Seiberg-Witten-like equations on 6−dimensional $SU(3)$−manifolds

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Abstract. It is known that Seiberg-Witten monopole equations are important for the investigations of smooth 4−manifolds. In this study we write the similar equations for 6−dimensional manifold M with structure group $SU(3)$. For Dirac equation we use the associated Spin^c−structure to the $SU(3)$−structure. For the curvature equation we make use of the decomposition $\Lambda^2(M) = \Lambda^2_1(M) \oplus \Lambda^2_6(M) \oplus \Lambda^2_8(M)$ [1]. We consider the part $\Lambda^2_1(M) \oplus \Lambda^2_6(M)$ as the bundle of self-dual 2−forms. Lastly, we give a global solution for these equations.

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1 Introduction

The Seiberg-Witten monopole equations, introduced by Witten in [12], play an important role in the topology of smooth 4−manifolds. Seiberg-Witten equations in dimension greater than four have been investigated by some authors [2, 3, 4, 7, 8]. In this paper, we are mainly interested in 6−dimensional manifolds with $SU(3)$−structure and write down Seiberg-Witten-like equations on these manifolds. The Seiberg-Witten equations consist of two equations. The first one is Dirac equation which is the harmonicity condition of spinor fields. The second one is called the curvature equation which couples the self-dual part of the curvature form with spinor field. In order to write down the Dirac equation the manifold must have a Spin^c−structure. 6−dimensional differentiable manifolds with $SU(3)$−structure have Spin^c−structure. Therefore, one can write down Dirac equation on such manifolds. On the other hand, to write down curvature equation one needs the self-duality notion of a 2−form. In 4−dimension self-duality of a 2−form is well known and this concept is being used in both mathematics and physics widely. We define self-duality of a 2−form on a 6−manifold with $SU(3)$−structure which is consistent with the other self-duality concepts in literature in 6−dimension [3, 10]. Thus, we achieve to write the curvature equation by means of this self-duality concept.

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The plan of this article is the following. In Section 2, we give some basic facts about $6-$dimensional $SU(3)-$manifolds and define self-dual $2-$forms with complex values by using $SU(3)-$action on the space of $2-$forms on such manifolds. In Section 3, we discuss Spin^c-structures and Dirac operator with respect to any given Spin^c-structure. In Section 4, we write down Seiberg-Witten-like equations on $6-$dimensional $SU(3)-$manifolds. In Section 5, we state these equations on $6-$ dimensional Euclidean space. Finally, we give a global solution to these equations.

2 Self-duality on $6-$dimensional $SU(3)-$manifolds

The space of $2-$forms splits into self-dual and anti-self-dual parts by using Hodge * operator on $4-$dimensional Riemannian manifolds. Any self dual $2-$form η satisfies $*\eta = \eta$. But this definition does not generalize to higher dimensional manifolds. Self-duality of a $2-$form has been studied on some specific dimensions [2, 3, 4]. In this section, we define self-duality of $2-$forms on $6-$dimensional $SU(3)-$manifolds.

A $6-$dimensional Riemannian manifold M is called a $SU(3)-$manifold if its structure group reduces to the Lie group $SU(3)$. A $SU(3)-$structure on M is determined by the choice of a non-degenerate $2-$form ω and a normalized positive $3-$form Ω. In fact such a pair (ω, Ω) induces an almost complex structure J on TM, a $J-$compatible hermitian metric g and a complex $(3,0)-$form ε of constant norm $2^{\frac{3}{2}}$. Then, J can be defined on the space of $1-$forms $T^*(M)$ in a natural manner and extended to its complexification $T^*(M) \otimes_{\mathbb{R}} \mathbb{C}$, denoted also by J. It satisfies the equation $J^2 = -Id$. The complexification $T^*(M) \otimes_{\mathbb{R}} \mathbb{C}$ splits into the $\pm i-$subspaces of J as follows:

$$\Lambda^1(M) = T^*(M) \otimes_{\mathbb{R}} \mathbb{C} = \Lambda^{1,0}(M) \oplus \Lambda^{0,1}(M)$$

where

$$\Lambda^{1,0}(M) = \{Z \in T^*(M^6) \otimes_{\mathbb{R}} \mathbb{C} | JZ = iZ\}$$
$$\Lambda^{0,1}(M) = \{Z \in T^*(M^6) \otimes_{\mathbb{R}} \mathbb{C} | JZ = -iZ\}.$$

