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**EXISTENCE, UNIQUENESS AND STATISTICAL THEORY OF
TURBULENT SOLUTIONS OF THE STOCHASTIC
NAVIER–STOKES EQUATION, IN THREE DIMENSIONS,
AN OVERVIEW**

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This paper is dedicated to Lars-Erik Persson on the occasion of his 60th birthday

Communicated by P. Wall

ABSTRACT. We discuss the proofs of the existence and uniqueness of solutions of the Navier–Stokes equation driven with additive noise in three dimensions, in the presence of a strong uni-directional mean flow with some rotation. We also discuss how the existence of a unique invariant measure is established and the properties of this measure are described. The invariant measure is used to prove Kolmogorov’s scaling in 3-dimensional turbulence including the celebrated $-5/3$ power law for the decay of the power spectrum of a turbulent 3-dimensional flow. Then we briefly describe the mathematical proof of Kolmogorov’s statistical theory of turbulence.

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