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## CHARACTERIZATIONS OF INNER PRODUCT SPACES BY STRONGLY CONVEX FUNCTIONS

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ABSTRACT. New characterizations of inner product spaces among normed spaces involving the notion of strong convexity are given. In particular, it is shown that the following conditions are equivalent: (1)  $(X, \|\cdot\|)$  is an inner product space; (2)  $f : X \rightarrow \mathbb{R}$  is strongly convex with modulus  $c > 0$  if and only if  $f - c\|\cdot\|^2$  is convex; (3)  $\|\cdot\|^2$  is strongly convex with modulus 1.

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