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REVERSE OF THE GRAND FURUTA INEQUALITY AND ITS APPLICATIONS

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This paper is dedicated to Professor J.E. Pečarić

Submitted by A. R. Villena

Abstract. We shall give a norm inequality equivalent to the grand Furuta inequality, and moreover show its reverse as follows: Let A and B be positive operators such that $0 < m \le B \le M$ for some scalars $0 < m < M$ and $h := \frac{M}{m} > 1$. Then

$$\begin{aligned} &\| A^{\frac{1}{2}}\{A^{-\frac{t}{2}}(A^{\frac{r}{2}}B^{\frac{(r-t)\{(p-t)s+r\}}{1-t+r}}A^{\frac{r}{2}})^{\frac{1}{s}}A^{-\frac{t}{2}}\}^{\frac{1}{p}}A^{\frac{1}{2}} \| \\ &\le \ K(h^{r-t}, \frac{(p-t)s+r}{1-t+r})^{\frac{1}{ps}} \| A^{\frac{1-t+r}{2}}B^{r-t}A^{\frac{1-t+r}{2}} \|^{\frac{(p-t)s+r}{ps(1-t+r)}} \end{aligned}$$

for $0 \le t \le 1$, $p \ge 1$, $s \ge 1$ and $r \ge t \ge 0$, where $K(h,p)$ is the generalized Kantorovich constant. As applications, we consider reverses related to the Ando-Hiai inequality.

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