

Banach J. Math. Anal. 6 (2012), no. 1, 147–154

BANACH JOURNAL OF MATHEMATICAL ANALYSIS ISSN: 1735-8787 (electronic) www.emis.de/journals/BJMA/

BISHOP'S PROPERTY (β) AND RIESZ IDEMPOTENT FOR *k*-QUASI-PARANORMAL OPERATORS

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Communicated by M. Fujii

ABSTRACT. The study of operators satisfying Bishop's property (β) is of significant interest and is currently being done by a number of mathematicians around the world. Recently Uchiyama and Tanahashi [Oper. Matrices 4 (2009), 517–524] showed that a paranormal operator has Bishop's property (β). In this paper we introduce a new class of operators which we call the class of k-quasi-paranormal operators. An operator T is said to be a k-quasi-paranormal operator if it satisfies $||T^{k+1}x||^2 \leq ||T^{k+2}x|||T^kx||$ for all $x \in H$ where k is a natural number. This class of operators contains the class of paranormal operators and the class of quasi-class A operators. We prove basic properties and give a structure theorem of k-quasi-paranormal operators. We also show that Bishop's property (β) holds for this class of operators. Finally, we prove that if E is the Riesz idempotent for a nonzero isolated point λ_0 of the spectrum of a k-quasi-paranormal operator T, then E is self-adjoint if and only if the null space of $T - \lambda_0$, ker $(T - \lambda_0) \subseteq \text{ker}(T^* - \overline{\lambda_0})$.

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Date: Received: 6 August 2011; Accepted: 10 November 2011.

2010 Mathematics Subject Classification. Primary 47B47; Secondary 47B30, 47B20.

Key words and phrases. k-quasi-paranormal operator, paranormal operator, Riesz idempotent, Bishop's property (β) , .