are* $\{0, 1, A, B\}$ *and* $\{0, 1, F, G\}$ *respectively. Let us define a function* f *from* (X, τ_1) *to* (Y, τ_2) *by* $f(a) = x$,

$f(b) = x$, $f(c) = y$ and $f(d) = z$. ps-ro α-open fuzzy set on Y are 0, 1, T, F and G where $F \leq T \leq G$. Also, $f^{-1}(U)$ is ps-ro open fuzzy set on X for all ps-ro α-open fuzzy set U on Y. Hence, f is ps-ro fuzzy strongly α-irresolute. H is fuzzy α-open set on Y, but $f^{-1}(H)$ is not fuzzy open set on X proving f is not fuzzy strongly α-irresolute.

Remark 4 *From Remark 3 and Example 3, it follows that ps-ro fuzzy strongly α-irresolute and fuzzy strongly α-irresolute functions are two independent concepts.*

Theorem 1 *If $f : X \to Y$ and $g : Y \to Z$ be two functions where X, Y and Z are three fts, then the following hold:*

(i) *if f and g are ps-ro fuzzy strongly α-irresolute function then gof is also so.*
(ii) *if f is ps-ro fuzzy strongly α-irresolute and g is ps-ro fuzzy α-continuous, then gof is ps-ro fuzzy continuous.*
(iii) *if f is ps-ro fuzzy continuous and g is ps-ro fuzzy strongly α-irresolute, then gof is ps-ro fuzzy strongly α-irresolute.*

Proof.

(i) Let A be a ps-ro α-open fuzzy set on Z. By given conditions, $g^{-1}(A)$ is ps-ro open and hence ps-ro α-open fuzzy set on Y. $f^{-1}(g^{-1}(A))$ is ps-ro open fuzzy set on X. As, $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$, $(gof)^{-1}(A)$ is ps-ro open fuzzy set on X, showing gof is ps-ro fuzzy strongly α-irresolute.
(ii) Let B be any ps-ro open fuzzy set on Z. By given hypothesis, $g^{-1}(B)$ is ps-ro α-open fuzzy set on Y and $f^{-1}(g^{-1}(B))$ is ps-ro open fuzzy set on X. As, $(gof)^{-1}(B) = f^{-1}(g^{-1}(B))$, $(gof)^{-1}(B)$ is ps-ro open fuzzy set on X and hence gof is ps-ro fuzzy continuous.
(iii) Let B be any ps-ro α-open fuzzy set on Z. $g^{-1}(B)$ is ps-ro open fuzzy set on Y and $f^{-1}(g^{-1}(B))$ is ps-ro open fuzzy set on X. Using $(gof)^{-1}(B) = f^{-1}(g^{-1}(B))$, $(gof)^{-1}(B)$ is ps-ro open fuzzy set on X. So, gof is a ps-ro fuzzy strongly α-irresolute.

□

Theorem 2 *A function f from a fts (X, τ_1) to another fts (Y, τ_2) is ps-ro fuzzy strongly α-irresolute iff for any fuzzy point x_α of X and any ps-ro α-open fuzzy set V on Y with $f(x_\alpha)qV$, there exist a ps-ro open fuzzy set U on X such that $x_\alpha qU \leq f^{-1}(V)$.*

Proof. Let f be ps-ro fuzzy strongly α-irresolute. Let x_α be a fuzzy point on X and V be ps-ro α-open fuzzy set on Y such that $f(x_\alpha)qV$. $f^{-1}(V)$ is ps-ro open fuzzy set on X and $Vf(x) + \alpha > 1$. So, $x_\alpha q f^{-1}(V)$. Taking $f^{-1}(V) = U$, the result follows.

