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QUOTIENT MEAN SERIES

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ABSTRACT. The well–known Mathieu series

$$S_M(r) = \sum_{n=1}^{\infty} \frac{2n}{(n^2+r^2)^2} \qquad (r>0),$$

can be transformed into the form

$$S_M(r) = \frac{1}{2r}\sum_{n=1}^{\infty} \frac{\sqrt{nr}^2}{\left(\sqrt{\frac{n^2+r^2}{2}}\right)^4} = \frac{1}{2r}\sum_{n=1}^{\infty}\frac{G^2(n,r)}{Q^4(n,r)},$$

where $G(n,r)$ and $Q(n,r)$ denote the Geometric and Quadratic mean of $n \in \mathbb{N}$ and $r > 0$. This connection leads us to the idea to introduce and research the so–called Quotient mean series as a be a generalizations of Mathieu's and Mathieu–type series. We give an integral representation of such series and their alternating variant, together with associated inequalities. Also, special cases of quotient mean series, involving Bessel function of the first kind, have been studied in detail.

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