POSITIVITY OF OPERATOR-MATRICES OF HUA-TYPE

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This paper is dedicated to Professor Josip E. Pečarić

Submitted by F. Kittaneh

ABSTRACT. Let A_j $(j = 1, 2, \ldots, n)$ be strict contractions on a Hilbert space. We study an $n \times n$ operator-matrix:

$$\mathbf{H}_n(A_1, A_2, \ldots, A_n) = [(I - A_j^* A_i)^{-1}]_{i,j=1}^n.$$

For the case $n = 2$, Hua [Inequalities involving determinants, Acta Math. Sinica, 5 (1955), 463–470 (in Chinese)] proved positivity, i.e., positive semi-definiteness of $\mathbf{H}_2(A_1, A_2)$. This is, however, not always true for $n = 3$. First we generalize a known condition which guarantees positivity of $\mathbf{H}_n$. Our main result is that positivity of $\mathbf{H}_n$ is preserved under the operator Möbius map of the open unit disc $\mathcal{D}$ of strict contractions.

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