The space $\Lambda^{p,q}(M)$ is defined by

$$\Lambda^{p,q}(M) = span\{u \wedge v | u \in \Lambda^p(\Lambda^{1,0}(M)), v \in \Lambda^q(\Lambda^{0,1}(M))\}.$$

Then, we have

$$\Lambda^r(M) = \sum_{p+q=r} \Lambda^{p,q}(M).$$

Note that the endomorphism J of TM also induces an endomorphism on $\Lambda^r(M)$, again denoted by J. This satisfies the identity $J^2 = (-1)^r I$. In particular, J acts on a $2-$form η by

$$(J\eta)(X, Y) = \eta(JX, JY).$$

Hence, we have the following:

$$\Lambda^{1,1}(M) = \{\eta \in \Lambda^2(M) : J\eta = \eta\}$$
$$\Lambda^{2,0}(M) \oplus \Lambda^{0,2}(M) = \{\eta \in \Lambda^2(M) : J\eta = -\eta\}$$

If we consider the natural action of $SU(3)$ on space of $2-$forms $\Lambda^2(M)$, then $\Lambda^2(M)$ decomposes as follows:

$$\Lambda^2(M) = \Lambda^2_1(M) \oplus \Lambda^2_6(M) \oplus \Lambda^2_8(M) \tag{2.1}$$

where

$$\begin{aligned}\Lambda^2_1(M) &= \{r\omega : r \in \mathbb{R}\}\\ \Lambda^2_6(M) &= \{\eta \in \Lambda^2(M) : J(\eta) = -\eta\}\\ \Lambda^2_8(M) &= \{\eta \in \Lambda^2(M) : J(\eta) = \eta \text{ and } \eta \wedge \omega \wedge \omega = 0\}.\end{aligned}$$

(See [1] for more details.)

Any 2−form with complex values can be written as follows:

$$\Lambda^2(M) \otimes_{\mathbb{R}} \mathbb{C} = \Lambda^{2,0}(M) \oplus \Lambda^{0,2}(M) \oplus \Lambda^{1,1}(M). \tag{2.2}$$

By complexifying the space of 2−forms we get the following:

$$\Lambda^2(M) \otimes_{\mathbb{R}} \mathbb{C} = \mathbb{C}\omega \oplus (\Lambda^2_6(M) \otimes_{\mathbb{R}} \mathbb{C}) \oplus (\Lambda^2_8(M) \otimes_{\mathbb{R}} \mathbb{C}). \tag{2.3}$$

Using (2.2) and (2.3) we deduce that

$$\Lambda^{2,0}(M) \oplus \Lambda^{0,2}(M) \oplus \Lambda^{1,1}(M) = \mathbb{C}\omega \oplus (\Lambda^2_6(M) \otimes_{\mathbb{R}} \mathbb{C}) \oplus (\Lambda^2_8(M) \otimes_{\mathbb{R}} \mathbb{C}). \tag{2.4}$$

A direct calculation yields

$$\begin{aligned}\Lambda^{1,1}(M) &= \mathbb{C}\omega \oplus (\Lambda^2_8(M) \otimes_{\mathbb{R}} \mathbb{C})\\ \Lambda^{2,0}(M) \oplus \Lambda^{0,2}(M) &= \Lambda^2_6(M) \otimes_{\mathbb{R}} \mathbb{C}.\end{aligned} \tag{2.5}$$

Definition 2.1. If $F \in \Lambda^2(M, \mathbb{C})$, then we may decompose the 2−form F as

$$F = F^{2,0} + F^{0,2} + (F^0)^{1,1} + \mathbb{C}\omega$$

where $F^{2,0}$ is of type $(2,0)$ and $(F^0)^{1,1}$ is of type $(1,1)$ but with zero ω−trace. Then, the self-dual part of F is $F^{2,0} + F^{0,2} + \mathbb{C}\omega$, denoted by F^+ and the anti-self-dual part of F is $(F^0)^{1,1}$, denoted by F^-.

Some authors make use of the decomposition of F in Definition (2.1) to define anti-self-dual instantons [11].

From (2.5) the space of self-dual 2−forms is given by

$$\Lambda^2_+ = \mathbb{C}\omega \oplus (\Lambda^2_6(M) \otimes_{\mathbb{R}} \mathbb{C})$$

and the space of anti-self-dual 2−forms is given by

$$\Lambda^2_- = \Lambda^2_8(M) \otimes_{\mathbb{R}} \mathbb{C}.$$

3 Spin^c−structure and Dirac operator

In this section, we recall the main definitions concerning Spin^c−structure and the associated Dirac operator.