Conversely, let V be a ps-ro α-open fuzzy set on Y and x_α be a fuzzy point on $f^{-1}(V)$. Then, $x_\alpha \le f^{-1}(V)$, $f(x_\alpha) \le f(f^{-1}(V)) \le V$. Choosing a fuzzy point x'_α with $x'_\alpha(x) = 1 - x_\alpha(x)$, we have $V(y) + f(x'_\alpha)(y) = V(y) + f(1 - x_\alpha)(y) \ge V(y) + (1 - V)(y) = 1$. So, $f(x'_\alpha)qV$. Then there exists a ps-ro open fuzzy set U on X such that $x'_\alpha qV \le f^{-1}(V)$. Since $x'_\alpha qV$, $x'_\alpha(x) + U(x) = 1 - x_\alpha(x) + U(x) > 1$. So, $x_\alpha \le U$. Hence, $x_\alpha \le U \le f^{-1}(V)$. x_α being arbitrary, taking union of all such relations, $\vee\{U : x_\alpha \in f^{-1}(V)\} = f^{-1}(V)$, proving $f^{-1}(V)$ is ps-ro open fuzzy set on X and hence, f is ps-ro fuzzy strongly α-irresolute. □

Definition 2 *For any fuzzy set* A *on* fts (X, τ)*, the smallest* ps-ro α*-closed fuzzy set containg* A *is called* ps-αcl(A) *and the largest* ps-ro α*-open fuzzy set contained in* A *is called* ps-αint(A)*.*

Theorem 3 *For a function* f *from a* fts (X, τ_1) *to another* fts (Y, τ_2) *the following statements are equivalent.*

(a) f *is* ps-ro *fuzzy strongly* α*-irresolute.*

(b) *the inverse image of each* ps-ro α*-closed fuzzy set on* Y *is* ps-ro *closed fuzzy set on* X*.*

(c) *for each fuzzy point* x_α *on* X *and each* ps-ro α*-open fuzzy set* B *on* Y *and* $f(x_\alpha) \in B$*, there exists* ps-ro *open fuzzy set* A *on* X *such that* $x_\alpha \in A$ *and* $f(A) \le B$*.*

(d) $f(\text{ps-cl}(A)) \le \text{ps-}\alpha\text{cl}f(A)$ $\forall$ *fuzzy set* A *on* X*.*

(e) $\text{ps-cl}(f^{-1}(B)) \le f^{-1}(\text{ps-}\alpha\text{cl}(B))$ $\forall$ *fuzzy set* B *on* Y*.*

(f) $f^{-1}(\text{ps-}\alpha\text{int}B) \le \text{ps-int}(f^{-1}(B))$ $\forall$ *fuzzy set* B *on* Y*.*

Proof. (a) ⇒ (b) Let f be ps-ro fuzzy strongly α-irresolute function. Let B be ps-ro α-closed fuzzy set on Y. Then $(1 - B)$ is ps-ro α-open fuzzy set on Y. Since f is ps-ro fuzzy strongly α-irresolute, $f^{-1}(1 - B)$ is ps-ro open fuzzy set on X. As $f^{-1}(1 - B) = 1 - f^{-1}(B)$, the result follows.

(b) ⇒ (a) Let B be ps-ro α-open fuzzy set on Y. Then $f^{-1}(1 - B)$ is ps-ro closed fuzzy set on X. As, $f^{-1}(1 - B) = 1 - f^{-1}(B)$, $f^{-1}(B)$ is ps-ro open fuzzy set on X. Therefore f is ps-ro fuzzy strongly α-irresolute.

(a) ⇒ (c) Let x_α be any fuzzy point on X and B be any ps-ro α-open fuzzy set on Y such that $f(x_\alpha) \in B$. Since f is ps-ro fuzzy strongly α-irresolute function,

