



REVERSES OF THE GOLDEN–THOMPSON TYPE INEQUALITIES DUE TO ANDO-HIAI-PETZ

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This paper is dedicated to Professor Josip E. Pečarić

Submitted by M. Frank

ABSTRACT. In this paper, we show reverses of the Golden–Thompson type inequalities due to Ando, Hiai and Petz: Let H and K be Hermitian matrices such that $mI \leq H, K \leq MI$ for some scalars $m \leq M$, and let $\alpha \in [0, 1]$. Then for every unitarily invariant norm

$$\|e^{(1-\alpha)H+\alpha K}\| \leq S(e^{p(M-m)})^{\frac{1}{p}} \| (e^{pH} \#_{\alpha} e^{pK})^{\frac{1}{p}} \|$$

holds for all $p > 0$ and the right-hand side converges to the left-hand side as $p \downarrow 0$, where $S(a)$ is the Specht ratio and the α -geometric mean $X \#_{\alpha} Y$ is defined as

$$X \#_{\alpha} Y = X^{\frac{1}{2}} \left(X^{-\frac{1}{2}} Y X^{-\frac{1}{2}} \right)^{\alpha} X^{\frac{1}{2}} \quad \text{for all } 0 \leq \alpha \leq 1$$

for positive definite X and Y .

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