Let M be an n−dimensional differentiable manifold with structure group $SO(n)$. Then, there is an open covering $\{U_\alpha\}_{\alpha\in A}$ of M and transition functions $g_{\alpha\beta} : U_\alpha \cap U_\beta \to SO(n)$ for TM. If there exists another collection of transition functions

$$\widetilde{g}_{\alpha\beta} : U_\alpha \cap U_\beta \to Spin^c(n)$$

such that following diagram commutes

$$\begin{array}{ccc} & & Spin^c(n) \\ & \overset{\widetilde{g}_{\alpha\beta}}{\nearrow} & \Big\downarrow {\scriptstyle Ad \; 2:1} \\ U_\alpha \cap U_\beta & \xrightarrow[g_{\alpha\beta}]{} & SO(n) \end{array}$$

that is, $Ad \circ \widetilde{g}_{\alpha\beta} = g_{\alpha\beta}$ and the cocycle condition $\widetilde{g}_{\alpha\beta}\widetilde{g}_{\beta\gamma} = \widetilde{g}_{\alpha\gamma}$ on $U_\alpha \cap U_\beta \cap U_\gamma$ is satisfied then M is called a Spin^c manifold. Then one can construct a principal Spin$^c(n)-$bundle $P_{Spin^c(n)}$ on M and a $2-1$ bundle map $\Lambda : P_{Spin^c(n)} \to P_{SO(n)}$.

Let $(P_{Spin^c(n)}, \Lambda)$ be a Spin$^c-$structure on M. We can construct a new associated complex vector bundle

$$S = P_{Spin^c(n)} \times_\kappa \Delta_n$$

where $\kappa : Spin^c(n) \to Aut(\Delta_n)$ is the spinor representation of Spin$^c(n)$. This complex vector bundle is called spinor bundle for a given Spin$^c-$structure on M and sections of S are called spinor fields. The principal bundle $P_{Spin^c(n)}$ and the spinor bundle S have been studied extensively [6, 9]. The spinor bundle S splits into a direct sum

$$S = S^+ \oplus S^- \text{ where } S^\pm = P_{Spin^c(n)} \times_{\kappa^\pm} \Delta_n^\pm.$$

The exact sequence $1 \to Spin(n) \to Spin^c(n) \xrightarrow{l} S^1 \to 1$ implies $Spin^c(n)/_{Spin(n)} = S^1$. Then, we deduce that $P_{S^1} = P_{Spin^c(n)}/Spin(n)$ is an S^1-bundle over M. Hence,

$$L := P_{Spin^c(n)} \times_l \mathbb{C} = P_{S^1} \times_{U(1)} \mathbb{C}$$

is a determinant line bundle.

Now, fix a connection $A : TP_{S^1} \to i\mathbb{R}$ in the principal $U(1)-$bundle P_{S^1}. By using this connection and the Levi-Civita connection ∇ on TM we can obtain a connection

$$\nabla^A : \Gamma(S) \to \Gamma(T^*M \otimes S)$$

on S, which is called spinor covariant derivative operator and it satisfies

$$\nabla^A_V(W \cdot \psi) = W \cdot \nabla^A_V \psi + (\nabla_V W) \cdot \psi$$

where $V, W \in \Gamma(TM)$ and ψ is a spinor, a section of S. At this point we can define the associated Dirac operator $D_A : \Gamma(S) \to \Gamma(S)$ locally by

$$D_A(\psi) = \sum_{i=1}^n \kappa(e_i)\nabla^A_{e_i}(\psi).$$

where $\{e_1, e_2, \dots, e_n\}$ is any positively oriented local orthonormal frame of TM. The Dirac operator decomposes into the sum of two operators $D_A^\pm : \Gamma(S^\pm) \to \Gamma(S^\mp)$.

4 Seiberg-Witten-like equations on $6-$dimensional $SU(3)-$manifolds

Let M be $6-$dimensional $SU(3)-$manifold. Fix a Spin$^c-$structure and a connection A in the principal $U(1)-$bundle P_{S^1} associated to the Spin$^c-$structure. The spinor bundle S on a Spin^c manifold M is defined by associated complex vector bundle

$$S = P_{Spin^c(6)} \times_\kappa \Delta_6$$

where $\kappa : Spin^c(6) \to Aut(\Delta_6)$ is the spinor representation of Spin$^c(6)$. This vector bundle splits into the sum of two subbundles S^+ and S^-. Namely, $S = S^+ \oplus S^-$, $S^\pm = P_{Spin^c(6)} \times_{\kappa^\pm} \Delta_6^\pm$ For a spinor $\psi \in S^+$ we define an imaginary valued 2−form $\sigma(\psi)$ by the formula

$$\sigma(\psi)(X,Y) =< X \cdot Y \cdot \psi, \psi > + < X, Y > |\psi|^2 \tag{4.1}$$

where $X, Y \in \Gamma(TM)$.

