



Banach J. Math. Anal. 3 (2009), no. 2, 64–76

**BANACH JOURNAL OF MATHEMATICAL ANALYSIS**

ISSN: 1735-8787 (electronic)

<http://www.math-analysis.org>

## ON A GEOMETRIC PROPERTY OF POSITIVE DEFINITE MATRICES CONE

MASATOSHI ITO<sup>1</sup>, YUKI SEO<sup>2</sup>, TAKEAKI YAMAZAKI<sup>3\*</sup> AND  
MASAHIRO YANAGIDA<sup>4</sup>

Communicated by M. S. Moslehian

**ABSTRACT.** We shall discuss the matrix geometric mean for the positive definite matrices. The set of all  $n \times n$  matrices with a suitable inner product will be a Hilbert space, and the matrix geometric mean can be considered as a path between two positive matrices. In this paper, we shall obtain a matrix geometric mean inequality, and as an application of it, a property of Riemannian metric space is given. We also obtain some examples related to our result.

<sup>1</sup> DEPARTMENT OF INTEGRATED DESIGN ENGINEERING, MAEBASHI INSTITUTE OF TECHNOLOGY, MAEBASHI 371-0816, JAPAN

*E-mail address:* [m-ito@maebashi-it.ac.jp](mailto:m-ito@maebashi-it.ac.jp)

<sup>2</sup> FACULTY OF ENGINEERING, SHIBAURA INSTITUTE OF TECHNOLOGY, SAITAMA 337-8570, JAPAN

*E-mail address:* [yukis@sic.shibaura-it.ac.jp](mailto:yukis@sic.shibaura-it.ac.jp)

<sup>3</sup> DEPARTMENT OF MATHEMATICS, KANAGAWA UNIVERSITY, YOKOHAMA 221-8686, JAPAN

*E-mail address:* [yamazt26@kanagawa-u.ac.jp](mailto:yamazt26@kanagawa-u.ac.jp)

<sup>4</sup> DEPARTMENT OF MATHEMATICAL INFORMATION SCIENCE, FACULTY OF SCIENCE, TOKYO UNIVERSITY OF SCIENCE, TOKYO 162-8601, JAPAN

*E-mail address:* [yanagida@rs.kagu.tus.ac.jp](mailto:yanagida@rs.kagu.tus.ac.jp)

---

*Date:* Received: 2 April 2009; Accepted: 18 June 2009.

\* Corresponding author.

2000 *Mathematics Subject Classification.* Primary 47A64. Secondary 47A63, 47L25.

*Key words and phrases.* Positive matrix, Riemannian metric, geometric mean.