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## THE HYPERBOLIC SQUARE AND MÖBIUS TRANSFORMATIONS

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*This paper is dedicated to Professor Themistocles M. Rassias.*

Submitted by J. M. Isidro Gómez

**ABSTRACT.** Professor Themistocles M. Rassias' special predilection and contribution to the study of Möbius transformations is well known. Möbius transformations of the open unit disc of the complex plane and, more generally, of the open unit ball of any real inner product space, give rise to Möbius addition in the ball. The latter, in turn, gives rise to Möbius gyrovector spaces that enable the Poincaré ball model of hyperbolic geometry to be approached by gyrovector spaces, in full analogy with the common vector space approach to the standard model of Euclidean geometry. The purpose of this paper, dedicated to Professor Themistocles M. Rassias, is to employ the Möbius gyrovector spaces for the introduction of the hyperbolic square in the Poincaré ball model of hyperbolic geometry. We will find that the hyperbolic square is richer in structure than its Euclidean counterpart.

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