Bianchi Identities, Yang-Mills and Higgs Field Produced on $\tilde{S}^{(2)}M$-Deformed Bundle

P.C.Stavrinos

Abstract

The Bianchi equations are determined for the deformed spinor bundle $\tilde{S}^{(2)}M = S^{(2)}M \times R$. Also, the Yang-Mills-Higgs equations are derived, and a geometrical interpretation of the Higgs field is given.

Mathematics Subject Classification: 81R25, 81T13, 53B30, 81T20, 53B50
Key words: deformed spinor bundle, Bianchi identities, Yang-Mills-Higgs equations, Higgs field.

1 Introduction

In our previous works [1],[2] the concepts of spinor bundle $S^{(2)}M$ as well as of deformed spinor bundle $\tilde{S}^{(2)}M$ of order two, were introduced in the framework of a geometrical generalization of the proper spinor bundles as they have been studied from different authors e.g. [3],[4],[8].
The study of fundamental geometrical subjects as well as the gauge covariant derivatives, connections field equations e.t.c. in a deformed spinor bundle $\tilde{S}^{(2)}M$, has been developed in a sufficiently generalized approach [2]. In these spaces the internal variables or the gauge variables of fibration have been substituted by the internal (Dirac) variables $\omega = (\xi, \bar{\xi})$. In addition, another central point of our consideration is that of the internal fibres C^4.
The initial spinor bundle $(S^{(2)}M, \pi, F)$, $\pi : S^{(2)}M \longrightarrow M$ was constructed from the one of the principal fibre bundles with fibre $F = C^4$ (C^4 denotes the complex space) and M the base manifold of space-time events of signature $(+, -, -, -)$.
A spinor in $x \in M$ is an element of the spinor bundle $S^{(2)}M$ [1],

$$(x^{\mu}, \xi_{\alpha}, \bar{\xi}^{\alpha}) \in S^{(2)}M.$$

A spinor field is section of $S^{(2)}M$. A generalization of the spinor bundle $S^{(2)}M$ in a internal deformed system, has been given in the work [2]. The form of this bundle determined as

Balkan Journal of Geometry and Its Applications, Vol.1, No.1, 1996, pp. 75-82

$$\tilde{S}^{(2)}M = S^{(2)}M \times R$$

where R represents the internal on dimension fibre of deformation. The metrical structure in the deformed spinor bundle $\tilde{S}^{(2)}M$ has the form:

$$G = g_{\mu\nu}(x,\xi,\bar{\xi})dx^{\mu} \otimes dx^{\nu} + g_{\alpha\beta}(x,\xi,\bar{\xi},\lambda)D\bar{\xi}^{\alpha} \otimes D\bar{\xi}^{*\beta} + \tag{1.1}$$
$$+ g^{\alpha\beta}(x,\xi,\bar{\xi},\lambda)D\xi_{\alpha} \otimes D\xi^{*}_{\beta} + g_{0,0}(x,\xi,\bar{\xi},\lambda)D\lambda \otimes D\lambda.$$

where "*" denotes Hermitean conjugation. The local adapted frame is given by:

$$\frac{\delta}{\delta\zeta^{A}} = \{\frac{\delta}{\delta x^{\lambda}}, \frac{\delta}{\delta\xi_{\alpha}}, \frac{\delta}{\delta\bar{\xi}^{\alpha}}, \frac{\partial}{\partial\lambda}\} \tag{1.2}$$

and the associated dual frame:

$$\delta\zeta^{A} = \{Dx^{K} \equiv dx^{K}, D\xi_{\beta}D\bar{\xi}_{\beta}, D_{0\lambda}\} \tag{1.3}$$

where the terms $\frac{\delta}{\delta^{\lambda}}$, $\frac{\delta}{\delta\xi_{\alpha}}$, $D_0\lambda$, Dx^K, $D\xi_{\beta}$, $D\bar{\xi}^{\beta}$ are provided by the relations (6)-(7) of [2].
The considered connection in $\tilde{S}^{(2)}M$ is a d-connection [5] having with respect to the adapted basis the coefficients (cf. [2]).

