Ricci solitons and gradient Ricci solitons in a D-homothetically deformed K-contact manifold

K. Venu, H.G. Nagaraja, D.L. Kiran Kumar

Abstract. The object of this paper is to study Ricci solitons and gradient Ricci solitons in D_a-homothetically deformed K-contact and $N(k)$-contact metric manifolds.

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1 Introduction

A Ricci soliton is a natural generalization of an Einstein metric, and is defined on a Riemannian manifold by [7]

$$(\mathscr{L}_{\rm V}g)(X,Y)+2S(X,Y)+2\lambda g(X,Y)=0, \tag{1.1}$$

where $\mathscr{L}_{\rm V}$ denotes the Lie-derivative of g along a vector field V, λ a constant, g a Riemannian metric, S the Ricci tensor and X, Y arbitrary vector fields on M. The Ricci soliton is said to be shrinking, steady and expanding according as $\lambda>0$, $\lambda=0$ and $\lambda<0$, respectively. If the vector field V is the gradient of a potential function $-f$, then g is called a gradient Ricci soliton and equation (1.1) assumes the form

$$\nabla\nabla f=S+\lambda g.$$

Ricci solitons are also of interest to physicists who refer to them as quasi-Einstein metrics(for example, see Friedan [3]).
On the other hand, the roots of contact geometry lie in differential equations as in 1872, Sophus Lie introduced the notion of contact transformation as geometric tool to study systems of differential equations. This subject has manifold connections with the other fields of pure mathematics, and substantial applications in applied areas such as mechanics, optics, phase space of a dynamical system, thermodynamics and control theory.
It is well known [12] that the tangent sphere bundle T_1M of a Riemannian manifold

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M admits a contact metric structure. If M is of constant curvature $c = 1$ then T_1M is Sasakian [17], and if $c = 0$ then the curvature tensor R satisfies $R(X,Y)\xi = 0$ [1]. As a generalization of these two cases, in [2], Blair et.al., started the study of the class of contact metric manifolds, in which the structure vector field ξ satisfies the (k,μ)-nullity condition. A contact metric manifold belonging to this class is called a (k,μ)-manifold. Such a structure was first obtained by Koufogiorgos [8] by applying a D_a-homothetic deformation [15] on a contact metric manifold satisfying $R(X,Y)\xi = 0$. In particular, a $(k,0)$-manifold is called an $N(k)$-contact metric manifold [16] and such manifold reduces to Sasakian manifold if $k = 1$ and K-contact manifold when vector field ξ is Killing. Also Nagaraja and Premalatha in [10] studied D_a-homothetically deformed K-contact manifold.

Sharma [13] initiated the study of Ricci solitons in contact Riemannian geometry. Later Tripathi [18], Nagaraja and Premalatha [9] and Ghosh et al. in their papers ([5, 4, 6, 14]) extensively studied Ricci solitons in contact metric manifolds. In particular, on K-contact and Sasakian manifolds. Motivated by these works, in this paper we study Ricci solitons in D_a-homothetically deformed K-contact and $N(k)$- contact metric manifolds.

2 Preliminaries

A $(2n+1)$-dimensional smooth manifold is said to be a contact manifold if it has a global 1-from η such that $\eta \wedge (d\eta)^n \neq 0$ on M. For a contact 1-form η there exists a unique vector filed ξ such that $d\eta(\xi, X) = 0$ and $\eta(\xi) = 1$. Polarizing $d\eta$ on the contact subbundle $\eta = 0$, we obtain a Riemannian metric g and a $(1,1)$-tensor field ϕ such that

$$d\eta(X,Y) = g(X,\phi Y),\ \eta(X) = g(X,\xi), \tag{2.1}$$

$$\phi^2 = -I + \eta \otimes \xi,\ \ \eta(\xi) = 1,\ \ \phi\xi = 0,\ \ \eta \circ \phi = 0, \tag{2.2}$$

g is called an associated metric of η and (ϕ, ξ, η, g) a contact metric structure. In a contact metric manifold M, the $(1,1)$-tensor field h defined by $2h = \mathcal{L}_\xi \phi$, is symmetric and satisfies

$$h\xi = 0, \quad h\phi + \phi h = 0, \tag{2.3}$$

$$\nabla_X \xi = -\phi X - \phi h X,$$

where ∇ is the Levi-Civita connection. A contact metric manifold is called K-contact manifold if the characteristic vector field ξ is a Killing vector field. A contact metric manifold M is Sasakian if and only if the curvature tensor satisfies

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y, \tag{2.4}$$

for any vector fields $X, Y \in TM$. Further, a Sasakian manifold is K-contact and K-contact 3-manifold is Sasakian.