$f^{-1}(B)$ is ps-ro open fuzzy set on X which contains x_α. Taking $f^{-1}(B) = A$, the result follows.
(c) $\Rightarrow$ (a) Let the given condition hold and B be any ps-ro α-open fuzzy set on Y. If $f^{-1}(B) = 0$, then the result is true. If $f^{-1}(B) \neq 0$, then there exist fuzzy point x_α on $f^{-1}(B)$. So, there exist ps-ro open fuzzy set U_{x_α} on X which contains x_α such that $x_\alpha \in U_{x_\alpha} \leq f^{-1}(B)$. Since x_α is arbitrary, taking union of all such relations, we get $f^{-1}(B) = \vee\{x_\alpha : x_\alpha \in f^{-1}(B)\} \leq \vee\{U_{x_\alpha} : x_\alpha \in f^{-1}(B)\} \leq f^{-1}(B)$. This shows that $f^{-1}(B) = \vee\{U_{x_\alpha} : x_\alpha \in f^{-1}(B)\}$ which imply that $f^{-1}(B)$ is ps-ro open fuzzy set on X. Therefore f is ps-ro fuzzy strongly α-irresolute.
(b) $\Rightarrow$ (d) Let A be fuzzy set on X. Then $A \leq f^{-1}(f(A)) \leq f^{-1}(ps\text{-}\alpha cl(f(A)))$. Since $ps\text{-}\alpha cl(f(A))$ is ps-ro α-closed fuzzy set on Y, $f^{-1}(ps\text{-}\alpha cl(f(A)))$ is ps-ro closed fuzzy set on X. Now, $ps\text{-}cl(A) \leq f^{-1}(ps\text{-}\alpha cl(f(A)))$ and $f(ps\text{-}cl(A)) \leq f(f^{-1}(ps\text{-}\alpha cl(f(A))) \leq ps\text{-}\alpha cl(f(A))$. Thus, $f(ps\text{-}clA) \leq ps\text{-}\alpha clf(A)$.
(d) $\Rightarrow$ (e) For any fuzzy set B on Y, let $A = f^{-1}(B)$. By hypothesis, we have $f(ps\text{-}cl(f^{-1}(B))) \leq ps\text{-}\alpha clf(f^{-1}(B)) \leq ps\text{-}\alpha cl(B)$ and $ps\text{-}cl(f^{-1}(B)) \leq f^{-1}(f(ps\text{-}cl(f^{-1}(B)))) \leq f^{-1}(ps\text{-}\alpha cl(B))$. Thus $ps\text{-}cl(f^{-1}(B) \leq f^{-1}(ps\text{-}\alpha cl(B))$.
(e) $\Rightarrow$ (f) $f^{-1}(ps\text{-}\alpha intB) = f^{-1}(1 - ps\text{-}\alpha cl(1 - B))$
$= 1 - f^{-1}(ps\text{-}\alpha cl(1 - B))$
$\leq 1 - ps\text{-}cl(f^{-1}(1 - B))$
$= 1 - ps\text{-}cl(1 - f^{-1}(B))$
$= (1 - (1 - ps\text{-}int(f^{-1}(B))))$
$= ps\text{-}int(f^{-1}(B))$. So, $f^{-1}(ps\text{-}\alpha intB) \leq ps\text{-}int(f^{-1}(B))$.
(f) $\Rightarrow$ (a) Let B be any ps-ro α-open fuzzy set on Y. Then $B = ps\text{-}\alpha intB$. Then $f^{-1}(ps\text{-}\alpha intB) = f^{-1}(B) \leq ps\text{-}int(f^{-1}(B))$. Also, $ps\text{-}int(f^{-1}(B)) \leq f^{-1}(B)$. Therefore, $ps\text{-}int(f^{-1}(B)) = f^{-1}(B)$ which imply that $f^{-1}(B)$ is ps-ro open fuzzy set on X. Thus, f is ps-ro fuzzy strongly α-irresolute function. □

Theorem 4 *A bijective function* f *from a* fts (X, τ_1) *to another* fts (Y, τ_2) *is* ps-ro *fuzzy strongly* α*-irresolute iff* $ps\text{-}\alpha int(f(A)) \leq f(ps\text{-}int(A))$, *for every fuzzy set* A *on* X.

