BISHOP'S PROPERTY (β) AND RIESZ IDEMPOTENT FOR k-QUASI-PARANORMAL OPERATORS

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Communicated by M. Fujii

ABSTRACT. The study of operators satisfying Bishop's property (β) is of significant interest and is currently being done by a number of mathematicians around the world. Recently Uchiyama and Tanahashi [Oper. Matrices 4 (2009), 517–524] showed that a paranormal operator has Bishop's property (β). In this paper we introduce a new class of operators which we call the class of k-quasi-paranormal operators. An operator T is said to be a k-quasi-paranormal operator if it satisfies $||T^{k+1}x||^2 \leq ||T^{k+2}x|| ||T^k x||$ for all $x \in H$ where k is a natural number. This class of operators contains the class of paranormal operators and the class of quasi-class A operators. We prove basic properties and give a structure theorem of k-quasi-paranormal operators. We also show that Bishop's property (β) holds for this class of operators. Finally, we prove that if E is the Riesz idempotent for a nonzero isolated point λ_0 of the spectrum of a k-quasi-paranormal operator T, then E is self-adjoint if and only if the null space of $T - \lambda_0$, $\ker(T - \lambda_0) \subseteq \ker(T^* - \overline{\lambda_0})$.

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