If M is $(2n+1)$-dimensional K-contact Riemannian manifold, then besides (2.1), (2.2), (2.3) and (2.4) the following relations hold.

$$\nabla_X \xi = -\phi X,\ \ (\nabla_X \eta)(Y) = -g(\phi X, Y),$$

$$S(X,\xi) = 2n\eta(X), \tag{2.5}$$

$$\eta(R(X,Y)Z) = g(Y,Z)\eta(X) - g(X,Z)\eta(Y),$$

$$R(\xi,X)Y = g(X,Y)\xi - \eta(Y)X, \tag{2.6}$$

for any vector fields $X, Y \in TM$.

A contact metric manifold M is said to be η-Einstein([11]) if the Ricci tensor S satisfies $S = ag + b\eta \otimes \eta$, where α and β are some smooth functions on the manifold. In particular if $\beta = 0$, then M becomes an Einstein manifold.

3 D_a-homothetic deformation and Ricci solitons on K-contact manifold

Let $M(\phi, \xi, \eta, g)$ be a $(2n+1)$-dimensional almost contact metric manifold. A D_a-homothetic deformation is defined by

$$\overline{\phi} = \phi,\ \overline{\xi} = \frac{1}{a}\xi,\ \ \overline{\eta} = a\eta,\ \ \overline{g} = ag + a(a-1)\eta \otimes \eta, \tag{3.1}$$

where a being a positive constant.
It is clear that $M(\overline{\phi}, \overline{\xi}, \overline{\eta}, \overline{g})$ is also an almost contact metric manifold. So, K-contact manifold under D_a-homothetic deformation is invariant. The Riemannian curvature $\overline{R}$ of deformed K-contact metric manifold is given by

$$\begin{aligned}\overline{R}(X,Y)Z =& R(X,Y)Z - (a-1)\{g(\phi Y,Z)\phi X + g(X,\phi Z)\phi Y\\ &+ 2g(X,\phi Y)\phi Z\} + (a-1)\big\{[g(Y,Z)\xi - \eta(Z)Y]\eta(X)\\ &- [g(X,Z)\xi - \eta(Z)X]\eta(Y)\big\}\\ &+ a(a-1)\{\eta(Y)X - \eta(X)Y\}\eta(Z).\end{aligned} \tag{3.2}$$

By contraction of (3.2), we get

$$\overline{S}(Y,Z) = S(Y,Z) - 2(a-1)g(Y,Z) + 2(a-1)[na+n+1]\eta(Y)\eta(Z) \tag{3.3}$$

$$\begin{aligned}&\overline{S}(Y,\xi) = 2n[a^2 - 2a + 2]\eta(Y),\\ &\overline{Q}Y = QY - 2(a-1)Y + 2(a-1)[na+n+1]\eta(Y)\xi,\\ &\overline{r} = r + 2n(a-1)^2,\end{aligned}$$

where $\overline{S}$ is the Ricci tensor, $\overline{Q}Y$ is the Ricci operator, $\overline{r}$ is the scalar curvature of $M(\overline{\phi}, \overline{\xi}, \overline{\eta}, \overline{g})$ and r is the scalar curvature of $M(\phi, \xi, \eta, g)$. Let us assume that $\overline{g}$ is a Ricci soliton on M.

$$(\mathscr{L}_V \overline{g})(X,Y) + 2\overline{S}(X,Y) = 2\lambda\overline{g}(X,Y). \tag{3.4}$$

Now, take the Lie-derivative of $\overline{g} = ag + a(a-1)\eta \otimes \eta$ along V and using the hypothesis, (3.1) and (3.3) in (3.4), we have

$$
\begin{aligned}
&a(\mathscr{L}_V g)(X,Y) + a(a-1)[(\mathscr{L}_V \eta)(X)\eta(Y) + \eta(X)(\mathscr{L}_V \eta)(Y)] \\
&+ 2S(X,Y) = \{2\lambda a + 4(a-1)\} g(X,Y) \\
&+ 2(a-1)\{\lambda a - 2[na+n+1]\}\eta(X)\eta(Y).
\end{aligned} \tag{3.5}
$$

Employing $(\mathscr{L}_V \eta)(X) = (\lambda - 2n)\eta(X)$ and $S(X,Y) = g(QX,Y)$, the Equation (3.5) becomes

$$
\begin{aligned}
&a(\mathscr{L}_V g)(X,Y) + 2a(a-1)[(\lambda - 2n)\eta(X)\eta(Y)] \\
&+ 2g(QX,Y) = \{2\lambda a + 4(a-1)\} g(X,Y) \\
&+ 2(a-1)\{\lambda a - 2[na+n+1]\}\eta(X)\eta(Y).
\end{aligned} \tag{3.6}
$$

Now we prove our main results.

