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## POSITIVITY OF OPERATOR-MATRICES OF HUA-TYPE

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*This paper is dedicated to Professor Josip E. Pečarić*

Submitted by F. Kittaneh

ABSTRACT. Let  $A_j$  ( $j = 1, 2, \dots, n$ ) be strict contractions on a Hilbert space. We study an  $n \times n$  operator-matrix:

$$\mathbf{H}_n(A_1, A_2, \dots, A_n) = [(I - A_j^* A_i)^{-1}]_{i,j=1}^n.$$

For the case  $n = 2$ , Hua [Inequalities involving determinants, Acta Math. Sinica, 5 (1955), 463–470 (in Chinese)] proved positivity, i.e., positive semi-definiteness of  $\mathbf{H}_2(A_1, A_2)$ . This is, however, not always true for  $n = 3$ . First we generalize a known condition which guarantees positivity of  $\mathbf{H}_n$ . Our main result is that positivity of  $\mathbf{H}_n$  is preserved under the operator Möbius map of the open unit disc  $\mathcal{D}$  of strict contractions.

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