



Banach J. Math. Anal. 2 (2008), no. 2, 59–67

**BANACH JOURNAL OF MATHEMATICAL ANALYSIS**

ISSN: 1735-8787 (electronic)

<http://www.math-analysis.org>

## OPERATOR-VALUED INNER PRODUCT AND OPERATOR INEQUALITIES

JUN ICHI FUJII<sup>1</sup>

*This paper is dedicated to Professor Josip E. Pečarić*

Submitted by M. S. Moslehian

ABSTRACT. The Schwarz inequality and Jensen's one are fundamental in a Hilbert space. Regarding a sesquilinear map  $B(X, Y) = Y^*X$  as an operator-valued inner product, we discuss operator versions for the above inequalities and give simple conditions that the equalities hold.

<sup>1</sup> DEPARTMENT OF ARTS AND SCIENCES (INFORMATION SCIENCE), OSAKA KYOIKU UNIVERSITY, ASAHIGAOKA, KASHIWARA, OSAKA 582-8582, JAPAN.

*E-mail address:* [fujii@cc.osaka-kyoiku.ac.jp](mailto:fujii@cc.osaka-kyoiku.ac.jp)

*Date:* Received: 29 March 2008; Accepted 13 May 2008.

*2000 Mathematics Subject Classification.* Primary 47A63; Secondary 47A75, 47A80.

*Key words and phrases.* Schwarz inequality, Jensen inequality, Operator inequality.