Theorem 3.1. *If the metric $\overline{g}$ of a D- homothetically deformed K- contact manifold $(M, \overline{g})$ is gradient soliton, then g is a shrinking soliton and $(M, \overline{g})$ is η-Einstein.*

Proof. If $V = grad\ f$ in (3.6), it follows that

$$
\begin{aligned}
2ag(\nabla_X Df, Y) =& 2g(QX,Y) - 2[\lambda a + 2(a-1)]g(X,Y) \\
&+ 4(a-1)(n+1)\eta(X)\xi.
\end{aligned} \tag{3.7}
$$

Then (3.7) can be written as

$$
\nabla_X Df = \frac{1}{a}\{QX - [\lambda a + 2(a-1)]X + 2(a-1)(n+1)\eta(X)\eta(Y)\}, \tag{3.8}
$$

for all vector fields X in M, where D denotes gradient operator of g. From (3.8) it follows that

$$
\begin{aligned}
R(X,Y)Df =& \frac{1}{a}\left[(\nabla_X Q)Y - (\nabla_Y Q)X\right] \\
&- \frac{4(a-1)(n+1)}{a} g(\phi X, Y)\xi \\
&+ \frac{2(a-1)(n+1)}{a}[\eta(X)\phi Y - \eta(Y)\phi X].
\end{aligned} \tag{3.9}
$$

By substituting $X = \xi$ in (3.9) and taking inner product with ξ, we get

$$
g(R(\xi,Y)Df,\xi) = \frac{1}{a} g(\left[(\nabla_\xi Q)Y - (\nabla_Y Q)\xi\right], \xi).
$$

In any K-contact manifold [5], we have $g(((\nabla_\xi Q)Y - (\nabla_Y Q)\xi), \xi) = 0$.

$$
i.e.,\ \ g(R(\xi,Y)Df,\xi) = 0. \tag{3.10}
$$

Next, In view of (2.6), we get

$$
g(R(\xi,Y)Df,\xi) = g((Df - (\xi f)\xi), Y). \tag{3.11}
$$

So, from (3.10) and (3.11), it follows that

$$Df = (\xi f)\xi. \tag{3.12}$$

Using (3.12) in (3.8), we get

$$\begin{aligned} S(X,Y) = \ & [\lambda a + 2(a-1)]g(X,Y) - 2(a-1)(n+1)\eta(X)\eta(Y) \\ & -a[X(\xi f)\eta(Y) + (\xi f)g(\phi X, Y)]. \end{aligned}$$

Symmetrizing this with respect to X and Y we obtain

(3.13)

$$2S(X,Y) = 2[\lambda a + 2(a-1)]g(X,Y) - 4(a-1)(n+1)\eta(X)\eta(Y) + X(\xi f)\eta(Y) + Y(\xi f)\eta(X).$$

Substituting ξ for Y in (3.13), we get

$$X(\xi f) = 2(2n - \lambda)\eta(X). \tag{3.14}$$

From (3.14) and (3.13), we obtain

$$S(X,Y) = [2a(2n-\lambda) - 2(a-1)(n+1)]\eta(X)\eta(Y) - [\lambda a + 2(a-1)]g(X,Y). \tag{3.15}$$

Using (3.15) in (3.8), we get

$$\nabla_X Df = 2(2n-\lambda)\eta(X)\xi. \tag{3.16}$$

Using (3.16), we compute $R(X,Y)Df$ and obtain

$$g(R(X,Y)(\xi f), \xi) = 2(2n-\lambda)d\eta(X,Y).$$

Then we infer

$$2n - \lambda = 0.$$

Hence from (3.14) we have

$$X(\xi f) = 0, \quad X \in TM,$$

that is, $\xi f = c$, where c is a constant. Thus (3.12) gives

$$df = c\eta.$$

Its exterior derivative implies that $c\, d\eta = 0$. Hence $c = 0$ and thus f is constant. Consequently, the equation (3.8) reduces to

$$S(X,Y) = [\lambda a + 2(a-1)]g(X,Y) - 2(a-1)(n+1)\eta(X)\eta(Y),$$

i.e., g is η Einstein. Also as $\lambda = 2n$ is positive, g is shrinking. This completes the proof. □