Definition 4.1. Let M be 6−dimensional $SU(3)$−manifold. Fix a Spin$^c(6)$−structure and a connection A in the $U(1)$−principal bundle P_{S^1} associated with the Spin^c structure. For $\psi \in \Gamma(S^+)$ Seiberg-Witten-like equations are defined by

$$\begin{aligned} D_A\psi &= 0 \\ F_A^+ &= -\frac{1}{4}\sigma(\psi)^+. \end{aligned} \tag{4.2}$$

where F_A^+ is the self-dual part of the curvature F_A and $\sigma(\psi)^+$ is the self-dual part of the 2−form $\sigma(\psi)$ corresponding to the spinor $\psi \in \Gamma(S^+)$.

5 Local interpretations of Seiberg-Witten-like equations

Dirac equation which is the first one of Seiberg-Witten equations can be written on any $2n$−dimensional Spin^c manifold. Firstly consider a Spin^c−structure κ on $\mathbb{R}^6$ which is coming from the representation of the complex Clifford algebra $\mathbb{C}l_6$. The Spin^c connection ∇^A on $\mathbb{R}^6$ is given by

$$\nabla_j^A \Psi = \frac{\partial \Psi}{\partial x_j} + A_j \Psi,$$

where $A_j : \mathbb{R}^6 \longrightarrow i\mathbb{R}$ and $\Psi : \mathbb{R}^6 \longrightarrow \mathbb{C}^4$ are smooth maps. Then, the associated connection on the line bundle $L = \mathbb{R}^6 \times \mathbb{C}$ is the connection 1−form

$$A = \sum_{i=1}^{6} A_i dx_i \in \Omega^1\left(\mathbb{R}^6, i\mathbb{R}\right)$$

and its curvature 2-form is given by

$$F_A = dA = \sum_{i<j} F_{ij} dx_i \wedge dx_j \in \Omega^2\left(\mathbb{R}^6, i\mathbb{R}\right),$$

where $F_{ij} = \dfrac{\partial A_j}{\partial x_i} - \dfrac{\partial A_i}{\partial x_j}$ for $i, j = 1, \ldots, 6$. Now we can write the Dirac operator D_A on $\mathbb{R}^6$ with respect to a given Spin^c−structure and Spin^c−connection ∇^A. The Dirac equation can be expressed as

$$D_A \Psi = 0.$$

Firstly we consider the following decompositions of 2−forms on $\mathbb{R}^6$. We denote by $\{e_1, e_2, e_3, e_4, e_5, e_6\}$ the standard basis of $\mathbb{R}^6$ and by $\{e^1, e^2, e^3, e^4, e^5, e^6\}$ the dual one. Fix on $\mathbb{R}^6$ the standard symplectic form

$$\omega_0 = e^1 \wedge e^2 + e^3 \wedge e^4 + e^5 \wedge e^6$$

and the standard complex volume form

$$\varphi_0 = (e^1 + ie^2) \wedge (e^3 + ie^4) \wedge (e^5 + ie^6)$$

and the complex structure J_0 by

$$J_0(e_1) = e_2 \quad J_0(e_3) = e_4 \quad J_0(e_5) = e_6.$$

Any 2−form $F = \sum_{i<j} F_{ij} e^i \wedge e^j \in \Omega^2(\mathbb{R}^6, \mathbb{C})$ can be decomposed into three part, we call the one belonging to $\mathbb{C}\omega_0 \oplus (\Lambda_6^2(\mathbb{R}^6) \otimes \mathbb{C})$ the self-dual part of F and we denote it by F^+. We call $\mathbb{C}\omega_0 \oplus (\Lambda_6^2(\mathbb{R}^6) \otimes \mathbb{C})$ as the space of self-dual 2−forms the following 2−forms constitute a basis for this space

$$\begin{aligned}
f_1 &= e^1 \wedge e^3 - e^2 \wedge e^4 \\
f_2 &= e^1 \wedge e^4 + e^2 \wedge e^3 \\
f_3 &= e^1 \wedge e^5 - e^2 \wedge e^6 \\
f_4 &= e^1 \wedge e^6 + e^2 \wedge e^5 \\
f_5 &= e^3 \wedge e^5 - e^4 \wedge e^6 \\
f_6 &= e^3 \wedge e^6 + e^4 \wedge e^5 \\
f_7 &= \omega_0 = e^1 \wedge e^2 + e^3 \wedge e^4 + e^5 \wedge e^6.
\end{aligned}$$