$$\Gamma^{A}_{BC} = \{\Gamma^{(*)\mu}_{\nu\rho}, C^{\mu}_{\nu\alpha}, \bar{C}^{\mu\alpha}_{\nu}, \Gamma^{(*)\mu}_{\nu 0}, \bar{\Gamma}^{(*)\alpha}_{\beta\lambda}, \tilde{C}^{\beta}_{\alpha\gamma}, \tilde{C}^{\beta\alpha}_{\gamma}, \tilde{\Gamma}^{(*)\beta}_{\alpha\gamma}, \Gamma^{(*)\beta}_{\alpha\nu}, C^{\gamma}_{\beta\alpha},$$

$$C^{\gamma\alpha}_{\beta}, C^{\alpha}_{\beta 0}, \Gamma^{(*)0}_{0\mu}, \bar{C}^{0\alpha}_{)}, C^{0}_{0\alpha}, L^{0}_{00}\}. \tag{1.4}$$

The metric G of relation (1.1) could be considered as a definite physical application like the one given by R. Miron and G. Atanasiu for Lagrange spaces [6] for the case of spinor bundles of order two. According to our approach on $\tilde{S}^{(2)}M$ the internal variables ξ and $\bar{\xi}$ play a crucial role similar to the variables $y^{(1)}$, $y^{(2)}$ of the vector bundles of order two.
The non-linear connection on $\tilde{S}^{(2)}M$ is defined analogously to the vector bundles at order two (cf. [6]) but in a gauge covariant form:

$$T(\tilde{S}^{(2)}M) = H(\tilde{S}^{(2)}M) \oplus F^{(1)}(\tilde{S}^{(2)}M) \oplus F^{(2)}(\tilde{S}^{(2)}M) \oplus R \tag{1.5}$$

where $H, F^{(1)}, F^{(2)}, R$ represent the horizontal vertical normal and deformation distributions respectively.
In the following, in section 2, we study the Bianchi identities and Yang-Mills-Higgs fields on $\tilde{S}^{(2)}M$ bundle in section 3.
Bianchi Identities In order to study of Bianchi Identities (kinematic constraints) it is necessary to use the Jacobi identities:

$$S_{(XYZ)}[\tilde{D}^{(*)}_{X}, [\tilde{D}^{(*)}_{Y}, \tilde{D}^{(*)}_{Z}]] = 0. \tag{1.6}$$

There are forty-eight Bianchi relations derived from twenty-four different types of Jacobi identities. Two of these relations are identically zero. Therefore remain forty-six Bianchi relations. We will give now some characteristic cases of the Bianchi identities.

Similarly to our previous work [2], the gauge covariant derivative will take the form

$$\tilde{D}_\mu^{(*)} = \frac{\tilde{\delta}}{\delta x^\mu} + \frac{1}{2}\omega_\mu^{(*)ab} J_{ab} \tag{1.7}$$

where

$$\frac{\tilde{\delta}}{\delta x^\mu} = \frac{\partial}{\partial x^\mu} - N_{\alpha\mu}\frac{\partial}{\partial \xi_\alpha} - \bar{N}_\mu^\alpha \frac{\partial}{\partial \bar{\xi}^\alpha} - N_\mu^0 \frac{\partial}{\partial \lambda}$$

or

$$\frac{\tilde{\delta}}{\delta x^\mu} = \tilde{A}_\mu^a P_a$$

with

$$\tilde{A}_\mu^a = A_\mu^a - N_\mu^0 L_0^k L_k^a$$

$$P_a = \frac{\partial}{\partial x^a}$$

$$A_\mu^a = h_\mu^a - N_{\alpha\mu}\bar{\psi}^{\alpha a} - \bar{N}_\mu^\alpha \psi_\alpha^a.$$

