



## THREE-PARAMETER WEIGHTED HARDY TYPE INEQUALITIES

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*This paper is dedicated to Professor Josip E. Pečarić*

Submitted by S.-M. Jung

ABSTRACT. For  $0 < r < \infty$  and  $1 \leq p \leq q < \infty$  we find necessary and sufficient conditions for the validity of the following inequality:

$$\left( \int_a^b u(x) \left( \int_a^x |g(x) - g(t)|^r w(t) dt \right)^{\frac{q}{r}} dx \right)^{\frac{1}{q}} \leq C \left( \int_a^b v(x) |g'(x)|^p dx \right)^{\frac{1}{p}},$$

where  $u(\cdot)$ ,  $v(\cdot)$ , and  $w(\cdot)$  are weight functions.

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