Let F_A be the curvature form of the i$\mathbb{R}$-valued connection 1-form A and F_A^+ be its self-dual part. Then,

$$\begin{aligned}
F_A^+ = \sum_{i=1}^{7} < F_A, f_i > \frac{f_i}{|f_i|^2} = \; & \tfrac{1}{2}[(F_{13} - F_{24})f_1 + (F_{14} + F_{23})f_2 + (F_{15} - F_{26})f_3 \\
& +(F_{16} + F_{25})f_4 + (F_{35} - F_{46})f_5 + (F_{36} + F_{45})f_6] \\
& +\tfrac{1}{3}(F_{12} + F_{34} + F_{56})f_7
\end{aligned}$$

Now we calculate the 2−form $\sigma(\psi)^+$. Let $\{e_1, e_2, e_3, e_4, e_5, e_6\}$ be the standard basis of $\mathbb{R}^6$ and $\{e^1, e^2, e^3, e^4, e^5, e^6\}$ the dual one. Then $\sigma(\psi)$ can be written in the following way:

$$\sigma(\psi) = \sum_{i<j} < e_i e_j \psi, \psi > e^i \wedge e^j$$

The projection onto the subspace Λ_2^+ is given by

$$\sigma(\psi)^+ = \sum_{i=1}^{7} < \sigma(\psi), f_i > \frac{f_i}{|f_i|^2}.$$

If $\sigma(\psi)^+$ is calculated explicitly, then we obtain the following identity:

$$\begin{aligned}
\sigma(\psi)^+ \;=\; & i(\psi_4\overline{\psi}_3 + \psi_3\overline{\psi}_4)f_1 + (-\psi_4\overline{\psi}_3 + \psi_3\overline{\psi}_4)f_2 + (\psi_4\overline{\psi}_1 - \psi_1\overline{\psi}_4)f_3 + i(\psi_4\overline{\psi}_1 + \psi_1\overline{\psi}_4)f_4 \\
& +i(\psi_4\overline{\psi}_2 + \psi_2\overline{\psi}_4)f_5 + (-\psi_4\overline{\psi}_2 + \psi_2\overline{\psi}_4)f_6 + \tfrac{i}{3}(|\psi_1|^2 + |\psi_2|^2 + |\psi_3|^2 - 3|\psi_4|^2)f_7
\end{aligned}$$

Hence, the curvature form can be defined as follows:

$$F_A^+ = -\frac{1}{4}\sigma(\psi)^+. \tag{5.1}$$

From the definition of $\sigma(\psi)$ we obtain the following theorem:

Theorem 5.1. $|\sigma(\psi)|^2 = 3|\psi|^4$

Corollary 5.2. *The curvature equation can be written in the following form:*

$$\begin{array}{l} < F_A, f_i >= -\frac{1}{2} < \psi\psi^*, \rho^+(f_i) > \\ < F_A, f_i >= -\frac{1}{4} < \psi, \rho^+(f_i)\psi > \end{array}$$

for $i = 1, 2, \dots, 7$.

Seiberg-Witten equations on $\mathbb{R}^6$ are studied with a different self-duality concept in [5].

6 A global solution to Seiberg-Witten-like equations on 6−dimensional $SU(3)$−manifolds

The 2−form ω acts as an endomorphism in the bundle S. The endomorphism $\omega : S \to S$ has the eigenvalues $3i$, i, $-i$ and $-3i$ and the corresponding eigensubspaces have dimension $1, 3, 3$ and 1, respectively. The spinor bundle S splits into

$$S = S(3i) \oplus S(i) \oplus S(-i) \oplus S(-3i)$$

where $S(k) = \{\psi \in S : \omega\psi = k\psi\}$, $(k = 3i, i, -i, -3i)$ are the corresponding subspaces. The subbundles S^+ and S^- are given by

$$S^+ = S(i) \oplus S(-3i), \qquad S^- = S(-i) \oplus S(3i),$$

respectively. Moreover, we have the following isomorphisms:

$$S^+ \cong \Lambda^{0,0} \oplus \Lambda^{0,2}, \qquad S^- \cong \Lambda^{0,1} \oplus \Lambda^{0,3}. \tag{6.1}$$