After some calculations we get:

$$[\tilde{D}_\mu^{(*)}, [\tilde{D}_\kappa^{(*)}, \tilde{D}_\lambda^{(*)}]] = (\frac{\delta \tilde{R}_{\kappa\lambda}^a}{\partial x^\mu} + \tilde{R}_{b\kappa\lambda}^a A_\mu^b + \omega_{\mu c}^{(*)ab} \tilde{R}_{\kappa\lambda}^c) P_a$$

$$+(\frac{1}{2}\frac{\delta \tilde{R}_{\kappa\lambda}^{ce}}{\partial x^\mu} + \omega_\mu^{(*)cd} \tilde{R}_{d\kappa\lambda}^e) J_{ce} \tag{1.8}$$

and $\omega_\mu^{(*)ab}$ represent the Lorentz-spin connection coefficients. We define also:

$$\tilde{D}_\mu \tilde{R}_{\kappa\lambda}^{ce} = \frac{1}{2}\frac{\tilde{\delta}\tilde{R}_{\kappa\lambda}^{ce}}{\delta x^\mu} + \omega_\mu^{(*)cd} \tilde{R}_{d\kappa\lambda}^e \tag{1.9}$$

$$\tilde{D}_\mu \tilde{R}_{\kappa\lambda}^a = \frac{\tilde{\delta}\tilde{R}_{\kappa\lambda}^\alpha}{\delta x^\mu} + \tilde{R}_{b\kappa\lambda}^a \tilde{A}_\mu^b + \omega_{\mu c}^{(*)a} \tilde{R}_{\kappa\lambda}^c. \tag{1.10}$$

By cyclic permutation of the independent generators J_{ce}, P_a we get the following Bianchi identities,

$$\tilde{D}_\mu \tilde{R}_{\kappa\lambda}^a + \tilde{D}_\kappa \tilde{R}_{\lambda\mu}^a + \tilde{D}_\lambda \tilde{R}_{\mu\kappa}^a = 0 \tag{1.11}$$

$$\tilde{D}_\mu \tilde{R}_{\kappa\lambda}^{ce} + \tilde{D}_\kappa \tilde{R}_{\lambda\mu}^{ce} + \tilde{D}_\lambda \tilde{R}_{\mu\kappa}^{ce} = 0. \tag{1.12}$$

Using the Jacobi identities $Q_{(\alpha,\beta,\gamma)}[\tilde{D}_\alpha^{(*)}, [\tilde{D}_\beta^{(*)}, \tilde{D}_\gamma^{(*)}]] = 0$ the Bianchi identities with respect to spinor quantities produce the relations,

$$\tilde{D}_\alpha Q_{\beta\gamma}^{ab} + \tilde{D}_\beta Q_{\gamma\alpha}^{ab} + \tilde{D}_\gamma Q_{\alpha\beta}^{ab} = 0. \tag{1.13}$$

$$\tilde{D}_\alpha Q^a_{\beta\gamma} + \tilde{D}_\beta Q^a_{\gamma\alpha} + \tilde{D}_\gamma Q^a_{\alpha\beta} = 0. \tag{1.14}$$

The new Jacobi identity, due to λ, has the form

$$[D_0^{(*)}, [D_0^{(*)}, D_0^{(*)}]] = 0 \tag{1.15}$$

which yields us no Bianchi identity.
Bianchi identities of mixed type give us the kinematic constraint which encompass space-time, spinors and deformed gauge covariant derivatives. In that case from the Jacobi identities

$$Q_{\mu\alpha 0}[\tilde{D}_\mu^{(*)}, [\tilde{D}_\alpha^{(*)}, \tilde{D}_0^{(*)}]] = 0.$$

we get the relations:

$$[\tilde{D}_\mu^{(*)}, [\tilde{D}_\alpha^{(*)}, \tilde{D}_0^{(*)}]] = (\frac{\tilde{\delta}\tilde{P}^d_{0\alpha}}{\delta x^\mu} + \tilde{P}^d_{c0\alpha}A^e_\mu + \omega^{(*)d}_{\mu a}\tilde{P}^a_{0\alpha})P_d + \tag{1.16}$$

