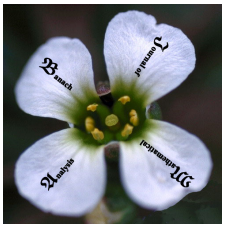

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FUNCTIONAL DECOMPOSITION OF STATE INDUCED C^*-MATRIX SPACES

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Abstract. A theorem of Dixmier states that each bounded linear functional f on the algebra of bounded linear operators on a separable Hilbert space is a direct sum of a trace functional g and a singular functional h, vanishing on the compact operators, such that $\|f\| = \|g\| + \|h\|$. We use elementary methods to construct, via the state space of a C^*-algebra, a Banach space of C^* matrices that contains a closed subspace on which a version of Dixmier's theorem is proved. When the C^*-algebra is taken to be the complex numbers our approach gives elementary and transparent proofs of Dixmier's theorem and the trace formula tr(AB) = tr(BA), without using the operator theoretical machineries used in the known proofs.

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