Now we give a global solution of the Seiberg-Witten-like equations. For this, let (M^6, J, g) be a Kahler manifold. Denote by Φ_0 the spinor $S(-3i) \cong \Lambda^{0,0}$ corresponding to the constant function 1. Hence, we have

$$\Phi_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{6.2}$$

in chosen coordinates. Here $\Phi_0 \in S(-3i)$ i.e., $\sigma(\Phi_0) = -i\omega$. The line bundle $L = \Lambda^2(TM)$ of the canonical Spin^c structure has the Levi-Civita connection A_0. Then, the corresponding Dirac operator $D_{A_0} : \Gamma(S^+) \to \Gamma(S^-)$ is given by

$$D_{A_0} = \sqrt{2}(\overline{\partial}_0 \oplus \overline{\partial}_2^*).$$

Now suppose that the scalar curvature s of the Kahler manifold (M^6, J, g) is negative and constant. Let $\Phi_1 = \sqrt{-\frac{2s}{3}}\Phi_0$. Then, Φ_1 is a spinor in $S(-3i)$ and

$$D_{A_0}\Phi_1 = 0 \tag{6.3}$$

$$\sigma(\Phi_1) = -i|\Phi_1|^2\omega = -i(-\frac{2s}{3})\omega = \frac{2}{3}\, is\omega. \tag{6.4}$$

Moreover, the curvature F_{A_0} in the line bundle $L = \Lambda^2(TM)$ is given by

$$F_{A_0} = i\rho \tag{6.5}$$

where ρ is the Ricci form, $\rho(X,Y) = g(X, JRicY)$ and $Ric : TM \to TM$ is the Ricci tensor. In local coordinates the almost complex structure J is given as follows:

$$J = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

Since $J \circ Ric = Ric \circ J$, we obtain the reduced form of the Ric in the following way:

$$Ric = \begin{pmatrix} R_{11} & 0 & R_{13} & R_{14} & R_{15} & R_{16} \\ 0 & R_{11} & -R_{14} & -R_{13} & -R_{16} & R_{15} \\ R_{13} & -R_{14} & R_{33} & 0 & R_{35} & R_{36} \\ R_{14} & -R_{13} & 0 & R_{33} & -R_{36} & R_{35} \\ R_{15} & -R_{16} & R_{35} & -R_{36} & R_{55} & 0 \\ R_{16} & R_{15} & R_{36} & R_{35} & 0 & R_{55} \end{pmatrix}.$$

Then, the Ricci form ρ can be written as follows:
$\rho = -R_{11}e^1\wedge e^2 - R_{33}e^3\wedge e^4 - R_{55}e^5\wedge e^6 + R_{13}(e^1\wedge e^4 - e^2\wedge e^3) - R_{15}(e^1\wedge e^6 - e^2\wedge e^5) + R_{14}(e^1\wedge e^3 + e^2\wedge e^4) + R_{16}(e^1\wedge e^5 + e^2\wedge e^6) + R_{36}(e^3\wedge e^5 + e^4\wedge e^6) - R_{35}(e^3\wedge e^6 - e^4\wedge e^5)$.
Moreover, the $2-$forms

$$\begin{gathered} e^1\wedge e^4 - e^2\wedge e^3 \\ e^1\wedge e^6 - e^2\wedge e^5 \\ e^1\wedge e^3 + e^2\wedge e^4 \\ e^1\wedge e^5 + e^2\wedge e^6 \\ e^3\wedge e^5 + e^4\wedge e^6 \\ e^3\wedge e^6 - e^4\wedge e^5 \end{gathered}$$

are anti-self-dual $2-$forms. The projection of ρ onto the subbundle Λ^2_+ is given by the formula

$$\rho^+ = <\rho, \omega> \frac{\omega}{|\omega|^2} = -\frac{R_{11} + R_{44} + R_{66}}{3}\omega = -\frac{s}{6}\omega.$$

By using (6.5) and (6.4) we obtain

$$F^+_{A_0} = i\rho^+ = -\frac{is\omega}{6} = -\frac{1}{4}\sigma(\Phi_1). \tag{6.6}$$

Since $\sigma(\Phi_1)$ is a self-dual 2−form we have $\sigma(\Phi_1)^+ = \sigma(\Phi_1)$. From the identities (6.3) and (6.6), the pairs $(A_0, \Phi_1) = (A_0, \sqrt{-\frac{2s}{3}}\Phi_0)$ is a solution of Seiberg-Witten-like equations in (4.2).

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