$$+(\frac{1}{2}\frac{\tilde{\delta}\tilde{P}^{cd}_{0\alpha}}{\delta x^\mu} + \omega^{(*)c}_{\mu\alpha}\tilde{P}^{ad}_{0\alpha})J_{cd}$$

$$[\tilde{D}_\alpha^{(*)}, [\tilde{D}_0^{(*)}, \tilde{D}_\mu^{(*)}]] = (\frac{\partial\tilde{P}^d_{\mu 0}}{\partial\bar{\xi}^\alpha} + \tilde{P}^d_{c\mu 0}A^c_\alpha + *\omega^{(*)d}_{\alpha a}\tilde{P}^a_{\mu 0})P_d + \tag{1.17}$$

$$+(\frac{1}{2}\frac{\partial\tilde{P}^{cd}_{\mu 0}}{\partial\bar{\xi}^\alpha} + \omega^{(*)c}_{\alpha a}\tilde{P}^{ad}_{\mu 0})J_{cd}$$

$$[\tilde{D}_0^{(*)}, [\tilde{D}_\mu^{(*)}, \tilde{D}_\alpha^{(*)}]] = (\frac{\partial\tilde{P}^d_{\alpha\mu}}{\partial\lambda} + \tilde{P}^d_{c\alpha\mu}A^c_0 + \omega^{(*)d}_{0a}\tilde{P}^a_{\alpha\mu})P_d + \tag{1.18}$$

$$+(\frac{1}{2}\frac{\partial\tilde{P}^{cd}_{\alpha\mu}}{\partial\lambda} + \omega^{(*)c}_{0a}\tilde{P}^{ad}_{\alpha\mu})J_{cd}$$

where,

$$\tilde{D}_\mu^{(*)} = \frac{\tilde{\delta}}{\delta x^\mu} + \frac{1}{2}\omega^{(*)ab}_\mu J_{ab}$$

$$\tilde{D}_\alpha^{(*)} = \frac{\partial}{\partial\bar{\xi}^\alpha} + \frac{1}{2}\Theta^{(*)ab}_\alpha J_{ab}$$

$$\tilde{D}_0^{(*)} = \frac{\partial}{\partial\lambda} + \omega^{ab}_0 J_{ab}$$

$$\frac{\partial}{\partial\lambda} = L^\mu_0 h^a_\mu P_a$$

$$\frac{\partial}{\partial\bar{\xi}^\alpha} = \psi^a_\alpha P_a.$$

Now we put,

(1.19) $$\tilde{D}_\mu \tilde{P}^d_{0\alpha} = \frac{\tilde{\delta}\tilde{P}^d_{0\alpha}}{\delta x^\mu} + \tilde{P}^d_{c\mu 0} A^c_\mu + \omega^{(*)d}_{\alpha a} \tilde{P}^a_{\mu 0}$$

(1.20) $$\tilde{D}_\alpha \tilde{P}^d_{\mu 0} = (\frac{\partial \tilde{P}^d_{\mu 0}}{\partial \bar{\xi}^\alpha} + \tilde{P}^d_{c\mu 0} A^c_\mu + \omega^{(*)d}_{\alpha a} \tilde{P}^a_{\mu 0})$$

(1.21) $$\tilde{D}_0 \tilde{P}^d_{\alpha\mu} = \frac{\partial \tilde{P}^d_{\alpha\mu}}{\partial \lambda} + \tilde{P}^d_{c\alpha\mu} A^c_0 + \omega^{(*)d}_{0a} \tilde{P}^a_{\alpha\mu}$$

in virtue of [14],[15],[16] we get the identity

(1.22) $$\tilde{D}_\mu \tilde{P}^d_{0\alpha} + \tilde{D}_\alpha \tilde{P}^d_{\mu 0} + \tilde{D}_0 \tilde{P}^d_{\alpha\mu} = 0.$$