Theorem 3.2. *If a K-contact metric $\overline{g}$ obtained by a D-homothetic deformation of (M,g) is a Ricci soliton with V point-wise collinear with ξ, then V is a constant multiple of ξ and (M,g) is η-Einstein.*

Proof. Let V be point-wise collinear with ξ. i.e., $V = \alpha\xi$ where α is a function on M. Then from (3.6), we have

$$\begin{aligned} &a(\mathscr{L}_{\alpha\xi} g)(X,Y) + 2a(a-1)[(\lambda - 2n)\eta(X)\eta(Y)] + 2g(QX,Y) \\ &= \{2\lambda a + 4(a-1)\}g(X,Y) + 2(a-1)\{\lambda a - 2[na+n+1]\}\eta(X)\eta(Y). \end{aligned}$$

Applying the properties of the Lie derivative and of the Levi-Civita connection, we have

$$\begin{aligned} a[(X\alpha)\eta(Y) + (Y\alpha)\eta(X)] + 2S(X,Y) =& \{2\lambda a + 4(a-1)\}g(X,Y) \\ &- \{4(a-1)(n+1)\}\eta(X)\eta(Y). \end{aligned} \tag{3.17}$$

By replacing Y by ξ and by using (2.5) in (3.17), we get

$$a[(X\alpha) + (\xi\alpha)\eta(X) - 2[\lambda - 2n]\eta(X)] = 0, \quad a > 0$$

So,

$$(X\alpha) + (\xi\alpha)\eta(X) - 2[\lambda - 2n]\eta(X) = 0. \tag{3.18}$$

Again putting $X = \xi$ in (3.18), we obtain

$$(\xi\alpha) = \lambda - 2n. \tag{3.19}$$

Using (3.19) in (3.18), we have

$$(X\alpha) = [\lambda - 2n]\eta(X). \tag{3.20}$$

Applying exterior derivative on (3.20), we get

$$\lambda - 2n = 0, \quad \text{since } d\eta \neq 0. \tag{3.21}$$

Using (3.21) in (3.20), we obtain

$$X\alpha = 0,$$

which implies α is constant. Then (3.17) becomes

$$S(X,Y) = [\lambda a + 2(a-1)]g(X,Y) - [2(a-1)(n+1)]\eta(X)\eta(Y).$$

□

4 D_a-homothetic deformation and Ricci solitons on $N(k)$-contact manifold

The Ricci tensor of D- homothetically deformed $N(k)$- contact metric manifold is

$$\begin{aligned} \overline{S}(Y,Z) =& S(Y,Z) - 2(a-1)g(Y,Z) + 2(a-1)[na+n+1]\eta(Y)\eta(Z) \\ &+ \frac{a-1}{a}\{g(hX,Y) - g(hX,hY)\}. \end{aligned} \tag{4.1}$$

By using (3.1) and (4.1) in (3.4), we have

$$\begin{aligned}(4.2)\quad & a(\mathscr{L}_V g)(X,Y) + a(a-1)[(\mathscr{L}_V \eta)(X)\eta(Y) + \eta(X)(\mathscr{L}_V \eta)(Y)] \\ & + 2S(X,Y) = \{2\lambda a + 4(a-1)\}g(X,Y) + 2(a-1)\{\lambda a - 2[na+n+1]\}\eta(X)\eta(Y) \\ & - 2\left(\frac{a-1}{a}\right)\{g(hX,Y) - g(hX,hY)\}.\end{aligned}$$

Using (3.4) in (4.2), the equation (4.2) becomes

$$\begin{aligned} & 2(1-a)g(X,Y) + a(a-1)[(\mathscr{L}_V \eta)(X)\eta(Y) + \eta(X)(\mathscr{L}_V \eta)(Y)] \\ & = 4(a-1)g(X,Y) + (a-1)\{2\lambda a - 4na - 4n - 4]\}\eta(X)\eta(Y) \\ & - 2(\frac{a-1}{a})\{g(hX,Y) - g(hX,hY)\}.\end{aligned} \tag{4.3}$$

We know that $g(\xi,\xi) = \eta(\xi) = 1$.
Lie-differentiate the above equation along V, we get

$$(\mathscr{L}_V \eta)\xi + \eta(\mathscr{L}_V \xi) = 0.$$

$$i.e.,\ (\mathscr{L}_V g)(\xi,\xi) + 2g(\mathscr{L}_V \xi,\xi) = 0. \tag{4.4}$$

Also we have $g(\mathscr{L}_V \xi,\xi) = \eta(\mathscr{L}_V \xi)$. From (3.4), we get

$$\begin{aligned} & (\mathscr{L}_V g)(\xi,\xi) + 2S(\xi,\xi) = 2\lambda g(\xi,\xi) \\ & i.e.,\ \ (\mathscr{L}_V g)(\xi,\xi) = 2\lambda - 4nk.\end{aligned}$$

