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ON STRICT INCLUSION RELATIONS BETWEEN APPROXIMATION AND INTERPOLATION SPACES

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ABSTRACT. Approximation spaces, in their many presentations, are well known mathematical objects and many authors have studied them for long time. They were introduced by Butzer and Scherer in 1968 and, independently, by Y. Brudnyi and N. Kruglyak in 1978, and popularized by Pietsch in his seminal paper of 1981. Pietsch was interested in the parallelism that exists between the theories of approximation spaces and interpolation spaces, so that he proved embedding, reiteration and representation results for approximation spaces. In particular, embedding results are a natural part of the theory since its inception. The main goal of this paper is to prove that, for certain classes of approximation schemes $(X, \{A_n\})$ and sequence spaces S , if $S_1 \subset S_2 \subset c_0$ (with strict inclusions) then the approximation space $A(X, S_1, \{A_n\})$ is properly contained into $A(X, S_2, \{A_n\})$. We also initiate a study of strict inclusions between interpolation spaces, for Petree's real interpolation method.

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