Similarly we define

(1.23) $$\tilde{D}_\mu \tilde{P}^{cd}_{0\alpha} = \frac{1}{2} \frac{\partial \tilde{P}^{cd}_{0\alpha}}{\partial x^\mu} + \omega^{(*)c}_{\mu a} \tilde{P}^{ad}_{0\alpha}$$

(1.24) $$\tilde{D}_\alpha \tilde{P}^{cd}_{\mu 0} = \frac{1}{2} \frac{\partial \tilde{P}^{cd}_{\mu 0}}{\partial \bar{\xi}^\alpha} + \omega^{(*)c}_{\alpha a} \tilde{P}^{ad}_{\mu 0}$$

(1.25) $$\tilde{D}_0 \tilde{P}^{cd}_{\alpha\mu} = \frac{1}{2} \frac{\partial \tilde{P}^{cd}_{\alpha\mu}}{\partial \lambda} + \omega^{(*)c}_{0\alpha} \tilde{P}^{ad}_{\alpha\mu}$$

From (1.18)-(1.20) we get

(1.26) $$\tilde{D}_\mu \tilde{P}^{cd}_{0\alpha} + \tilde{D}_\alpha \tilde{P}^{cd}_{\mu 0} + \tilde{D}_0 \tilde{P}^{cd}_{\alpha\mu} = 0$$

2 Yang-Mills-Higgs equations. A geometrical interpretation of Higgs Field.

The study of Yang-Mills-Higgs equations within the framework of the geometrical structure of $\tilde{S}^{(2)}(M)$-bundle that contains the one-dimensional fibre as an internal deformed system can characterize the Higgs field which is studied in the elementary particle physics. In our description we are allowed to choose a scalar from the internal deformed fibre of the spinor bundle $\tilde{S}^{(2)}(M)$. Its contribution to the Lagrangian density provides us with the generated Yang-Mills-Higgs equations.
In the following we define a gauge potential $\tilde{A} = (A_\mu, A_\alpha, \bar{A}^\alpha, \phi)$ with space-time and spinor components, $\phi : R \longrightarrow g$ which takes its values in a Lie Algebra g.

$$\tilde{A} : \tilde{S}(M) \longrightarrow g$$

$$\tilde{A}_X = A^i_X \tau_i, [\tau_i, \tau_j] = C^k_{ij} \tau_k$$

$$\tilde{A}_X = \{A_\mu, A_\alpha, \bar{A}^\alpha, \phi\}$$

where the elements τ_i are the components which satisfy the commutation relations of the Lie algebra. Then $\tilde{A}$ is called a *g-valued spinor gauge potential.* We can define the gauge covariant derivatives:

$$\hat{D}_\mu = \tilde{D}_\mu + iA_\mu$$

$$\hat{D}_\alpha = \tilde{D}_\alpha + iA_\alpha \tag{2.1}$$

$$\hat{D}^\alpha = \bar{\tilde{D}}^\alpha + iA_\mu.$$

In virtue of the preceding relations we can get the following theorem:
Theorem 2.1. *The commutators of gauge covariant derivatives on a $\tilde{S}^{(2)}M$ deformed bundle are given by the relations:*

$$\text{a) } [\hat{D}_\mu, \hat{D}_\nu] = [\tilde{D}_\mu, \tilde{D}_\nu] + i\tilde{F}_{\mu\nu}$$

$$\text{b) } [\hat{D}_\mu, \hat{\bar{D}}^\alpha] = [\tilde{D}_\mu, \bar{\tilde{D}}^\alpha] + i\tilde{F}^\alpha_\mu$$

$$\text{c) } [\hat{D}_\alpha, \hat{\bar{D}}^\beta] = [\tilde{D}_\alpha, \bar{\tilde{D}}^\beta] + i\tilde{F}^\beta_\alpha \tag{2.2}$$

