



Banach J. Math. Anal. 2 (2008), no. 2, 94–106

**BANACH JOURNAL OF MATHEMATICAL ANALYSIS**

ISSN: 1735-8787 (electronic)

<http://www.math-analysis.org>

## ON INEQUALITIES OF HARDY–SOBOLEV TYPE

A. BALINSKY<sup>1</sup>, W. D. EVANS<sup>2\*</sup>, D. HUNDERTMARK<sup>3</sup> AND R. T. LEWIS<sup>4</sup>

*This paper is dedicated to Professor Josip E. Pečarić*

Submitted by T. Riedel

ABSTRACT. Hardy–Sobolev–type inequalities associated with the operator  $L := \mathbf{x} \cdot \nabla$  are established, using an improvement to the Sobolev embedding theorem obtained by M. Ledoux. The analysis involves the determination of the operator semigroup  $\{e^{-tL^*L}\}_{t>0}$ .

<sup>1,2</sup> SCHOOL OF MATHEMATICS, CARDIFF UNIVERSITY, 23 SENGHENNYDD ROAD, CARDIFF CF24 4AG, UK.

*E-mail address:* [BalinskyA@cardiff.ac.uk](mailto:BalinskyA@cardiff.ac.uk) , [EvansWD@cardiff.ac.uk](mailto:EvansWD@cardiff.ac.uk)

<sup>3</sup> DEPARTMENT OF MATHEMATICS, UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN, 1409 W. GREEN STREET, URBANA, ILLINOIS 61801, USA.

*E-mail address:* [dirk@math.uiuc.edu](mailto:dirk@math.uiuc.edu)

<sup>4</sup> DEPARTMENT OF MATHEMATICS, UNIVERSITY OF ALABAMA AT BIRMINGHAM, BIRMINGHAM, AL 35294-1170, USA.

*E-mail address:* [lewis@math.uab.edu](mailto:lewis@math.uab.edu)

---

*Date:* Received: 11 April; Accepted: 20 June 2008.

\* Corresponding author.

2000 *Mathematics Subject Classification.* Primary 46E35; Secondary 35K05.

*Key words and phrases.* Hardy’s inequality, Sobolev’s inequality, heat semigroup, Ledoux’s inequality.