Proof. Let f be ps-ro fuzzy strongly α-irresolute and A be any fuzzy set on X. $ps\text{-}\alpha int(f(A))$ being ps-ro α-open fuzzy set on Y, $f^{-1}(ps\alpha int(f(A))$ is ps-ro open fuzzy set on X. Now, $f^{-1}(ps\alpha int(f(A)) \leq ps\text{-}int(f^{-1}(f(A))) = ps\text{-}int(A)$, as f is one-to-one, which gives $f(f^{-1}(ps\alpha int(f(A))) \leq f(ps\text{-}int(A))$ and hence, $ps\alpha int(f(A)) \leq f(ps\text{-}int(A))$, as f is onto.

Conversely, let B be any ps-ro α-open fuzzy set on Y. Then, $B = ps\text{-}\alpha int(B) = ps\text{-}\alpha int(f(f^{-1}(B))$, (since f is onto). By given condition, ps-αint

$(f(f^{-1}(B)) \leq f(\text{ps-int}(f^{-1}(B))$ so, $f^{-1}(B) \leq f^{-1}(f(\text{ps-int}(f^{-1}(B)))) = \text{ps-int}(f^{-1}(B))$,(since f is one-to-one). But $\text{ps-int}(f^{-1}(B)) \leq f^{-1}(B)$. Therefore, $\text{ps-int}(f^{-1}(B)) = f^{-1}(B)$ proving that $f^{-1}(B)$ is ps-ro open fuzzy set on X. Hence, f is ps-ro fuzzy strongly α-irresolute function. □

Lemma 1 [1] *Let* $g : X \to X \times Y$ *be the graph of a function* $f : (X, \tau_1) \to (Y, \tau_2)$ *defined by* $g(x) = (x, f(x))$. *If* A *and* B *are fuzzy sets on* X *and* Y *respectively, then* $g^{-1}(A \times B) = A \wedge f^{-1}(B)$.

Theorem 5 *For a function* f *from a* fts (X, τ_1) *to another* fts (Y, τ_2), *if the graph* $g : X \to X \times Y$ *of* f *is* ps-ro *fuzzy strongly* α*-irresolute then* f *is also* ps-ro *fuzzy strongly* α*-irresolute.*

Proof. Let B be any ps-ro α-open fuzzy set on Y. Using Lemma 1 we get, $f^{-1}(B) = 1 \wedge f^{-1}(B) = g^{-1}(1 \times B)$. Now, $(1 \times B)$ is ps-ro α-open fuzzy set on $(X \times Y)$ and since g is ps-ro fuzzy strongly α-irresolute function, $g^{-1}(1 \times B)$ is ps-ro open fuzzy set on X. Thus, $f^{-1}(B)$ is ps-ro open fuzzy set on X. Hence f is ps-ro fuzzy strongly α-irresolute. □

Definition 3 *A fuzzy set* A *on a* fts (X, τ) *is called* ps-ro *fuzzy dense set if* $\text{ps-cl}(A) = 1$ *and* A *is called nowhere* ps-ro *fuzzy dense set if* $\text{ps-int}(\text{ps-cl}(A)) = 0$.

Theorem 6 *If a function* $f : (X, \tau_1) \to (Y, \tau_2)$ *is* ps-ro *fuzzy strongly* α*-irresolute, then* $f^{-1}(A)$ *is* ps-ro *closed fuzzy set on* X *for any nowhere* ps-ro *fuzzy dense set* A *on* Y.

Proof. Let A be any nowhere ps-ro fuzzy dense set on Y. Then, $1 - \text{ps-int}(\text{ps-cl}A) = 1$ i.e., $\text{ps-cl}(\text{ps-int}(1 - A)) = 1$. As $\text{ps-int}1 = 1$, $\text{ps-int}(\text{ps-cl}(\text{ps-int}(1-A))) = \text{ps-int}1 = 1$. $1-A \leq 1 = \text{ps-int}(\text{ps}-\text{cl}(\text{ps}-\text{int}(1-A)))$ which shows that $1 - A$ is ps-ro α-open fuzzy set on Y. f being ps-ro fuzzy α-irresolute, $f^{-1}(1 - A)$ is ps-ro open fuzzy set on X. Using $f^{-1}(1 - A) = 1 - f^{-1}(A)$, $f^{-1}(A)$ is ps-ro closed fuzzy set on X. □

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