$$\text{d) } [\hat{D}_\alpha, \hat{\bar{D}}_\beta] = [\tilde{D}_\alpha, \bar{\tilde{D}}_\beta] + i\tilde{F}_{\alpha\beta}$$

$$\text{e) } [\hat{D}_\mu, \hat{\bar{D}}_\alpha] = [\tilde{D}_\mu, \bar{\tilde{D}}_\alpha] + i\tilde{F}_{\mu\alpha}$$

$$\text{f) } [\hat{\bar{D}}^\alpha, \hat{\bar{D}}^\beta] = [\bar{\tilde{D}}^\alpha, \bar{\tilde{D}}^\beta] + i\bar{\tilde{F}}^{\alpha\beta}.$$

The curvature two-forms $\tilde{F}_{XY}, \tilde{F}^{XY}, F^X_Y$ $X, Y = \{\alpha, \beta, \mu, \nu\}$ are the g-valued field strengths on $\tilde{S}^{(2)}M$ and they have the following form:

$$\tilde{F}_{\mu\nu} = \tilde{D}_\mu A_\nu - \tilde{D}_\nu A_\mu + i[A_\mu, A_\nu]$$

$$\tilde{F}_{\mu\alpha} = \tilde{D}_\mu A_\alpha - \tilde{D}_\alpha A_\mu + i[A_\alpha, A_\mu]$$

$$\bar{\tilde{F}}^\beta_\alpha = \tilde{D}_\alpha \bar{A}^\beta - \bar{\tilde{D}}^\beta \bar{A}_\alpha + i[A_\alpha, \bar{A}^\beta]$$

$$\bar{\tilde{F}}^\alpha_\mu = \tilde{D}_\mu \bar{A}^\alpha - \bar{\tilde{D}}_\alpha A_\mu + i[A_\mu, \bar{A}^\alpha] \tag{2.3}$$

$$\tilde{F}_{\alpha\beta} = \tilde{D}_\alpha A_\beta - \tilde{D}_\beta A_\alpha + i[A_\alpha, A_\beta]$$

$$\bar{\tilde{F}}^{\alpha\beta} = \bar{\tilde{D}}^\alpha \bar{A}^\beta - \bar{\tilde{D}}^\beta \bar{A}_\alpha + i[\bar{A}^\alpha, \bar{A}^\beta]$$

The appropriate Lagrangian density of Yang-Mills (Higgs) can be written in the form

$$\tilde{L} = tr(\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} + tr(\tilde{F}_{\mu\alpha}\bar{\tilde{F}}^{\mu\alpha} + tr(\tilde{F}_{\alpha\beta}\bar{\tilde{F}}^{\alpha\beta} + tr(\bar{\tilde{F}}^\beta_\alpha \bar{\tilde{F}}^\alpha_\beta \tag{2.4}$$

$$+\frac{1}{2}m^2\phi^2-\frac{1}{2}tr[(\hat{D}_\mu\phi)(\hat{D}^\mu\phi)]-\frac{1}{2}tr[(\hat{D}_\alpha\phi)(\hat{D}^\mu\phi)]$$

In our case the Yang-Mills(Higgs) generalized action can be written in the form

$$\tilde{\mathcal{I}}_{YM(H)}=\int\tilde{\mathcal{L}}d^4xd^4\xi d^4\bar{\xi}d\lambda. \tag{2.5}$$

From the Euler-Lagrange equations

$$\frac{\delta\tilde{L}}{\delta A_Y}=\tilde{D}_X(\frac{\partial\tilde{L}}{\partial(\tilde{D}_XA_Y)})-\frac{\partial\tilde{L}}{\partial A_Y}=0 \tag{2.6}$$

the variation of $\tilde{L}$ with respect to A_λ is

$$\tilde{D}_k(\frac{\partial\tilde{L}}{\partial(\tilde{D}_kA_\lambda)})+\tilde{D}_\beta(\frac{\partial\tilde{L}}{\partial(\tilde{D}_\beta A_\lambda)})+\tilde{\bar{D}}^\beta(\frac{\partial\tilde{L}}{\partial(\tilde{\bar{D}}_\beta A_\lambda)})-\frac{\partial\tilde{L}}{\partial A_\lambda}=0 \tag{2.7}$$

and it will give us after some straightforward calculations the equation:

$$\hat{D}_k\tilde{F}^{k\lambda}+\hat{D}_\beta\tilde{\bar{F}}^{\lambda\beta}+\hat{\bar{D}}^\beta\tilde{F}^\lambda_\beta+[\phi,\hat{D}^\lambda\phi]=0 \tag{2.8}$$