Therefore (4.4) implies

$$\eta(L_V \xi) = 2nk - \lambda.$$

Using this in (4.4) we get

$$(\mathscr{L}_V \eta)\xi = \lambda - 2nk. \tag{4.5}$$

Now substituting ξ for Y in (4.3) and using (4.5), we get

$$(\mathscr{L}_V \eta)(X) = \left[\lambda - \frac{4(a+1)n}{a} + 2nk(\frac{a+2}{a})\right]\eta(X). \tag{4.6}$$

By using (4.6) in (4.3), we find

$$\begin{aligned} S(X,Y) = & 2[n\{a(k-1) + 2k - 1\} + 1]\eta(X)\eta(Y) - 2g(X,Y) \\ & + \frac{1}{a}[g(hX,Y) - g(hX,hY)].\end{aligned} \tag{4.7}$$

Putting $X = Y = e_i$ in (4.7) and summing over $i = 1,...,2n+1$, we get

$$r = 2[n\{a(k-1) + 2k - 1\} + 1] - 2(2n+1) + \frac{2n(k-1)}{a}.$$

Let us now use the formula

$$\mathscr{L}_V r = -\triangle r + 2R_{ij}R^{ij} - 2\lambda r.$$

As r is a constant, we get $R_{ij}R^{ij} = \lambda r$. So,

$$R_{ij}R^{ij} = \lambda\{2[n\{a(k-1)+2k-1\}+1] - 2(2n+1) + \frac{2n(k-1)}{a}\}. \tag{4.8}$$

From (4.7), we obtain

$$\begin{aligned} R_{ij}R^{ij} =& 4[4\{a(k-1)+2k-1\}+1]^2 - 8[n\{a(k-1)+2k-1\}+1] + 4(2n+1) \\ &- \frac{8n(k-1)}{a} - \frac{2(k-1)}{a^2}[k(n-1)+1]. \end{aligned} \tag{4.9}$$

From (4.8) and (4.9), we have

$$\begin{aligned} \lambda =& \frac{4[4\{a(k-1)+2k-1\}+1]^2 - 8[n\{a(k-1)+2k-1\}+1]}{2[n\{a(k-1)+2k-1\}+1] - 2(2n+1) + \frac{2n(k-1)}{a}} \\ &+ \frac{4(2n+1) - \frac{8n(k-1)}{a} - \frac{2(k-1)}{a^2}[k(n-1)+1]}{2[n\{a(k-1)+2k-1\}+1] - 2(2n+1) + \frac{2n(k-1)}{a}}. \end{aligned} \tag{4.10}$$

If $a = \frac{1}{k-1}$, Eq. (4.10) reduces to

$$\lambda = \frac{4[8k+1]^2 - 8[2nk+1] + 4(2n+1) - 8n(k-1)^2 - 2(k-1)^3[k(n-1)+1]}{2n(k^2-1)}. \tag{4.11}$$

Now we take $k = \frac{-1}{n}$ in (4.11), we have

$$\lambda = \frac{4(\frac{n-8}{n})^2 + 8 + 4(2n+1) + 8n(\frac{n+1}{n})^2 + 2(\frac{1+n}{n})^3\frac{1}{n}}{2(\frac{1-n^2}{n})}.$$

Thus we state the following:

Theorem 4.1. *A Ricci soliton in a D-homothetically deformed $N(\frac{-1}{n})$ contact metric manifold is a shrinking soliton.*

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Authors' addresses:

Kumaraswamy Venu
Department of Mathematics and Statistics,
Faculty of Mathematical and Physical Sciences,
M.S. Ramaiah University of Applied Sciences,
Peenya Industrial Area, Peenya, Bengaluru, 560058, India.
E-mail: venuk.math@gmail.com

Halammanavar Gangadharappa Nagaraja
Department of Mathematics, Bangalore University,
Jnana Bharathi Campus, Bengaluru, 560056, India.
E-mail: hgnraj@yahoo.com

Devasandra Lakshmana Kiran Kumar
Department of Mathematics,
RV College of Engineering,
Bengaluru, 560059, India.
E-mail: kirankumar250791@gmail.com