



## COMPACT FAILURE OF MULTIPLICATIVITY FOR LINEAR MAPS BETWEEN BANACH ALGEBRAS

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**ABSTRACT.** We introduce notions of compactness and weak compactness for multilinear maps from a product of normed spaces to a normed space, and prove some general results about these notions. We then consider linear maps  $T : A \rightarrow B$  between Banach algebras that are “close to multiplicative” in the following senses: the failure of multiplicativity, defined by  $S_T(a, b) = T(a)T(b) - T(ab)$  ( $a, b \in A$ ), is compact [respectively weakly compact]. We call such maps cf-homomorphisms [respectively wcf-homomorphisms]. We also introduce a number of other, related definitions. We state and prove some general theorems about these maps when they are bounded, showing that they form categories and are closed under inversion of mappings and we give a variety of examples. We then turn our attention to commutative  $C^*$ -algebras and show that the behaviour of the various types of “close-to-multiplicative” maps depends on the existence of isolated points in the maximal ideal space. Finally, we look at the splitting of Banach extensions when considered in the category of Banach algebras with bounded cf-homomorphisms [respectively wcf-homomorphisms] as the arrows. This relates to the (weak) compactness of 2-cocycles in the Hochschild-Kamowitz cohomology complex. We prove “compact” analogues of a number of established results in the Hochschild-Kamowitz cohomology theory.

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