Similarly from the variation of $\tilde{L}$ with respect to A_α and $\bar{A}^\beta$ we associate the equations:

$$\hat{D}_k\tilde{\bar{F}}^{k\gamma}+\hat{D}_\delta\tilde{\bar{F}}^\gamma_\delta+\hat{\bar{D}}^\delta\tilde{\bar{F}}^\gamma_\delta+[\phi,\hat{D}^\gamma\phi]=0 \tag{2.9}$$

$$\hat{D}_k\tilde{F}^k_\gamma+\hat{D}_\delta\tilde{F}^\delta_\gamma+\hat{\bar{D}}^\delta\tilde{F}_{\delta\gamma}+[\phi,\hat{\bar{D}}_\gamma\phi]=0 \tag{2.10}$$

So we can state the following theorem:
Theorem 2.2. *The Yang-Mills-Higgs equations of $\tilde{S}^{(2)}M$-bundle are given by the relations [2.4]-[2.5].*

3 Conclusions

1. In this paper we studied the Bianchi identities choosing a Lagrangian density that contains the component ϕ of a g-valued spinor gauge field of mass $m\in R$. Also we derived the Yang-Mills-Higgs equations on $S^{(2)}M\times R$. When $m_0\in R$ the gauge symmetry is spontaneous broken which is connected with Higgs field.
2. The introduction of d-connections in the internal (spinor) structures on $\tilde{S}^{(2)}M$-bundle provides the presentation of parallelism of the spin components constraints which satisfy by the field strengths.
3. In the metric G (relation (1)) of the bundle $\tilde{S}^{(2)}M$, the term $g^{\alpha\beta}D\xi_\alpha D\xi^*_\beta$ has a physical meaning since it expresses the measure of the number of particles to same point of the space.
4. The above mentioned approach can be combined with the phase transformations of the fibre $U(1)$ on a bundle $S^{(2)}M\times U(1)$ in the Higgs mechanism. This will be the subject of our future study.

References

[1] P.C.Stavrinos and P.Manouselis, *Nonlocalized Field Theory over Spinor Bundles: Poincarè Gravity and Yang - Mills Fields.* Rep. Math. Phys. Vol 36 (95), No 2/3, pp. 293-306.

[2] P.C.Stavrinos, V. Balan, N. Prezas, P.Manouselis, *Spinor Bundle of order two on the internal Deformed System*, Proc. Int. Conf. of Differential Geometry, September 1995, Iasi, to appear.

[3] M. Carmelli, *Classical Fields: General Relativity and Gauge Theory*, John Wiley and Sons, New York, 1982.

[4] *General Relativity*, University of Chicago Press, Chicago, London, 1984.

[5] R.Miron, M.Anastasiei, *The Geometry of Lagrange Spaces: Theory and Applications*, Kluwer Acad. Publ., 1994.

[6] R.Miron, Gh.Atanasiu, *Lagrange Geometry of Second Order*, Math. Comput. Model - Vol 20, No 4/5, pp. 41-56, Pergamon Press, 1994.

[7] E.Witten, *An Introduction of Yang-Mills Theory*, Phys. Letter B(77), 1978, pp. 394-398.

[8] R.S.Ward, R.O. Wells, *Twistor Geometry and Field Theory*, Cambridge Univ. Press, 1950.

Department of Mathematics
University of Athens, 157 84
Panepistimiopolis, Athens, Greece